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Research on Prediction of Reverse Returned Logistics Based on Grey-Markov Model

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Abstract

In order to improve the prediction accuracy of reverse returned logistics, considering it has the characteristics of high volatility and uncertainty, the paper used the theory of Markov Chain to modify the result of Grey prediction. And a Grey-Markov prediction model was established. Several parallel region has been divided used the prediction curve of Grey prediction model as symmetric center. And each region was a state interval. A practical example show that the average relative error rate and the variance ratio of Grey-Markov prediction model was smaller, and the prediction accuracy is higher comparing with the Grey prediction model. The model is effective and feasible.

Keywords: *reverse logistics, Grey prediction, Markov Chain prediction, Grey-Markov prediction*

1. Introduction

With the increasingly social concern about environmental issues, the increase of landfill space and landfill costs, people gradually realized that the recycling of resources could be a way to solve the problem of environmental degradation and resource shortage. So more and more countries and regions have developed strict regulations of waste disposal and recycling products. Its core idea is to require producers complete a series of work include recovery, disposal, recycling waste products within the life of the product. As the first step of implementing reverse logistics, product recovery prediction is the most important. It plays a vital role for the smooth implementation of reverse logistics. The accurate of product recovery prediction will directly affect the recycling network design, recycling inventory planning and production planning arrangement. So products recovery prediction plays an important role in reverse logistics. Research on product recovery prediction have general guiding significance to reality of enterprise micro level or society macro level. As a kind of logistics system, the predict of reverse logistics have the commonness of logistics system, and their own characteristics. The key is to build an appropriate mathematical model to reflect the uncertainty of reverse logistics and to implement the activities.

The characteristics of logistics system demand mainly includes the space time, complexity, requirements regularity or regularity, dependent demand and independent demand. Because of there always are certain regularity or some characteristics, so the demand features can be predicted. The predicted principle mainly includes inertia, analogy principle, the related principle and principle of probability judgment. As a kind of logistics system reverse logistics, its demand and prediction has those characteristics mentioned above. At the same time, there are three kinds of uncertainty in reverse logistics: quality uncertainty, uncertainties of the number and time uncertainty. These uncertainties will affect the production plan, distribution planning and inventory management strategy of reverse logistics [1]. As a result, the reverse logistics prediction level is the key of reverse logistics organization.

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The research on reverse logistics prediction is not particularly deep at this stage. The methods mostly used for forward logistics [2-8]. The United States was the first country to study logistics forecast. A large number of logistics forecasting model is put forward in the 1970s [9]. In the past period of time, scholars study to product remanufacturing based on assumption that the demand and return back of products match the poison process or normal process [10-14]. J Marx-Gómez, C Rautenstrauch, A Nürnberger, R Kruse introduced an extended forecasting method to provide prognoses for return values (amount and time) of scrapped products to recycling is presented. Based on relevant influencing factors and product life cycle data the model has been applied to a case study (photocopiers) for evaluation and usability [15]. P Kelle, EA Silver introduce four different forecasting procedures, based upon different amounts of information. The methods are compared on a wide range of simulated data, including some cases based on empirical data obtained from industry [16]. P Georgiadis,D Vlachos,G Tagaras introduced how the lifecycles and return patterns of various products affect the optimal policies regarding expansion and contraction of collection and remanufacturing capacities. The model can be used to identify effective policies, to conduct various “what-if” analyses, and to answer questions about the long-term profitability of reverse supply chains with remanufacturing. The results of numerical examples with quite different lifecycle and return patterns show how the optimal collection expansion/contraction and remanufacturing contraction policies depend on the lifecycle type and the average usage time of the product [17].

The main problems about reverse logistics return prediction research are as follows: (1) Theory and practice is not consistent. The current study based on the assumption that the product mount of return has certain statistical regularity. But the actual product recycling is completely random and uncertain. It is difficult to determine the recycling is match a certain probability distribution. (2) The theory research about recovery system of reverse logistics is lack. Especially for the influence factors of recovery level analysis is inadequate. The importance of all the factors is not clear. (3) The Prediction model is lack of practicability. The promotion of the forecast model in theory research is not enough. The actual operation is mostly be determined by experience. The prediction precision is very low.

2. Mathematical Model

The paper will introduce three models applied in logistics prediction, include the Grey prediction model, Markov prediction model and Grey-Markov prediction model. Finally the paper will compare the three models, and the simulated test will prove the merits of the third model. Traditional GM (1,1) model is mainly used for short time, less data, small fluctuations, has the long-term trend prediction objects. But the degree of prediction fitting will be bed when the original data is volatile. Markov prediction is through reflect the influence degree of various random factors and the status of transfer between intrinsic regularity can be used to predict the future direction of the system, especially suitable for the raw data of the volatile sequence prediction. But it can only give the direction of the future, not the specific values. So is generally not used alone in the reverse logistics research. Grey-Markov prediction model could complement each other. It will be a very good fusion of two kinds of advantage.

2.1. The Grey Prediction Model

Gray theory was founded by the famous scholar professor Deng Julong in 1982 [18]. Its research object is uncertainty system with characteristics of partial information known and partial information unknown, small sample, less information and so on. It implement correct understanding and precise description to system operation rule through generation and using of partly known information. Then, it can be predicted scientifically.

Assume $X^{(0)}$ is the original sequence. $X^{(0)} = (X^{(0)}(1), X^{(0)}(2), \cdots, X^{(0)}(\mathrm{n}))$ generate the sequence $X^{(1)} = (X^{(1)}(1), X^{(1)}(2), \cdots, X^{(1)}(\mathrm{n}))$.

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i) \tag{1}$$

Formula 1 Upper marking 0 express raw data, 1 express once accumulation generation sequence. The process of generating sequence is 1 time accumulation generation number. The sequence is 1 time accumulation generation operator(1-AGO).

$$Z^{(1)}(k) = \frac{1}{2}[X^{(1)}(k) + X^{(1)}(k-1)], K = 2, 3, \cdots n \tag{2}$$

We called equation $X^{(0)}(k) + aZ^{(1)}(k) = \mathrm{b}$ as the original form of GM (1,1). GM is the abbreviation of the Grey Model. The first 1 is the order number of differential equation. The second 1 express the variable in the differential equation is 1.

$Z^{(1)} = (Z^{(1)}(2), Z^{(1)}(3), \cdots, Z^{(1)}(\mathrm{n}))$ is called close to mean generating sequence. In practice, the original form of GM (1,1) is the most commonly used.

In the differential equation, parameter a is the development coefficient, b is Grey action parameter. Assume $\hat{a} = (\mathrm{a}, \mathrm{b})^T$ is parameters sequence will be estimated. The least squares estimated parameter sequences of Grey differential equation $X^{(0)}(k) + aZ^{(1)}(k) = \mathrm{b}$ will meet:

$$\hat{a} = (\mathrm{B}^T B)^{-1} B^T Y, \mathrm{Y} = \begin{pmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \vdots \\ X^{(0)}(\mathrm{n}) \end{pmatrix}, \mathrm{B} = \begin{pmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -Z^{(1)}(n) & 1 \end{pmatrix} \tag{3}$$

The equation $dX^{(1)} / dt + aX = b$ is named of vernacular equation of the grey differential equation $X^{(0)}(k) + aZ^{(1)}(k) = \mathrm{b}$.

About the test of the GM(1,1) prediction model, assume the original sequence is $X^{(0)} = (X^{(0)}(1), X^{(0)}(2), \cdots, X^{(0)}(\mathrm{n}))$.

The prediction sequences is $\hat{X}^{(0)} = (\hat{X}^{(0)}(1), \hat{X}^{(0)}(2), \cdots, \hat{X}^{(0)}(\mathrm{n}))$.

The residual sequences is $\varepsilon^{(0)} = (\varepsilon(1), \varepsilon(2), \cdots, \varepsilon(\mathrm{n}))$

$= (X^{(0)}(1) - \hat{X}^{(0)}(1), X^{(0)}(2) - \hat{X}^{(0)}(2), \cdots, X^{(0)}(\mathrm{n}) - \hat{X}^{(0)}(\mathrm{n}))$.

The relative error sequence is $\Delta = (\frac{\varepsilon(1)}{X^{(0)}(1)}, \frac{\varepsilon(2)}{X^{(0)}(2)}, \cdots, \frac{\varepsilon(\mathrm{n})}{X^{(0)}(\mathrm{n})}) = \{\Delta_k\}_1^n$.

$\Delta_k = \frac{\varepsilon(k)}{X^{(0)}(k)}$ is prediction relative error of point k.

$\overline{\Delta} = (1/n)\sum_{k=1}^{n} \Delta_k$ is average relative error.

Given α, if $\overline{\Delta} < \alpha$, and $\Delta_n < \alpha$, the residual error test of model is up to standard.

$$\overline{X} = \frac{1}{n}\sum_{k=1}^{n} X^{(0)}(k) \tag{4}$$

$$S_1 = \frac{1}{n}\sum_{k=1}^{n} [X^{(0)}(k) - \overline{X}]^2 \tag{5}$$

$\overline{X}$ is the mean of $X^{(0)}$, and S_1 is the variance of $X^{(0)}$.

$$\overline{\varepsilon} = \frac{1}{n}\sum_{k=1}^{n} \varepsilon^{(0)}(k) \tag{6}$$

$$S_2 = \frac{1}{n}\sum_{k=1}^{n} [\varepsilon^{(0)}(k) - \overline{\varepsilon}]^2 \tag{7}$$

$\overline{\varepsilon}$ is the mean of $\varepsilon^{(0)}$, and S_2 is the variance of $\varepsilon^{(0)}$.

$C = S_2 / S_1$ is the mean square error ratio. Given $C_0 > 0$, if $C < C_0$, the variance ratio test is qualified.

Table 1 is used to precision inspection for level reference. It can be used as a reference for GM (1, 1) model test.

Table 1. Reference To Accuracy Test Level

Test Type	The Average Relative Error	Absolute Correlation Degree	The Mean Square Error Ratio	Small Error Probability
Critical Value Of Level 1	0. 01	0. 90	0. 35	0. 95
Critical Value Of Level 2	0. 05	0. 80	0. 50	0. 80
Critical Value Of Level 3	0. 10	0. 70	0. 65	0. 70
Critical Value Of Level 4	0. 20	0. 60	0. 80	0. 60

Generally it would be qualified if the average relative error is less than 0.2. The average relative error is smaller the accuracy of prediction model is higher. The absolute correlation degree express the degree of fitting between prediction model and actual model. The absolute correlation degree is higher, the fitting is better. Generally it should greater than 0.6. The mean square error ratio express the deviation from the mean of predicted and actual values. The ratio is smaller, the value is more concentrated, the oscillation amplitude is smaller. Generally it would be unqualified if the value is less than 0.8. The small error probability express the percentage of forecast data occurred in the normal probability range as the total number of original data. It is greater, the data deviation from the normal range is less and the forecast accurate is better. Generally it should greater than 0.6.

2.2. The Markov Prediction Model

Markov chain is a kind of random time series. Its main the feature is no aftereffect. Because the time and amount of the reverse logistics system are uncertain. In some cases, the future date is determined by The current data, the relationship between the future and the past is not close, therefore, Markov chain could be used to predict reverse logistics.

Markov chain can get the state probability distribution of next moment according the current state of system. Its basic principle is: according to the development of a system, time can be discrete for $n = 1, 2, 3, \cdots$. The state of every system can be expressed by available random and corresponding to certain probability named state corresponding to a certain probability. When the Markov process state is transferred from a moment to another moment with the transfer of probability named transfer probability. Markov chain prediction theory is based on Markov process. The analysis of the movement and the prediction mainly through the relations among A finite number of Markov states. The result of Markov prediction is a value range. And it is suitable for correcting description of prediction problem with random volatile.

Markov prediction model can be expressed as:

$$P_{t+1} = P_0[P^{(1)}]^{t+1} \tag{8}$$

Equation : P_{t+1} is the probability distribution of time $t+1$. P_0 is the unconditional probability distribution of initial moment. $P^{(1)}$ is the step transition probability matrix as equation (9).

$$P^{(1)} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix} \tag{9}$$

P_{ij} is the step transition probability that has nothing to do with the initial time. It means the probability transfer t_n moment state S_i to t_{n+1} moment state S_j by one step.

$$p_{ij} = P(X_{n+1} = S_j \mid X_n = S_i), 0 \le p_{ij} \le 1, \sum_{j=1}^{n} p_{ij} = 1 \tag{10}$$

The reverse logistics prediction model based on Markov process could be express by Figure 1.

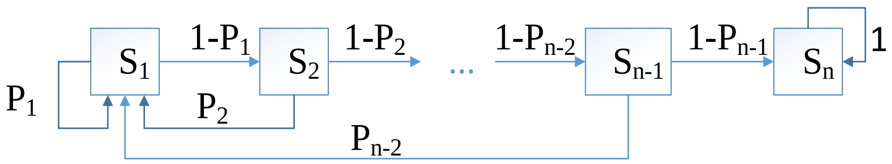

Figure 1. State Transition Process

The steps of Markov Chain prediction model are as follows:

(1) Dividing prediction object's state: This process should be according to actual circumstances. There is no fixed program. Some used the mean and variance of salvage value sequence include measured sequences and predicted sequences. Some used the relative value of the measured and predicted sequences. Some used the upper and lower boundary and the upper and lower limit line. In this paper the original value area will be divided into several parallel region as forecast curve the center of symmetry.

(2) Calculate the initial probability: Assume there are n status. Watch m periods, the state $S_i(i = 1, 2, \cdots, n)$ experienced m_i. So the frequency of S_i will be

$$f_i = m_i / m \tag{11}$$

We used $p_i = f_i$ as the probability frequency of S_i emergence.

(3) First calculate the frequency of state $S_i \to S_j$ (state S_i transfer to the state S_j).

$$f_{ij} = f(S_j \mid S_j) \tag{12}$$

Assume m_{ij} is the number of state S_i transfer to the state S_j ,

$$f_{ij} = m_{ij} \big/ m_i \tag{13}$$

Make frequency equal probability, $f_{ij} = p_{ij}$. A state transition probability matrix P could be obtained.

(4) Predict according to the state transition probability matrix in step (3): If the prediction object in state S_i, p_{ij} describes the probability in the future from S_i transfer to state $S_j(j = 1, 2, \cdots, n)$. Based on the principle of maximum probability, we choose the state match $Max\{p_{i1}, p_{i2}, \cdots, p_{in}\} = p_{ik}$ as prediction results. So we predict system will turn to next state S_k.

2.3. The Grey-Markov Prediction Model

The steps of Gray-Markov prediction model are as follows:

(1) Find the prediction curve According to shows the GM (1,1) model.

$$y(k) = \hat{X}^{(0)}(k+1) \tag{14}$$

(2) Divide the state: as forecast curve the center of symmetry, the original value area will be divided into several parallel region. Each region will form a state interval S_i. $S_i = [S_{1i}, S_{2i}](i = 1, 2, \cdots, n)$, $S_{1i} = y(k) \pm A_i$, $S_{2i} = y(k) \pm B_i$.

(3) Calculate the state transition probability.

$$p_{ij} = m_{ij} \big/ m_i \, (i, j = 1, 2, \cdots, n) \tag{15}$$

(4) p_{ij} is the probability of state S_i transfer to state S_j. n is the number of states be divided. m_i is the number of the sample original data fall state S_i according to certain probability. m_{ij} is the number of the original data sample from state S_i transfer to the state S_j. A state transition probability matrix P could be obtained. Then it could predict according to the transition probability.

(5) Determine the Gray-Markov prediction curve: When the next state be determined is S_k, the the predicted values change interval will be determined is $[S_{1k}, S_{2k}]$. We take the curve in the middle of the interval to be predict curve. The prediction curve equation is:

$$y(k+1) = \hat{X}^{(0)}(k+1) + (S_{1k} + S_{2k}) \big/ 2 \tag{16}$$

(6) Test the gray markov prediction curve: If it want the final result under the status of now, it can seek the ultimate state probability first. And the solution can be calculated according to the ultimate state probability. The ultimate state probability irefers to a

probability vector $\pi = (\pi_1, \pi_2, \cdots, \pi_n)$, and $\pi P = \pi$. $\pi_i \geq 0$, $\sum_{i=1}^{n} \pi_i = 1$. At this point, we called the probability vector ultimate state vector.

Calculation equation is as equation (17). The upper of the last column in the ultimate state is the maximum probability of next state appeared. Then it could get the eventually state transferred to.

$$\begin{cases} \sum_{i=1}^{n} \pi_i = 1 \\ \pi_i \geq 0, i = 1, 2, \cdots, n \\ \sum_{i=1}^{m} \pi_i p_{ik} = \pi_k, k = 1, 2, \cdots, n \end{cases} \tag{17}$$

3. Practical Example

About the calculating examples, the paper exclude the effect of industry factors to reverse logistics prediction. The simulation and verification of several prediction methods were made to the computer reverse logistics products recycling. Table 2 is the monthly summary data in 2014 of a company's computer reverse logistics.

Table 2. The Date of A Company's Computer Reverse Logistics

Month	1	2	3	4	5	6	7	8	9	10	11	12
Amount	1802	1531	1462	945	810	1023	1945	2286	842	943	1035	1382

Table 3 and Figure 1 is actual data and the two kinds of prediction results. It can be seen from the results that the Gray-Markov prediction method value is more close to the actual.

Table 3. The Prediction Result

Month	1	2	3	4	5	6	7	8	9	10	11	12
actual value	1802	1531	1462	945	810	1023	1945	2286	842	943	1035	1382
Grey	1802	1492	1368	1236	1024	986	1598	1753	1025	952	963	1207
Grey-Markov	1802	1501	1398	1012	977	993	1546	2073	984	954	992	1312

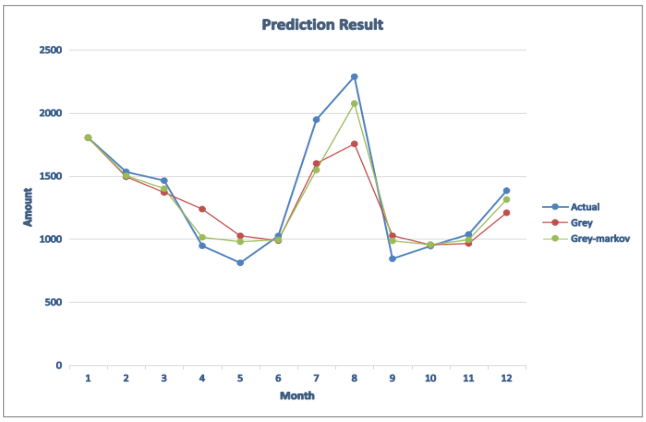

Figure 2. Prediction Results

Table 4-5 is the monthly residual error and error rate. It can be seen that the residual error and error rate of Gray-Markov prediction model is generally smaller than Gray prediction model.

Table 4. Residual Error

Month	1	2	3	4	5	6	7	8	9	10	11	12
Grey	0	39	94	−291	−214	37	347	533	−183	−9	72	175
Grey-Markov	0	30	64	−67	−167	30	399	213	−142	−11	43	70

Table 5. Relative Error Rate

Month	1	2	3	4	5	6
Grey(%)	0. 00	2. 55	6. 43	−30. 79	−26. 42	3. 62
Grey-Markov(%)	0. 00	1. 96	4. 38	−7. 09	−20. 62	2. 93
Month	7	8	9	10	11	12
Grey(%)	17. 84	23. 32	−21. 73	−0. 95	6. 96	12. 66
Grey-Markov(%)	20. 51	9. 32	−16. 86	−1. 17	4. 15	5. 07

Table 6. Test Result

Month	Average Relative Error	Mean Square Error Ratio
Grey	0. 12	0. 52
Grey-Markov	0. 08	0. 47

Test results of two kinds of prediction methods are listed in the table 6. Generally it would be unqualified if the average relative error is less than 0.2. The average relative error is smaller the accuracy of prediction model is higher. The mean square error ratio express the deviation from the mean of predicted and actual values. The ratio is smaller, the value is more concentrated, the oscillation amplitude is smaller. Generally it would be unqualified if the value is less than 0.8. It can be seen that two kinds of prediction methods are qualified. The Grey-Markov prediction model's accuracy is higher and the oscillation amplitude is smaller.

4. Conclusion

Considering the characteristics of Grey prediction and Markov Chain prediction, in order to improve the prediction accuracy of reverse returned logistics, the paper used the theory of Markov Chain to modify the result of Grey prediction. And a Grey-Markov prediction model was established. A practical example show that the average relative error rate and the variance ratio of Grey-Markov prediction model was smaller, and the prediction accuracy is higher comparing with the Grey prediction model. Further studies could be considered with combination with other prediction model, integrate optimization of prediction, inventory management and facility location.

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