

## Analysis of D2D Communication over $N^*$ Nakagami Channels

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### Abstract

*In this paper, performance of device-to-device (D2D) communication system over an  $N^*$ Nakagami channel is presented. The  $N^*$ Nakagami distribution can effectively model cascaded Nakagami fading channels. Multiple co-channel interferers are considered in the system. The channel of the co-channel interference is assumed to be Gamma distributed. The expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the signal-to-interference ratio (SIR) are presented. Using the PDF expression, expressions of the outage probability, success probability, outage capacity, channel capacity and symbol error rate (SER) are then presented. The expressions are functions of path-loss exponents, D2D and interference channel fading parameters, distance between interferers and the D2D receiver, and the distance between D2D pair. The effects of D2D fading channel parameters, interference channel fading conditions, and the path-loss conditions on the performance of D2D system are then analyzed and discussed.*

**Keywords:** Channel Capacity, Device-to-Device Communication, Co-channel Interference, Outage Capacity, Outage probability, Signal-to-Interference Ratio, Success Probability, Symbol-Error Rate

### 1. Introduction

The progression in technology of smart devices and their high data rate demand is one of the motivations to improve cellular communication systems. This demand of high data rate is due to omnipresence of numerous wireless devices. These wireless devices provide multimedia services, online gaming and content distribution services. Device-to-device (D2D) communication has been proposed as one of the solutions to these problems in 5G cellular communication technology. D2D allows nearby devices to transfer information via direct channel without routing through the cellular infrastructure [1-3]. D2D communication will provide high data rate, bandwidth efficiency and offloading of the base-stations (BS). However, D2D communication brings some challenges as well. The competition for the limited wireless channel frequency resources is a major challenge. Co-channel interference (CCI) occurs in the absence of proper coordination between various wireless devices in the system. Therefore, effects of CCI should be considered while analyzing the performance of D2D communication system [4-5].

In [6], authors proposed resource allocation scheme for multiple D2D cluster multicast communications and has analyzed the outage probability of the system under the proposed scheme. Success probability performance of a D2D system over Rician fading channel in the presence of Rayleigh faded interference is studied by authors in [7]. A centralized opportunistic access control scheme is proposed and analyzed in [7]. Authors in [8], studied outage capacity and channel capacity of decode and forward Relaying system over Rayleigh fading channel in the presence of Rayleigh distributed CCI. A

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support-vector-machine-based algorithm is proposed by authors in [9] for power control in D2D communication system. System symbol-error rate (SER) is analyzed under proposed algorithm.

In this paper, outage probability, success probability, outage capacity, channel capacity and SER performance metrics of a D2D communication system are analyzed. The D2D system is considered to be affected by various co-channel interferers. The channel for the desired D2D communication is assumed to be  $N$ \*Nakagami distributed.  $N$ \*Nakagami is a generic distribution based on the product of  $N$  independent Nakagami random variables.  $N$ \*Nakagami can be interpreted by considering the product of reflected rays via  $N$  independent scatterers [10]. In D2D communication system the received signal can be a product of  $N$  scattered signals. The multiple scattering occur due to many small objects in local environment of D2D sender and receiver node. The movement of sender and receiver of D2D pair can also case multiple scattering phenomena. The  $N$ \*Nakagami distribution is an efficient distribution for modeling of multiple scattering phenomena. The channel for the CCI signals are assumed to be Gamma distributed. The Gamma distribution is also a versatile fading distribution that can model different fading conditions. The rest of paper is organized as follows. In Section 2, system layout and expressions of outage probability, success probability, outage capacity, channel capacity and SER are presented. With help of numerical results, performance of D2D communication system is discussed in Section 3. This paper is concluded in Section 4.

## 2. System Model

Consider a device-to-device (D2D) pair communication over a direct link which is  $N$ \*Nakagami distributed. The system layout is shown in Figure. 1. As shown in Figure. 1, the D2D system is considered to be affected by many co-channel interferers. The interference channel is considered to be Gamma distributed. To keep the mathematical analysis simple, following assumptions are considered. The interferers are assumed to be independent and identically distributed. The distances between the interferers and the considered D2D receiver are assumed to be equal.

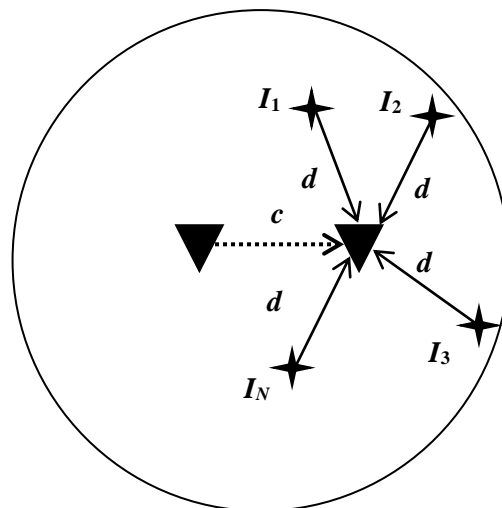






Figure 1. System Layout of a D2D Communication System

**Table 1. Nomenclature**

Desired D2D Pair	
Co-channel Interferers	
Desired D2D signal	
Co-channel Interference (CCI) Signal	
Distance Between the Desired D2D Pair	$c$
Distance Between the $i$ -th Interferer and the Receiver of D2D Pair	$d$

The probability density function (PDF) of the Gamma distribution is [11]

$$f_x(x) = \frac{e^{-\frac{x}{\sigma}} x^{\delta-1}}{\sigma^{\delta} \Gamma(\delta)}, \quad x \geq 0 \quad (1)$$

where  $\delta$  is the shape parameter and  $\sigma$  is the scale parameter of the Gamma distribution. Shape parameter of distribution shows the severity of the fading. The scale parameter is related to the average power of the fading channel. The PDF of  $N$ \*Nakagami distribution is [10]

$$f_Y(y) = \frac{1}{y \prod_{n=1}^N \Gamma(m_n)} G_{0,N}^{N,0} \left[ y \prod_{n=1}^N \left( \frac{m_n}{\Omega_n} \right) \middle| m_1, m_2, \dots, m_N \right] \quad (2)$$

where  $N$  is the number of independent Nakagami random variables.  $m_n$  is the shape parameter of the  $n$ -th Nakagami random variable. The shape parameter of fading controls the fading. The fading of  $\Omega_n$  is related to the average power of  $n$ -th Nakagami random variable.  $G_{g,h}^{e,f}[\cdot]$  is the Meijer's G-function [12] and  $\Gamma(\cdot)$  is the Gamma function [12]. To include the effects of path-loss, a simplified path-loss model is considered [13]. The D2D signal power at the D2D receiver is

$$S_s = P_1 \left( \frac{\lambda}{4\pi c_0} \right)^2 \left( \frac{c_0}{c} \right)^a \quad (3)$$

where  $P_1$  is the D2D signal power,  $\lambda$  is the wavelength,  $c$  is the distance between D2D pair,  $c_0$  is the reference distance (1 to 100 meters) and  $a$  ( $2 \leq a \leq 5$ ) is the path-loss exponent. Similarly, the power of the  $i$ -th CCI at the receiver of the D2D pair is

$$S_I = P_2 \left( \frac{\lambda}{4\pi d_0} \right)^2 \left( \frac{d_0}{d} \right)^b \quad (4)$$

where  $P_2$  is the CCI signal power,  $d$  is the distance between the receiver of the D2D pair and the  $i$ -th co-channel interferer,  $d_0$  is the reference distance (1 to 100 meters) and  $b$  ( $2 \leq b \leq 5$ ) is the path-loss exponent. The expression for the signal-to-interference ratio (SIR) of the D2D communication system is

$$\rho = \frac{\alpha}{\Phi \sum_{i=1}^L \beta_i}, \quad \Phi = \frac{P_2 \left( \frac{c^a}{b^d} \right) (c_0)^{2-a}}{P_1 (d_0)^{2-b}} \quad (5)$$

where  $\alpha$  is the independent  $N^*$ Nakagami fading variable,  $\beta_i$  is the independent Gamma variable of  $i$ -th CCI signal and  $L$  is the number of CCI signals. The PDF of the SIR of the D2D communication system is

$$f_{\rho}(r) = \int_0^{\infty} x \frac{1}{rx \prod_{n=1}^N \Gamma(m_n)} G_{0,N}^{N,0} \left[ rx \prod_{n=1}^N \left( \frac{m_n}{\Omega_n} \right) \middle| m_1, m_2, \dots, m_N \right] \frac{x^{\delta-1} e^{-\frac{x}{\Phi\sigma}}}{(\Phi\sigma)^{\delta} \Gamma(\delta)} dx$$

$$f_{\rho}(r) = \frac{1}{r \Gamma(\delta) \prod_{n=1}^N \Gamma(m_n)} G_{1,N}^{N,1} \left[ rU \middle| 1-\delta, m_1, m_2, \dots, m_N \right] \quad (6)$$

where  $U = \Phi\sigma \prod_{n=1}^N \left( \frac{m_n}{\Omega_n} \right)$ . The expression for the cumulative distribution function (CDF) of D2D communication system is

$$F(r) = \frac{1}{\Gamma(\delta) \prod_{n=1}^N \Gamma(m_n)} G_{2,N+1}^{N,2} \left[ rU \middle| 1-\delta, 1, m_1, m_2, \dots, m_N, 0 \right] \quad (7)$$

### 2.1. Outage Probability

The outage probability is described as the probability that the SIR of the system drops below a predefined threshold  $R$ . The expression for the outage probability of the D2D system with help of formula,  $P_{out} = \int_0^R f_{\rho}(r) dr$ , is given as

$$P_{out} = \frac{1}{\Gamma(\delta) \prod_{n=1}^N \Gamma(m_n)} G_{2,N+1}^{N,2} \left[ RU \middle| 1-\delta, 1, m_1, m_2, \dots, m_N, 0 \right] \quad (8)$$

### 2.2. Success Probability

The success probability  $P_S$  is defined as the probability that the SIR of the system remains above the given threshold  $R$  [14]. The expression for success probability of the D2D communication system is

$$P_S = \frac{1}{\Gamma(\delta) \prod_{n=1}^N \Gamma(m_n)} G_{2,N+1}^{N+1,1} \left[ RU \middle| 1-\delta, 1, 0, m_1, m_2, \dots, m_N \right] \quad (9)$$

### 2.3. Outage Capacity

The outage capacity  $C_{out}$  is termed as the probability that the instantaneous capacity  $C_{\rho}$ , where  $C_{\rho} = \log_2(1+r)$ , drops below a predefined capacity threshold  $C_{th}$  [15]. To obtain an analytical expression for outage capacity performance of the D2D communication system, firstly PFD  $f_{c_{\rho}}(c_{\rho})$  is shown below

$$f_{c_\rho}(c_\rho) = \frac{\ln(2)2^{c_\rho}}{(2^{c_\rho} - 1)\Gamma(\delta)\prod_{n=1}^N\Gamma(m_n)} G_{1,N}^{N,1} \left[ (2^{c_\rho} - 1)U \left| \begin{matrix} 1-\delta \\ m_1, m_2, \dots, m_N \end{matrix} \right. \right] \quad (10)$$

Now, by using expression in (10), the Expression for the outage capacity of the D2D communication system is

$$C_{out} = \frac{1}{\Gamma(\delta)\prod_{n=1}^N\Gamma(m_n)} G_{2,N+1}^{N,2} \left[ U (2^{C_{th}} - 1) \left| \begin{matrix} 1-\delta, 1 \\ m_1, m_2, \dots, m_N, 0 \end{matrix} \right. \right] \quad (11)$$

## 2.4. Channel Capacity

The channel capacity  $C_c$  is the maximum rate of information that can be reliably communicated over a channel. The expression for the channel capacity of the desired D2D communication system is

$$C_c = \frac{1}{\ln(2)\Gamma(\delta)\prod_{n=1}^N\Gamma(m_n)} G_{3,N+2}^{N+2,2} \left[ U \left| \begin{matrix} 1-\delta, 0, 1 \\ m_1, m_2, \dots, m_N, 0, 0 \end{matrix} \right. \right] \quad (12)$$

## 2.5. Symbol Error Rate

The expression for the  $M$ -ray phase-shift keying ( $M$ -PSK) SER performance for the D2D communication system is given as

$$SER = \frac{1}{\pi\Gamma(\delta)\prod_{n=1}^N\Gamma(m_n)} \int_0^{\left(\frac{M-1}{M}\right)\pi} G_{2,N}^{N,2} \left[ \left( \frac{\sin(\varphi)}{\sin\left(\frac{\pi}{M}\right)} \right)^2 U \left| \begin{matrix} 1-\delta, 1 \\ m_1, m_2, \dots, m_N \end{matrix} \right. \right] d\varphi \quad (13)$$

## 3. Numerical Results and Analysis

In this section, numerical results are presented. For the numerical analysis reference distances  $c_0$  and  $d_0$  are set to be 1 meters. Outage probability of D2D communication system with varying values of  $N$  and  $m_n$  is shown in Figure. 2. The values of D2D signal power  $P_1$ , path-loss exponent  $a$  and distance between D2D pair  $c$  are set to be 20 dBm, 2.5 and 15 meters, respectively. The power of CCI signal  $P_2$ , path-loss exponent of CCI  $b$ , distance between  $i$ -th interferer and the receiver of D2D pair  $d$  and fading shape parameter of CCI signal  $\delta$  are assumed to be 10 dBm, 4, 40 meters and 1, respectively. Number of interferers  $L$  is fixed at 5. From the figure it is observed that the outage performance is improved as the value of  $m_n$  is increased. It is due to the improvement of D2D channel fading conditions. Also, by increasing the values of  $N$  outage performance degrades. It is due to increase in the fading severity of the D2D system. Outage performance of D2D communication system for various values of fading parameter  $m_n$  and distance  $c$  is shown in Figure. 3. The values for  $P_1$ ,  $P_2$ ,  $a$ ,  $b$ ,  $d$ ,  $\delta$ ,  $N$  and  $L$  are considered to be 20 dBm, 10 dBm, 2.7, 3.3, 35 meters, 2, 2 and 5, respectively. Outage threshold  $R$  is set to be 10 dBm. From the figure, it is observed that as the D2D pair devices move away from each other, *i.e.*, value of  $c$  is increased. Outage performance of the D2D system degrades. It is due to weakening of the desired D2D signal at the D2D

receiver. It is because of the path-loss effects. It is also clear that the outage performance is improved when the fading conditions of the D2D system are improved, *i.e.*, values of  $m_n$  are increased.

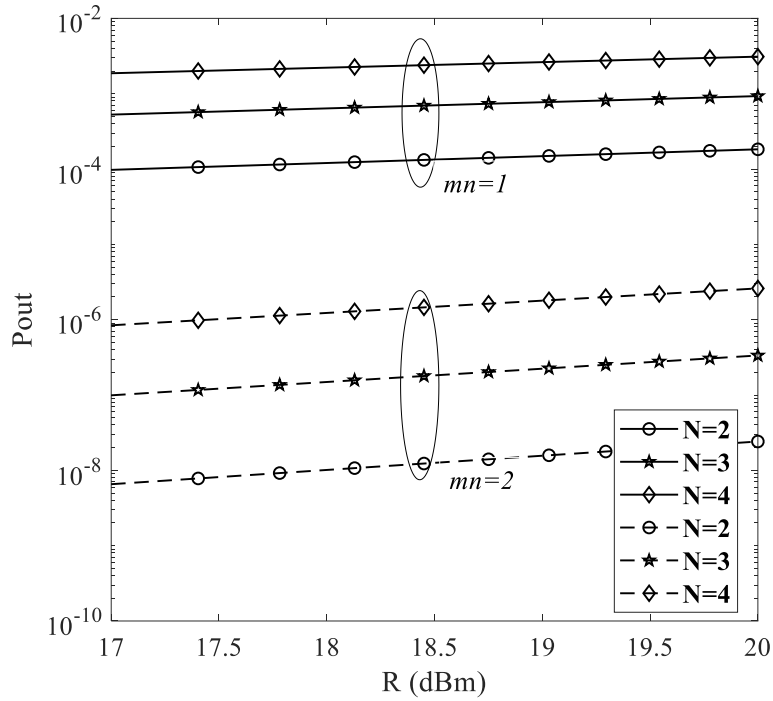


Figure 2. Outage Probability with Various Values of  $N$

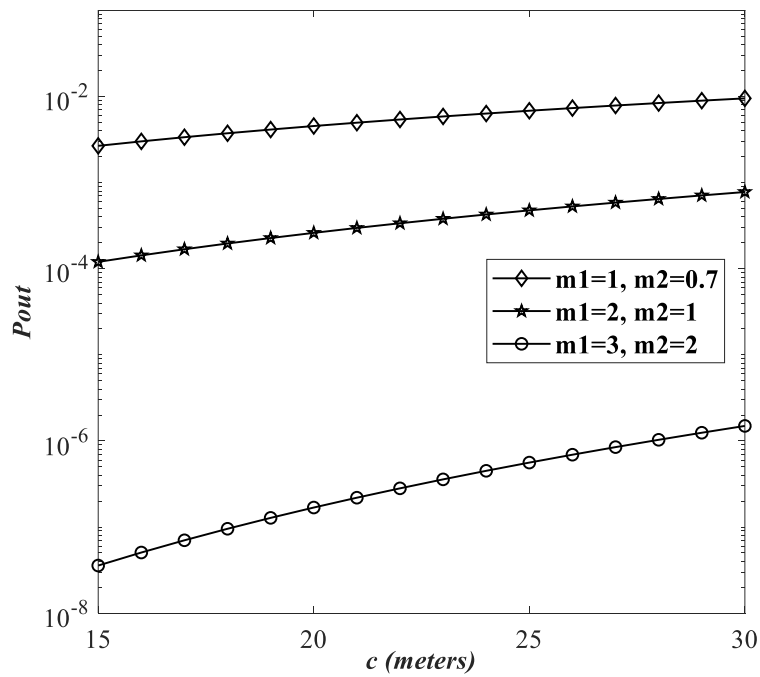
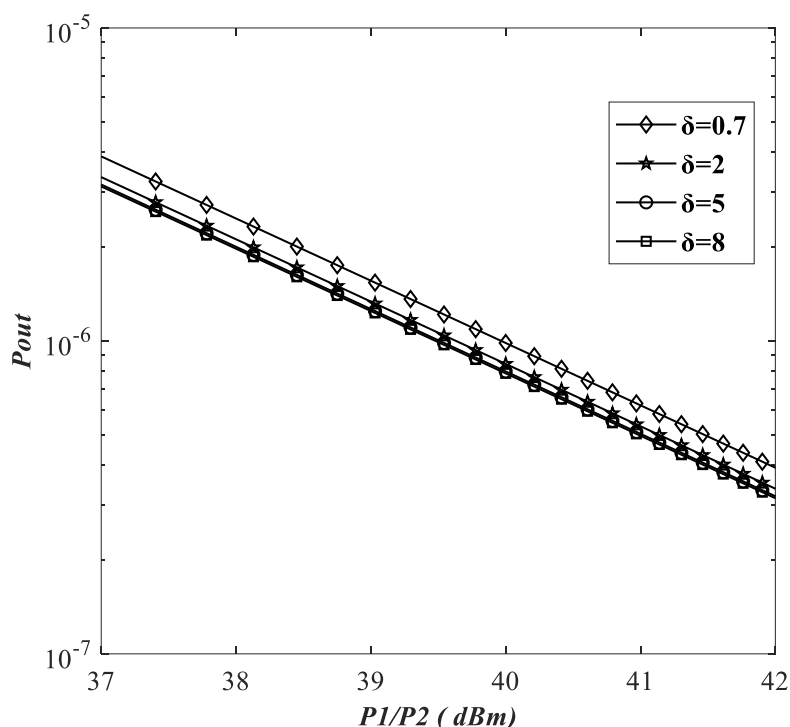


Figure 3. Outage Performance with Various Values of  $m_n$

Outage performance with various values of CCI fading parameter  $\delta$  is shown in Figure. 4. The values for  $P_2$ ,  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $m_1$ ,  $m_2$ ,  $R$ ,  $N$  and  $L$  are considered to be 10 dBm, 2.8, 3.3, 20 meters, 30 meters, 2, 3, 10 dBm, 2 and 5, respectively. From the figure, it is observed that outage performance of the D2D system is mostly insensitive to the variations of the fading conditions of CCI. It is also obvious from the figure that the outage performance is improved when the D2D signal power  $P_1$  is increased. It is due to better SIR conditions of D2D system.



**Figure. 4. Outage performance with various values of CCI fading parameter**

Success probability performance of D2D communication system with varying values of  $d$  is shown in Figure. 5. The values for  $P_1$ ,  $a$ ,  $b$ ,  $c$ ,  $\delta$ ,  $m_1$ ,  $m_2$ ,  $R$ ,  $N$  and  $L$  are considered to be 20 dBm, 3.5, 3, 25 meters, 5, 2, 3, 10 dBm, 2 and 5, respectively. From the figure, it is observed that as the interferers move away from the desired D2D receiver, i.e., value of  $d$  is increased. Success probability of the D2D system improves. It is due to weakening of the interference signals at the D2D receiver side. It is because of the path-loss effects. It is also noticed from the figure that the success probability of D2D system degrades when the interference power  $P_2$  is increased. The increase in  $P_2$  degrades SIR conditions. Outage capacity performance with various values of CCI path-loss exponent  $b$  is presented in Figure. 6. The values for  $P_1$ ,  $P_2$ ,  $a$ ,  $d$ ,  $\delta$ ,  $m_1$ ,  $m_2$ , threshold capacity  $C_{th}$ ,  $N$  and  $L$  are considered to be 20 dBm, 10 dBm, 2.7, 30 meters, 2, 2, 3, 0.1 bits/s/Hz, 2 and 5, respectively. From the figure, it is clear that as the value of  $b$  is increased. Outage capacity performance of the D2D system improves. It is because of the weakening of the interference signals at the D2D receiver. It is due to path-loss effects. From the figure, it is also observed that as the distance between D2D devices is increased, i.e., value of  $c$  is increased. Outage capacity performance of the D2D system deteriorates due to path-loss.

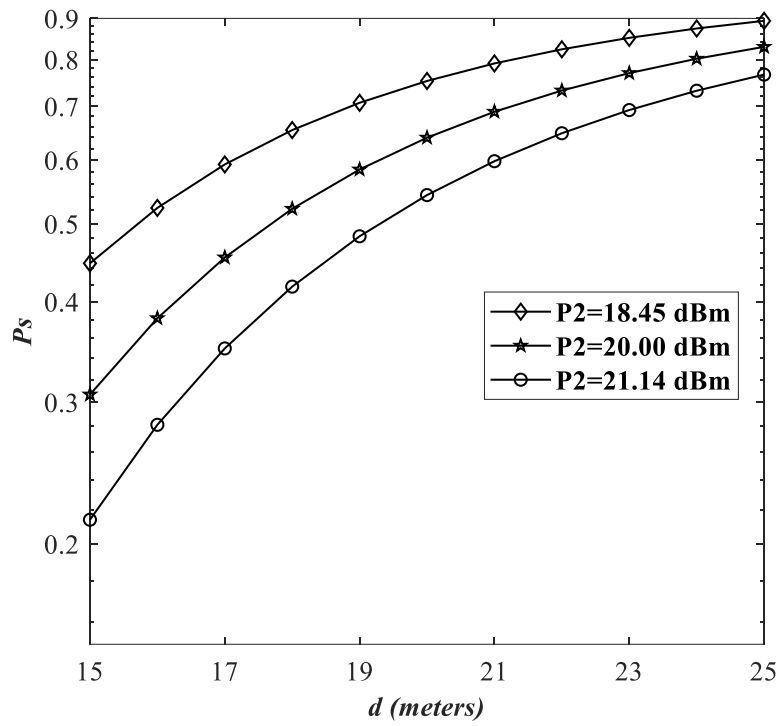


Figure 5. Success probability with various values of  $P_2$

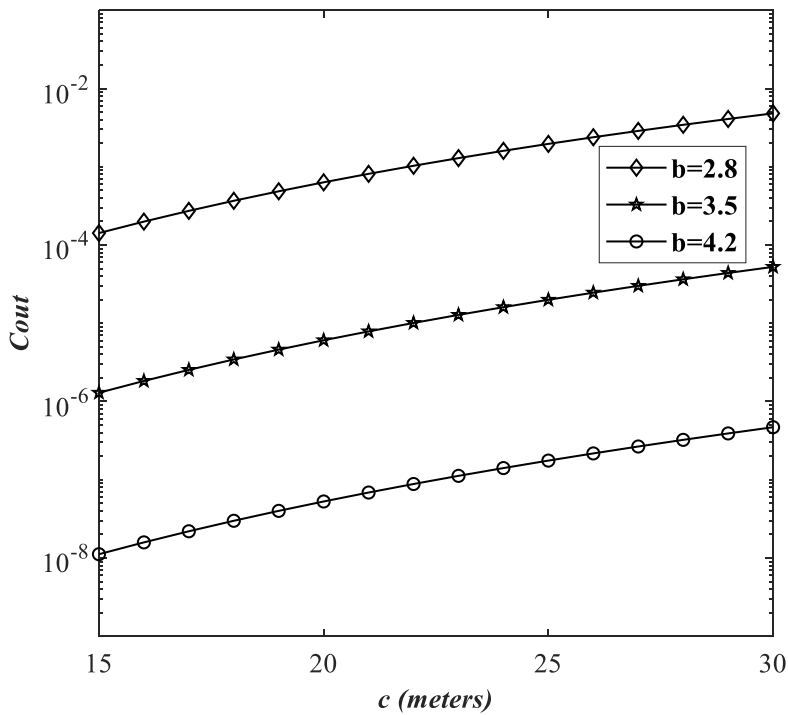
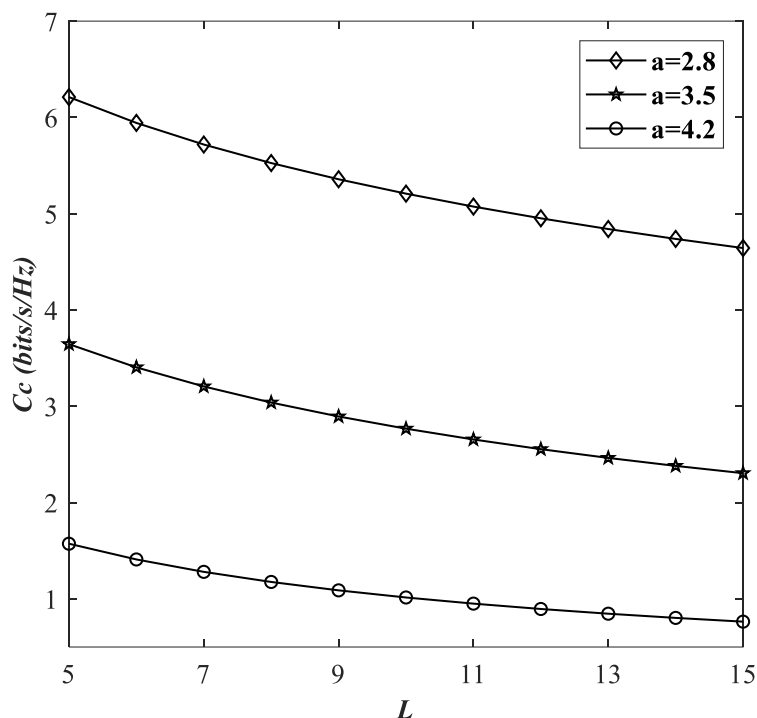


Figure 6. Outage capacity with various values of  $b$

Channel capacity performance with various D2D signal path-loss exponent values  $a$  is shown in Figure 7. The values for  $P_1$ ,  $P_2$ ,  $b$ ,  $d$ ,  $\delta$ ,  $m_1$ ,  $m_2$ ,  $N$  and  $c$  are considered to be 20



dBm, 10 dBm, 3.4, 30 meters, 2, 2, 3, 2 and 15 meters, respectively. From the figure, it is clear that as the value of  $a$  is increased. Capacity performance of the D2D system degrades. It is because of the weakening of the desired D2D signal at the D2D receiver. It is also observed that capacity performance of the D2D system degrades as the number of interferers, i.e.,  $L$  is increased.



**Figure. 7. Channel capacity with various values of  $a$**

Channel capacity performance with varying values of D2D signal power  $P_1$  is given in Figure. 8. The values for  $P_2$ ,  $a$ ,  $b$ ,  $d$ ,  $\delta$ ,  $m_1$ ,  $m_2$ ,  $N$  and  $L$  are considered to be 10 dBm, 2.7, 3.5, 20 meters, 5, 2, 3, 2 and 5, respectively. From the figure, it is observed that the capacity performance is improved as the D2D signal power is increased. From the figure, it can be seen that as the distance between D2D pair is increased. Channel capacity deteriorates due to path-loss effects. SER performance of 8-PSK D2D communication system with varying values of  $P_1$  is shown in Figure. 9. The values for  $P_2$ ,  $c$ ,  $b$ ,  $d$ ,  $\delta$ ,  $m_1$ ,  $m_2$ ,  $N$  and  $L$  are considered to be 10 dBm, 15 meters, 4.1, 40 meters, 2, 3, 4, 2 and 5, respectively. From the figure, it can be seen that the SER performance is improved as the value of  $P_1$  is increased. It is also observed that as the value of  $a$  is increased. SER performance degrades due to weakening of the desired D2D signal at the D2D receiver.

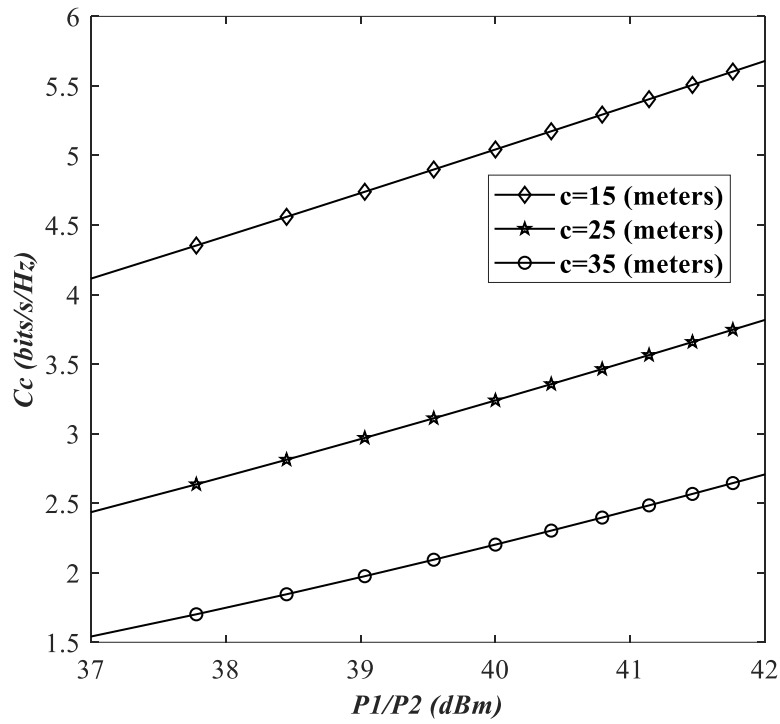


Figure 8. Channel capacity with various values of  $c$

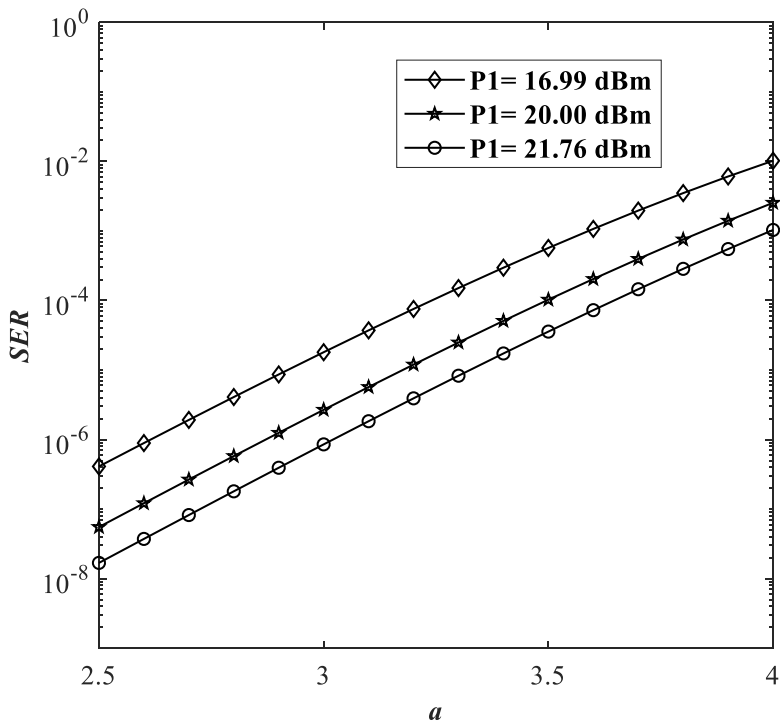


Figure 9. SER with various values of  $P_1$

## 4. Conclusion

In this paper, outage probability, success probability, outage capacity, channel capacity and symbol error rate (SER) performances of a D2D communication system are presented. The  $N^*$ Nakagami distribution is considered for the D2D system.  $N^*$ Nakagami is a general fading distribution based on the product of  $N$  independent Nakagami random variables. Effects of co-channel interference caused by various rogue wireless devices in the system are also considered. Expressions of the PDF and CDF of the SIR of the D2D communication system are presented. Based on the PDF expression, expressions of outage probability, success probability, outage capacity, channel capacity and symbol error rate (SER) are then presented. Numerical results highlighting the effects of various interference, path-loss and channel fading conditions on the performance of D2D system are then discussed. It is observed that higher values of D2D system path-loss exponent degrade its performances. However, increase in the path-loss exponent values of the interferers improve D2D system performances. Increase in the distance of the D2D pair degrades D2D system SIR conditions. As the interferers move away from the D2D receiver D2D system SIR conditions are improved. Furthermore, as the number of  $N$  increases system performance degrades.

## References

- [1] T. Rui, Z. Jihong and Q. Hua, "Distributed power control for energy conservation in hybrid cellular network with Device-to-Device communication", *China Communications*, vol. 11, no. 3, (2014), pp. 27-39.
- [2] M. Haus, M. Waqas, A. Y. Ding, Y. Li, S. Tarkoma and J. Ott, "Security and Privacy in Device-to-Device (D2D) Communication: A Review", *IEEE Communications Surveys & Tutorials*, vol. 19, no. 2, (2017), pp. 1054-1079.
- [3] S. H. Lee, D. R. Shin, H. W. Jeong and Y. H. Kim, "Distributed Bargaining Strategy for Downlink Virtual MIMO With Device-to-Device Communication", in *IEEE Transactions on Communications*, vol. 64, no. 4, (2016), pp. 1503-1516.
- [4] Y. J. Chun, S. L. Cotton, H. S. Dhillon, A. Ghayeb and M. O. Hasna, "A Stochastic Geometric Analysis of Device-to-Device Communications Operating Over Generalized Fading Channels", in *IEEE Transactions on Wireless Communications*, vol. 16, no. 7, (2017), pp. 4151-4165.
- [5] Z. Uykan and R. Jäntti, "Transmission-Order Optimization for Bidirectional Device-to-Device (D2D) Communications Underlying Cellular TDD Networks—A Graph Theoretic Approach", in *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 1, (2016), pp. 1-14.
- [6] J. Kim, J. Joung and J. W. Lee, "Resource Allocation for Multiple Device-to-Device Cluster Multicast Communications Underlay Cellular Networks", in *IEEE Communications Letters*, vol. 22, no. 2, (2018), pp. 412-415.
- [7] M. Peng, Y. Li, T. Q. S. Quek and C. Wang, "Device-to-Device Underlaid Cellular Networks under Rician Fading Channels", in *IEEE Transactions on Wireless Communications*, vol. 13, no. 8, (2014), pp. 4247-4259.
- [8] Y. Gu and S. Aïssa, "RF-Based Energy Harvesting in Decode-and-Forward Relaying Systems: Ergodic and Outage Capacities", in *IEEE Transactions on Wireless Communications*, vol. 14, no. 11, (2015), pp. 6425-6434.
- [9] M. Lin, J. Ouyang and W. Zhu, "Joint Beamforming and Power Control for Device-to-Device Communications Underlying Cellular Networks", in *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 1, (2016), pp. 138-150.
- [10] G. K. Karagiannidis, N. C. Sagias and P. T. Mathiopoulos, " $N^*$ Nakagami: A Novel Stochastic Model for Cascaded Fading Channels", in *IEEE Transactions on Communications*, vol. 55, no. 8, (2007), pp. 1453-1458.
- [11] Z. Hussain, A. R. Khan, H. Mehdi, S. M. Saleem, "Analysis of Device-to-Device Communication System in the Presence of Multiple Co-Channel Interference", *University of Sindh Journal of Information and Communication Technology*, vol. 2, no. 1, (2018), pp. 71-67.
- [12] I. S. Gradshteyn, I. M. Ryzhik, "Table of Integrals, Series, and Products", 7th ed.; Academic: San Diego, CA, USA, (2007).
- [13] Andrea Goldsmith, "Wireless Communications", Cambridge University Press: Cambridge, England, (2005).
- [14] M. Haenggi, "Outage, local throughput, and capacity of random wireless networks", in *IEEE Transactions on Wireless Communications*, vol. 8, no. 8, (2009), pp. 4350-4359.

- [15] J. A. Anastasov, M. Z. Nemanja and T. D. Goran, "Outage capacity evaluation of extended generalized-K fading channel in the presence of random blockage", *Journal of the Franklin Institute* vol. 352, no. 10 (2015), pp. 4610-4623.