A Class of Stabilizing PID Controllers for Position Control of Single-Link Robot

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Abstract

A class of stabilizing proportional-integral-derivative (PID) controllers for a nonlinear system is designed using different techniques. The ranges of controller parameters are obtained by a search technique (1) checking the stability of the system by solving an interval Lyapunov equation, (2) checking the quadratic stability, (3) checking the stability of the system by solving Lyapunov equation, (4) solving stability boundary equations.. This characterization is useful for designing an optimal set of controller parameters. A simulation example of a single-link robot manipulator is considered to demonstrate the applicability of the proposed control schemes.

Key words: *Lyapunov equation, boundary equation, interval matrix, quadratic stability, robot manipulator*

1. Introduction

A class of all stabilizing P, PI and PID controllers was designed by Bhattacharya et al [1] using generalized Hermite Biehler theorem. This characterization of all feedback gain values is useful for carrying out optimal designs with respect to various performance indices. Most of the industrial controllers even today are either proportional- integral (PI) or proportional-integral-derivative (PID) types [13] and hence the result of the above paper had great significance. One of the drawbacks of the above design method is that it can be applied only to linear system. Moreover, it has been a long standing belief that PID control is inadequate to cope with highly nonlinear system, since the design of the control law is based solely on linear models. A very important technology is necessary to introduce for industrial robot to realize a fine and fast positioning control. Most of the robots employed in industry are often equipped with conventional PID controllers due to their simplicity in structure and ease of design. In practice, a simple linear PID feedback controller with appropriate control gains may lead to the desired position without causing any steady-state error. This is the reason why PID controllers are still used in industrial robots. In true sense, however, PID controller does not ensure to achieve a desired level of control performance, while dynamic equations for mechanical manipulators are highly uncertain due to load changes and nonlinear friction effects.

In this paper, a class of stabilizing controllers for a single-link robot was found out using a simple design technique where the feedback gains are obtained by checking the stability of the closed-loop system. Subsequently we obtain the range of PID controllers for the single-link robot manipulator and it is discussed in detail.

This paper is organized as follows. In Section 2, we describe the dynamic model of a single-link manipulator. In Section 3, we obtain the complete characterization of all stabilizing PID controllers for a single-link manipulator using different methods. In Section 4, a simulation example is given to demonstrate the applicability of the proposed control schemes. An optimal selection of PID controller parameter for each method based on genetic algorithm is obtained from the stabilizing region of controller parameters and it is discussed in the same section. Finally, the conclusions are included in section 5.

2. Dynamic Model of a Robot and Problem Formulation

The dynamic equation of a single-link robot manipulator shown in Fig. 1, is described by [2]

$$\tau = J_m \ddot{\theta} + f(\dot{\theta}) + ml\, g \cos\theta \tag{1}$$

where θ is the link angle, m and J_m are the mass and the rotational inertia of the link, respectively, l is the distance from the joint axis to the link center mass, f is the friction function, and τ is the output torque of motor reflected to the joint axis. If the link is modeled as a point mass at the distal end, $J_m = ml^2$.

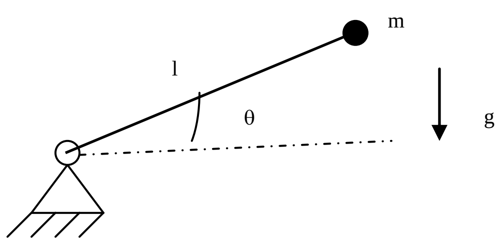

Fig.1 Schematic of one link revolute robot

Let θ_d denote the desired constant joint position and $\theta(0), \dot{\theta}(0)$ be any given initial condition. The classical PID control law can be written as

$$\tau = K_p e(t) + K_d \dot{e}(t) + K_i \int_0^t e(t)dt \tag{2}$$

where $e(t) = \theta_d - \theta(t)$ is the position error, K_p, K_i and K_d are respectively, the proportional, integral and derivative gains of the PID controller. The control problem is to provide a complete solution to the constant gain stabilizing control parameters K_p, K_i, K_d such that the position error $e(t)$ vanishes with time, i.e. $\lim_{t\to\infty} e(t) = 0$.

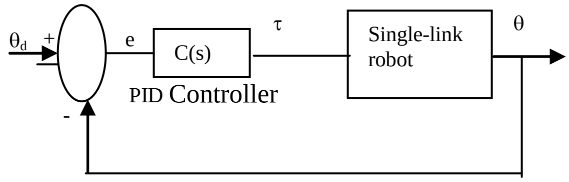

Fig. 2 Block diagram of single robot system

3. Design of all Stabilizing PID Controller Gains

Let us consider the feedback system shown in Fig.2 where θ_d is the desired command signal and θ is output. Now combining the dynamics of single-link robot acting in a friction free space ($f(\dot{\theta}) = 0$) and control law (equations 1 & 2) we obtain

$$J_m \dddot{e}(t) + K_d \ddot{e}(t) + [K_p - ml\,g \sin(\theta_d - e(t))]\dot{e}(t) + K_i\, e(t) = 0 \qquad (3)$$

Then the following state space form can represent the above error system dynamic equation

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_i / J_m & -(K_p - ml\,g \sin\theta)/J_m & -K_d / J_m \end{bmatrix} X(t) \qquad (4)$$

where $x_1(t) = e(t)$, $x_2(t) = \dot{e}(t)$ and $x_3(t) = \ddot{e}$

In this paper we will consider the different design techniques for characterization of all stabilizing PID controllers.

3.1 Method 1: Based on interval matrix approach

It may be noted that the system equation (4) can now be changed to an interval matrix by assigning the value of $\sin(\theta_d - x_1)$ in the range of [0 1] and more specifically the position of robot link can be placed any location in the range θ ($=\theta_d - x_1$) = 0 to 180°. Furthermore the system matrix given in equation (4) thus becomes an interval matrix as

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_i / J_m & 1/J_m[-K_p \quad -K_p + ml\,g] & -K_d / J_m \end{bmatrix} X(t)$$

$$= A_I X(t) = [A_C - \Delta,\ A_C + \Delta] X(t) \qquad (5)$$

where A_C is the center or nominal matrix and Δ is the perturbed matrix. In order to check the stability of the interval matrix A_I we use the result of Garofalo et al [3]-[5]. It is assumed that the nominal matrix A_C is asymptotically stable for the choice of PID controller parameters. An equivalent statement is that for any symmetric positive definite matrix Q_0 there exists a unique symmetric positive definite matrix P_0 [3] such that

$$A_C^{\mathrm{T}} P_0 + P_0 A_C = -Q_0 \qquad (6)$$

We focus our attention on the stability of the interval system (5) and this can be achieved by considering the same P_0 matrix as obtained from equation (6).This means that we could check the positive definiteness of the interval matrix

$$Q_I = -(A_I^{\mathrm{T}} P_0 + P_0 A_I) \qquad (7)$$

In general, it is difficult to check the positive definiteness of an interval matrix Q_I, but the structure of the matrix A_I can help in solving the equation (7) in a simpler way. Since the matrix A_I is an interval matrix, each entry of it is a continuous function over a compact set, with its maximum and minimum range. For this class of matrices, a sufficient condition for matrix Q_I defined by (7) to be positive definite [5] is that the matrices

$$Q^r = -({A^r}^{\mathrm{T}} P_0 + P_0 A^r) \quad (8)$$

be positive definite for all $r = 1, 2, \ldots, 2^{n^2}$, where n is the number of elements in A having interval range (for the present case, n = 1 and r = 1,2). The matrix A^r is the corner matrices of A_I and more specifically

$$A^1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_i & -K_p & -K_d \end{bmatrix} \text{and } A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_i & -K_p + ml\,g & -K_d \end{bmatrix}$$

In the proposed method we search for the range of PID controller parameters by checking the positive definiteness of Q^1 and Q^2 after solving P_0 [5] from the Lyapunov equation (6). The algorithmic steps involved to obtain the range of controller parameters are given in Appendix I and explained through flow diagram in fig. 20.

3.2 Method 2: Based on quadratic stability approach

The investigation of stability domains for state space models has been addressed in many papers during the last decades. The use of Lyapunov functions[14-17] is certainly the main approach for this kind of analysis, since bounds for the stability domains can be found in terms of the associated Lyapunov matrix and the allowed perturbation direction. For linear systems with uncertain parameters, a polytopic bounding approach was proposed in literature and the quadratic stability conditions over the entire uncertainty polytope have been derived. The results of quadratic stability conditions are conservative and many attempts have been made to reduce the conservatism by seeking parameter dependent techniques for determining the stability of system with polytopic type uncertainties. These techniques assign a Lyapunov function to each vertex of the uncertainty polytope and subsequently parameter dependent condition for the stability of linear systems with polytopic uncertainty has been introduced in [6] - [7]. It is worth of mentioning that the stability domains can be computed by evaluating all the vertices of the polytope.

The dynamic equation of the system (4) is a function of θ ($e = x_1 = \theta_d - \theta$) and it can be considered an uncertain time-varying system since θ is varying with respect to time. A time-varying system can be modeled as a polytopic system and this model is also called Polytopic Linear Differential Inclusions (PLDI) in the literature and it is given in Appendix II.

Consider the system (4) and the corresponding polytopic system is quadratically stable if and only if [6] there exits a common Lyapunov matrix $P > 0$ such that

$$A_i^{\mathrm{T}} P + P A_i < 0 \quad i = 1,2,\ldots. \tag{9}$$

where the system matrix of equation (4) ranges in the polytope by its vertices.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_i & -K_p & -K_d \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_i & -K_p + ml\,g & -K_d \end{bmatrix};$$

Each vertex is computed by taking alternatively the maximum and minimum values of each element of the system matrix of equation (4). We can find out the range of controller parameters by checking the quadratic stability of PLDI system using the MATLAB LMI Control Toolbox [8] and simultaneously adopting the procedure given in Appendix I.

3.3 Method 3: Based on Lyapunov function

The necessary conditions for stability of the closed-loop system (4) are given by the following expressions.

$$K_i > 0, \qquad K_d > 0 \qquad \text{and} \qquad K_p > mlg\sin\theta \tag{10}$$

In order to study the stability of the system (4) we introduce the Lyapunov function

$$V(X(t),t) = X^{\mathrm{T}}(t)PX(t) \tag{11}$$

where $P = (p_{ij})$ is a positive definite matrix and the diagonal elements of P are nonzero and positive (i.e. $p_{ii} > 0$, $i = 1, 2, 3$). The time derivative of V along the trajectories of (4) is given by

$$\dot{V} = \frac{\partial V}{\partial t} = \frac{\partial V}{\partial \mathrm{X}}\dot{X} \tag{12}$$

The above equation is now written in explicit form as

$$\begin{aligned}
\dot{V}(X(t),t) &= -2p_{13}K_i / J_m x_1^2 + 2(p_{12} + p_{23}(-K_p + ml\,g\sin\theta)/J_m)x_2^2 + 2(p_{23} - p_{33}K_d / J_m)x_3^2 \\
&+ 2(p_{11} + p_{13}(-K_p + ml\,g\sin\theta)/J_m - p_{23}K_i / J_m)x_1x_2 + 2(p_{12} - p_{33}K_i / J_m - p_{13}K_d / J_m)x_1x_3 \\
&+ 2(p_{22} + p_{13} - p_{23}K_d / J_m + p_{33}(-K_p + ml\,g\sin\theta)/J_m)x_2x_3 . \\
&\le -2p_{13}K_i / J_m x_1^2 + 2(p_{12} + p_{23}(-K_p + ml\,g\sin\theta)/J_m)x_2^2 + 2(p_{23} - p_{33}K_d / J_m)x_3^2 + \\
&(p_{11} + p_{13}(-K_p + ml\,g\sin\theta)/J_m - p_{23}K_i / J_m)(x_1^2 + x_2^2) + (p_{12} - p_{33}K_i / J_m - p_{13}K_d / J_m)(x_1^2 + x_3^2) \\
&+ (p_{22} + p_{13} - p_{23}K_d / J_m + p_{33}(-K_p + ml\,g\sin\theta)/J_m)(x_2^2 + x_3^2), \text{since} \\
&x_i^2 + x_j^2 \ge 2x_ix_j
\end{aligned} \tag{13}$$

It can be noted that the sign of the cross product terms x_ix_j are indefinite and the coefficients associated with these terms are assigned to zero to satisfy $\dot{V}(X(t),t) < 0$. This in turn, needs to satisfy the following six equations.

$$-p_{13}K_i / J_m < 0 \tag{14}$$

$$p_{12} + p_{23}(-K_p + ml\,g\sin\theta)/J_m < 0 \tag{15}$$

$$p_{23} - p_{33}K_d / J_m < 0 \tag{16}$$

$$p_{11} + p_{13}(-K_p + ml\,g\sin\theta)/J_m - p_{23}K_i / J_m = 0 \tag{17}$$

$$p_{12} - p_{33}K_i / J_m - p_{13}K_d / J_m = 0 \tag{18}$$

$$p_{22} + p_{13} - p_{23}K_d / J_m + p_{33}(-K_p + ml\,g\sin\theta)/J_m = 0 \tag{19}$$

Using the equation (10) and the properties of $P > 0$ ($p_{ii} > 0$, $i = 1, 2, 3$) in equations (12) - (19), one can easily arrive at the following necessary conditions by carefully examining the equations (12) - (19).

$p_{13} > 0, \quad p_{12} > 0 \quad \text{and} \quad p_{23} > 0$

Equations (14) – (19) are solved for wide choice of controller parameters (K_p, K_i, K_d) with $\theta = 90^\circ$ in order to achieve not only $\dot{V}(X(t),t) < 0,$ but also to satisfy $P > 0$.

It can be noted further from the equations (14)- (19) that the same value of P and the controller parameters that are obtained in previous step for $\theta = 90^\circ$ can also be used in equations (14)- (19) for $0 < \theta < 180^\circ$ to ensure $\dot{V}(X(t),t) < 0$. Equations (14)-(19) implies that the controller designed parameters for $\theta = 90^\circ$ will stabilize a family of plants while the desired position of the robot arm angle θ varies from $\theta = 0^\circ$ to $\theta = 180^\circ$.

3.4 Method 4: Based on stability boundary equation

The concepts of gain margin and phase margin of single input single output (SISO) systems are well defined and understood. They reflect on the performance and stability of the system and widely used for controller designs. A classical approach to consider model uncertainties is to design the closed-loop control system with sufficient gain and phase margins and it is then guaranteed that gain variation and phase delays do not lead to instability. A class of research workers has extended these SISO phase and gain margin concepts to multivariable systems and it can be found in literature [18-20].

The concept of stability equation method [9] is employed to obtain stability boundary of a system and simultaneously the gain margins (equal to 1) and phase margins (equal to zero) are maintained along the stability boundary in a controller parameter plane or parameter space. We recall the equation (1) and can be rewritten as

$J_m \ddot{\theta} = \tau - ml\, g \cos\theta$ (assuming that the robot is acting in friction free space)

The block diagram representation of the above system is considered here by taking $ml\, g \cos\theta$ as another input acting on the system.

Fig. 3 shows the s-domain representation of the system with a PID controller, where $R_1(s)$ and $R_2(s)$ are the inputs to the system, $Y(s)$ is the output and τ is the controller ($C(s) = K_p + K_i/s + s\, K_d$) output acting as an input to the system.

The system output for the system under consideration is given by

$$
\begin{aligned}
Y(s) &= Y_1(s) + Y_2(s) \\
&= \frac{G(s)C(s)}{1+G(s)C(s)} R_1(s) - \frac{G(s)}{1+G(s)C(s)} R_2(s) \\
&= \frac{1}{1+G(s)C(s)} \begin{bmatrix} G(s)C(s) & -G(s) \end{bmatrix} \begin{bmatrix} R_1(s) \\ R_2(s) \end{bmatrix} \qquad (20)
\end{aligned}
$$

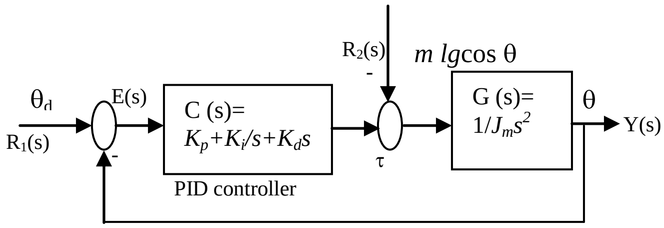

Fig 3 Another form of block diagram representation of a single-link robot

The open loop transfer function from each input ($R_1(s)$ or $R_2(s)$) to the output $Y(s)$ can be written as

$$G_C(s) = G(s)C(s) \tag{21}$$

For $s = j\omega$, then we have

$$G(j\omega)C(j\omega) = \alpha e^{j\beta} \tag{22}$$

where $|G(j\omega)C(j\omega)| = \alpha$ and $\angle G(j\omega)C(j\omega) = \beta$

Equation (22) can be written in the following form [21]

$$1 + \frac{1}{\alpha} e^{-j(180+\beta)} G(j\omega)C(j\omega) = 0$$

$$1 + Ae^{-j\gamma} G(j\omega)c(j\omega) = 0 \tag{23}$$

where $A = \frac{1}{\alpha} = \frac{1}{|G(j\omega)C(j\omega)|}$ and $\gamma = 180 + \beta$

It can be noted that A is the gain margin of the system when $\gamma = 0$ and γ is the phase margin when A=1. More specifically, one can easily determine the gain margin and phase margin of the system by adopting the gain-phase margin tester $Ae^{-j\gamma}$, which can be represented by an additional block in cascade with $G(s)C(s)$ and shown in fig. 4

Using the expression for $C(j\omega) = K_p + K_i/j\omega + K_d\, j\omega$ in eqn.(23) and evaluating the real and imaginary part to zero, we get

$$R = \mathrm{Re}\left[1 + Ae^{-j\gamma} G(j\omega)C(j\omega)\right] = 0 \tag{24}$$

$$I = \mathrm{Im}\left[1 + Ae^{-j\gamma} G(j\omega)C(j\omega)\right] = 0 \tag{25}$$

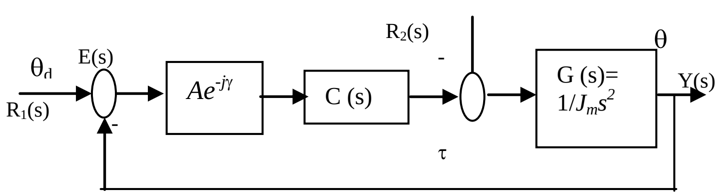

Fig. 4 Feedback control system with a gain and phase tester

It can be noted that the number of controller parameters are more than the number of equations and it is necessary to assign one of the controller parameter (say K_d) and the remaining controller parameters are solved from equations (24) and (25). To have the stable region in the parametric plane, one generally finds the stability boundary first, and then determines the stable region by the sign of J_j $(J_j = \partial R/\partial K_p . \partial I/\partial K_i - \partial R/\partial K_i . \partial I/\partial K_p)$. If the sign of J_j is positive (negative) facing the direction in which ω is increasing, the left (right) side of the stability boundary is the stable region [10]. Thus one can obtain the range of controller parameters (K_p, K_i) for a fixed value of K_d from the boundary of the stability region.

4. Simulation Results

To illustrate the proposed stabilizing control schemes, a simulation example is carried out for a one-link robot manipulator. For simplicity a single-link revolute joint robot operating in a friction-free space with point mass acting at the distal end of the link is considered. The dynamic equation of the one link robot manipulator is given by

$$\tau = ml^2\ddot{\theta} + mlg\cos\theta$$

The parameters are set as m = 1.0 kg, l = 1.0 m and g = 9.81m/s^2.

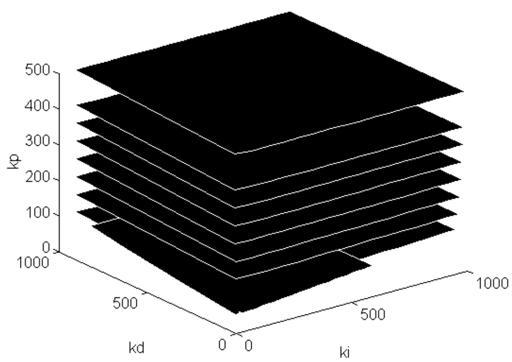

Fig. 5 The stabilizing set of (K_p,K_i,K_d) values by method 1

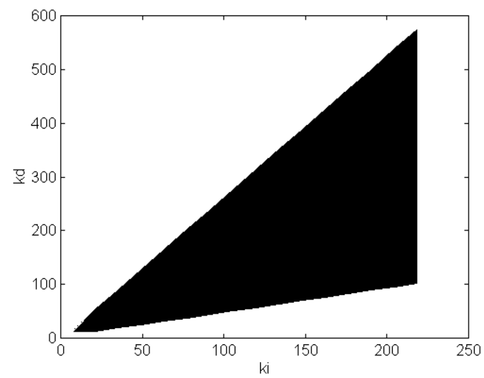

Fig. 6 The plane of (Ki, Kd) values for Kp = 25 by method 1

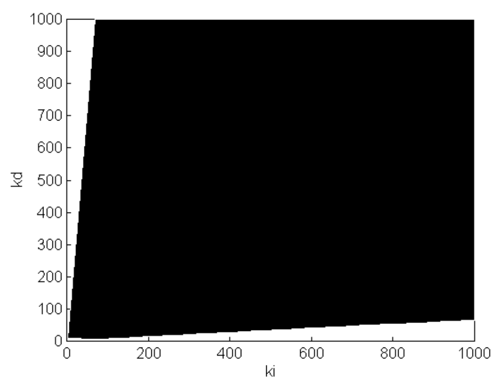

Fig. 7 The plane of (Ki, Kd) values for Kp = 100 by method 1

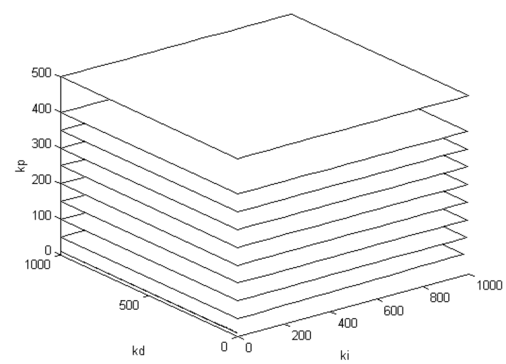

Fig.8 The stabilizing set of (Kp,Ki,Kd) values by method 2

The stabilizing set of K_p from 20.608 to ∞ is obtained by the method 1. The range of K_i, K_d values for K_p varying from 20.608 to 500 is shown in fig. 5. The stabilizing set of K_p obtained by method 2 and method 3 are same and is from 9.82 to ∞. Figs 8 and 11 show the stabilizing set (K_i, K_d) values for K_p values from 9.82 to 500 which is obtained by adopting the method 2 and method 3 respectively. The plane of K_i, K_d values for a particular K_p is also drawn for method 1, 2 and 3. (Fig 6 K_p= 25, method 1, Fig 7 K_p= 100, method 1, Fig 9 K_p= 25, method 2, Fig 10 K_p= 100, method 2, Fig 12 K_p= 25, method 3, Fig 13 K_p= 100, method 3).

In method 4 we are assuming K_d value and finding out the stabilizing region of K_p, K_i as ω varies from 0.01 to 22Hz. Fig. 14 shows the stabilizing set of K_p, K_i values for K_d from 1 to 1000. Figs 15 and 16 shows the region of stabilizing set of K_p, K_i obtained for K_d = 1 and 100 respectively. The region of stability is found out by evaluating J_j(here J_j is negative) and hence the right side of the stability boundary is the stable region , facing the direction in which ω is increasing. This is shown shaded in figs 15 and 16. It is observed that the stabilizing region of K_p, K_i, K_d obtained by method 1 is less than the method 2, 3 and 4. This is mainly due to the interval operations involved in method 1. Results obtained by method 1 are conservative compared to other methods discussed in this paper.

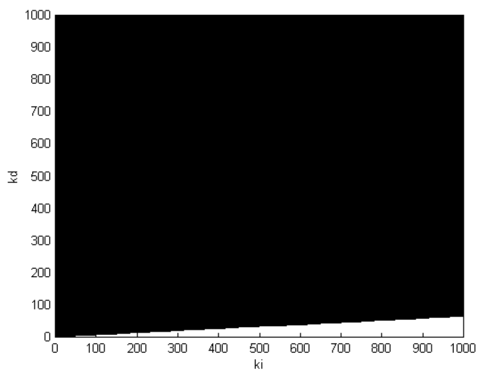

Fig. 9 The plane of (Ki, Kd) values for Kp = 25 by method 2

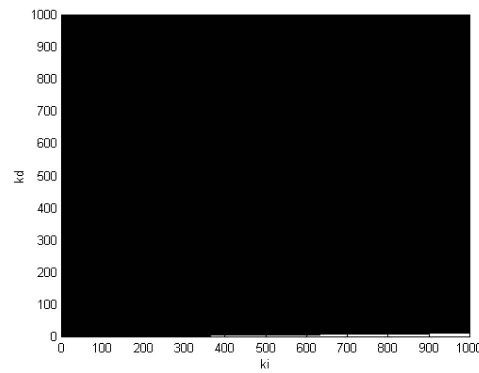

Fig. 10 The plane of (Ki, Kd) values for Kp = 100 by method 2

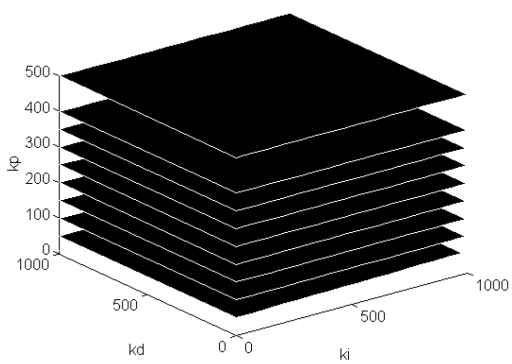

Fig. 11 The stabilizing set of (Kp,Ki,Kd) values by method 3

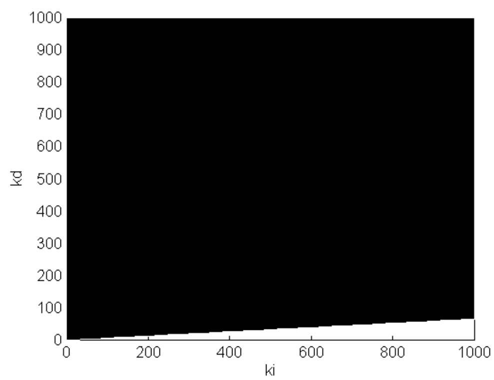

Fig. 12 The plane of (Ki, Kd) values for Kp = 25 by method 3

From the class of stabilizing controllers obtained by all the methods we tried to design an optimal controller parameter (K_p, K_i, K_d) by maximizing the fitness function J_f

$$J_f = \frac{1}{1+J} \tag{26}$$

where J is the performance index given by $J = \int_0^t e^2 dt$

The genetic algorithm based optimization technique [11] is used to obtain the optimal controller gains. The genetic operations applied were arithmetic crossover, uniform mutation and ranking selection. The GA was run for 25 generations with a population size of 80.

The initial conditions are chosen as $x_1(0) = 0.0$ rad, $x_2(0) = 0$, $x_3(0) = 0$. The desired angle is selected as 0.524 rad (30^o). The range of controller gains K_i = [0.002, $\propto$] and K_d = [0.002, $\propto$] for K_p = 25 are obtained using method 1 and the corresponding optimum controller gains K_i^* = 57.03 and K_d^* = 15.11 for K_p = 25 are obtained by adopting genetic algorithm. The output and corresponding performance index value J_1 (is the performance index by method 1) is 10.93 are shown in fig. 17. Similarly graph 2, 3, 4 indicates the output response of the system for the optimal controller gains [(K_p = 25, K_i^* = 43.01, K_d^* = 5.23), (K_p = 25, K_i^* = 71.95, K_d^* = 9.18), (K_p = 25, K_i^* = 53.59, K_d^* = 7.24)] obtained from a search space of stabilizing controller parameters by method 2, 3, 4 respectively and the corresponding cost functions J_2 = 7.07, J_3 = 8.08, J_4 = 8.03 are obtained.

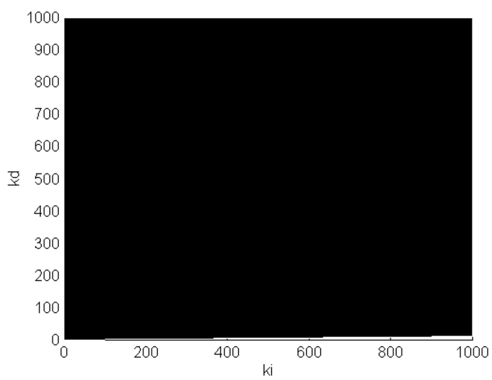

Fig. 13 The plane of (Ki, Kd) values for Kp =100 by method 3

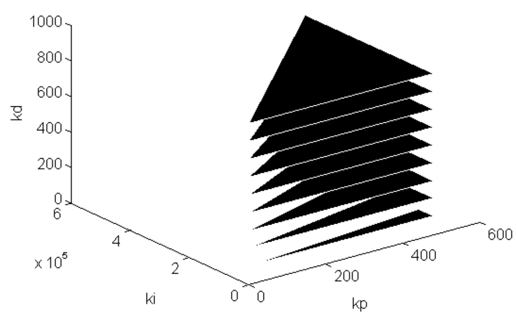

Fig. 14 The stabilizing set of (Kp,Ki,Kd) values by method 4

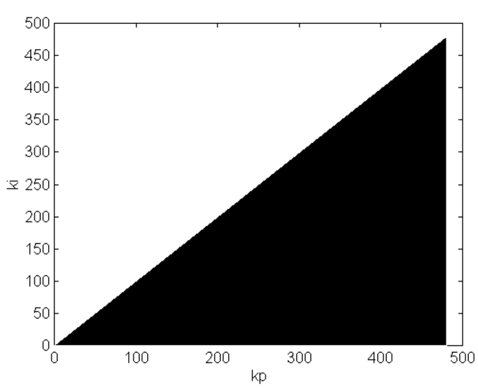

Fig. 15 The plane of (Kp, Ki) values for Kd = 1 by method 4

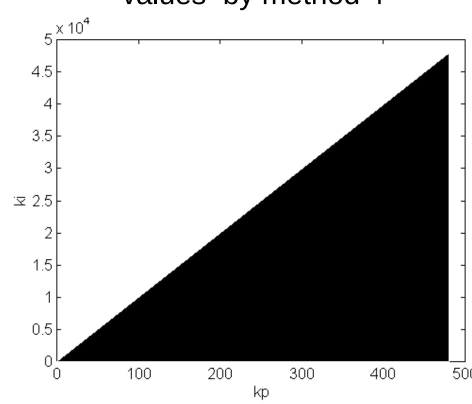

Fig. 16 The plane of (Kp, Ki) values for Kd = 100 by method 4

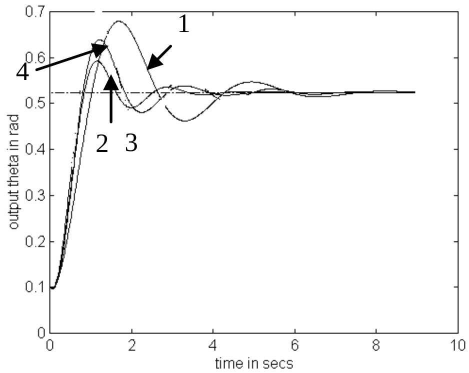

Fig. 17 Output θ of the system

We used the range of controller parameters (K_p = [9.82, $\propto$] K_i = [0.002, $\propto$] and K_d = [0.002, $\propto$]) obtained by method 3 which is also same as method 2 for obtaining an optimal controller parameter. Fig. 18 shows the output response of the system for fixed desired position θ_d = 30° with optimum value of controller gains [K_p^*=488.31, K_i^*=248.85, K_d^*= 22.80]. Fig 19 shows the output of the system for time-varying desired position θ_d = 1-cos(t) with optimum value of controller gains [K_p^*=499.03, K_i^*=493.39, K_d^*= 66.86].

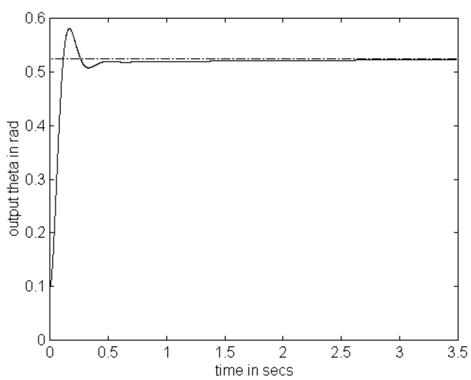

Fig. 18 Output θ of the system for θd= 0.524 rad Kp*=488.31, Ki*=248.85, Kd*= 22.80 J= 8.49.

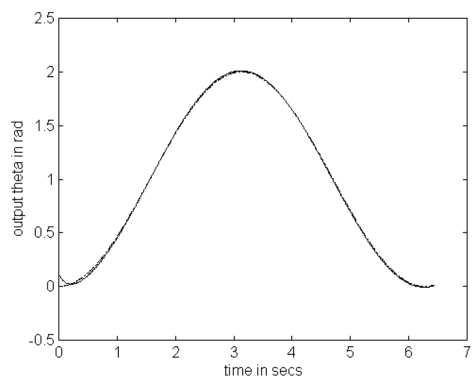

Fig. 19 Output θ of the system for θd= 1-cos(t) Kp*=499.03, Ki*=493.39, Kd*= 66.86, J= 0.14.

5. Conclusions

A class of stabilizing PID controllers for a single-link robot system is obtained by using different techniques. Concept of interval matrix, quadratic stability, stability boundary approach and Lyapunov function method are utilized to check the stability and subsequently the range of controller parameters are obtained using the procedure given in appendix I. The range of controllers obtained was used to obtain optimal controller parameters by employing genetic algorithm based optimization technique. Range of stabilizing controller parameters K_i and K_d for a fixed K_p obtained by using method 1 are conservative compared to other methods.

Appendix I

Steps to obtain the range of controller parameters (K_p, K_i, K_d).

Step (i) Any two values are assigned for K_p as 'a_L' (lower bound) and 'a_U'(upper bound).

Step (ii) K_i and K_d values are assigned arbitrarily from '-b' to 'b' and from '-c' to 'c' in steps of 1 respectively for the two values of K_p = 'a_L' and K_p = 'a_U'.

Step (iii) For these two set of controller gains($K_p = a_L$ & a_U) we need to check the following condition depending upon the methods adopted

- Method 1 Solve the Lyapunov equation (6), taking Q_0 as the identity matrix and obtain the P_0 matrix. Check for positive definiteness of Q^1 and Q^2 obtained by equation (8).
- Method 2: Check for quadratic stability by computing the vertices of PLDI systems satisfying equation (9).
- Method 3: Assume Q as identity matrix, obtain the P matrix solving the Lyapunov equation, $A^T P + PA = -Q$ for the system (4) with $\theta = 90^\circ$ which is same as solving equations (14) - (19) with $\theta = 90^\circ$. Check for positive definiteness of matrix P.

If step (iii) is satisfied K_p value is retained as a_L or a_U and proceed to step(vi) else go to step (iv).

Step (iv) If we are not getting a K_p value for the above range of K_i and K_d values we change the K_i value to -r*b to r*b and K_d value to -r*c to r*c excluding the previously checked range of -b to b and -c to c. (r > 1 is an integer quantity) Go to step (iii).

Step (v) If step (iii) is not satisfied after repeating step (iv), K_p is assigned new value of $a_L/2$ or $a_U/2$.

Step (vi) To get the exact value of K_p we assign two new values of K_p as $a_L - \Delta_a$ and $a_U + \Delta_a$ where $\Delta_a = 10^p$ and p is an integer. Numerical values of 'p' varies from positive to negative values (say p = …,-3,-2,-1,0,1,2,3,…). For each Δ_a starting from positive large values we repeat steps (ii) to (iv) until the step (iii) is
satisfied (ie till Q^1 and Q^2 matrices are negative definite for method 1, the systems defined by the corner matrices are quadratically stable for method 2, the P matrix is negative definite for method 3).

Step (vii) After obtaining K_p values the lower limit as well as upper limit we check whether K_p values exist from lower to upper limit. To do that steps ii to iv is repeated for K_p equal to lower value to K_p equal to upper value in small steps of length Δ_{Kp}. If the interval matrix P is positive definite the value of K_p is stored in an array. If the difference between each element of the above array is greater than Δ_{Kp} the range of K_p values is discontinuous. Steps (i) – (vii) are shown in flow-chart diagram.

Step (viii) Once K_p range is known the same steps (i) to (vi) are repeated to obtain the K_i values by arbitrarily assigning K_d values.

Step (ix) Once K_p and K_i values are known the same steps are repeated to get K_d values.

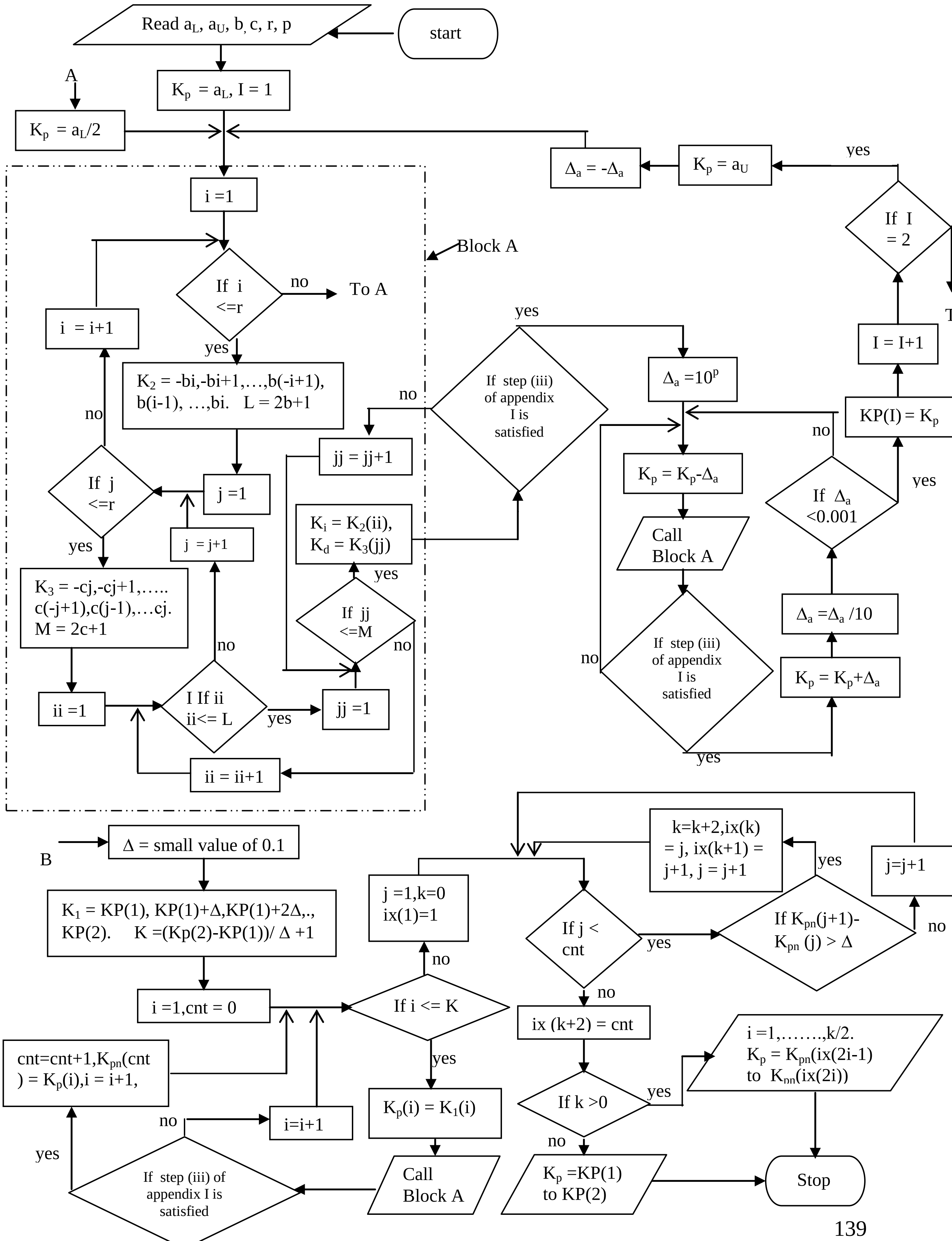

Fig. 20 Flow chart to obtain K_p range

Appendix II : Using polytopes to describe time-varying systems [12]
Let the strictly –proper linear time-varying system represented by the state variable realization

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t)$$

(A)

have k time-varying parameters $q_i(t)$, where $q_i^- \leq q_i(t) \leq q_i^+$, i = 1,….k
The system represented by {A(t),B(t),C(t)} in (A) is a polytope of linear systems and can defined by its vertices $\{(A_1, B_1, C_1), (A_2, B_2, C_2), \ldots\ldots\ldots.(A_L, B_L, C_L)\}$ where $L = 2^k$. For each time there is a set of L values $\{p_j\}$, such that

$$A = \sum_{j=1}^{L} p_j A_j , \qquad B = \sum_{j=1}^{L} p_j B_j , \quad C = \sum_{j=1}^{L} p_j C_j , \qquad \sum_{j=1}^{L} p_j = 1$$

and $p_j \geq 0$ for $j = 1,2….L$. the parameter space is a convex set, where the vertices are extreme values.

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