

Method for Group Decision-Making of Multi-Stakeholders with Category Preference on Alternatives

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Abstract

This paper studies the issue of decision making on some alternatives with category preference of multiple decision makers in which interest conflicts exist and the weights are unknown. It puts forward the method to firstly calculate the attribute weight in each decision maker's circumstance through case-based learning and thus meet subjective preferences of different decision makers. Considering the interest conflicts between multiple decision makers, the fuzzy set theory is then used to optimize the weight of each decision makers to comply with objective requirements of group decision-making process. Finally, the total order relation of the alternative which does not only conform to the subjective preference information of the decision maker but also consist with the objective fact of group decision-making is obtained based on the calculation result of overall off-target distance for the alternative. The case indicates application procedures and method feasibility.

Keywords: Multiple decision makers; alternatives with category preference; multi-attribute; group decision-making; grey target decision making

1. Introduction

With the development of modern society, decision-making problems in social and economic activities have become more and more complicated and volatile. It is increasingly difficult to make effective decisions simply relying on a single decision maker [1-2]. More and more decision-making activities are dependent on joint involvement and determination of multiple decision makers. The shared decision-making of multi-stakeholders exists in various fields including large construction projects and public affairs management. Such group decision-making problems usually have the following three main features: ① Multiple stakeholders are involved. Multiple decision makers will face complicated uncertainties derived from the event itself or internal and external environment during the decision making, then each decision maker can identify the categories and make judgments relatively clear for some alternatives, or familiar fields and focus areas; ② Such group decision-making problems require cross-department, cross-industry and cross-discipline subjects to make group decisions. Due to the differences in physiology, psychology, behavior, background and culture, *etc.*, different decision makers have different understandings, thoughts and judgments on the same problem, so the difference existing in evaluation results. A certain degree of antagonism or concession may exist in the process of decision making; ③ The final decision result depends on the evaluation of each decision maker. It is unknown whether there is any master-slave relation, dependency or ordering relation between multiple decision makers before the formation of an objective and reasonable evaluation result. That is, the relationship or weights of the so-called multiple decision makers are uncertain or vague. With respect to the group decision-making problem of multi-stakeholders on alternatives

with category preference, it is urgent for group decision makers, for the purpose of development both in theory and practice to find out a decision-making method based on the correspondence between subjectivity and objectivity and seek for a satisfactory alternative with subjective concession and objective consistency for multiple subjects to coordinate the conflicts between multiple decision makers and help multiple decision makers head for the “dialogue” from “confrontation”, provide a foundation for correct, scientific and reasonable group decision-making.

As for practice study, it is essentially an indication of information uncertainty and interest conflict when multiple stakeholders make decisions simultaneously. The applications of such problems include sustainable development problems of large infrastructures [3], E-Business risk evaluation [4], urban planning and filtration of schemes [5], supply chain selection and optimization [6], project selection of social industrial projects [7], *etc.* It can be seen that the game method is often used to study the decision-making of multi-stakeholders in terms of practice study. There are few quantitative researches of data. In terms of theoretical study, similar problems have gained much attention from scholars. Firstly, regarding the study on category preference information: ① Focus on the study of preference information provided by decision makers for all alternatives (or evaluation objects) or some alternatives, for example, scholars have studied various methods of classification or sorting for alternatives in the field of multi-attribute decision-making according to the situations of comparison based on the reference [8], reference point and critical value based on different types of alternatives [9], strength and weakness relationship based on different alternative advantages [10], precise comparison [11] and fuzzy comparison [12] of different alternatives, *etc.*; ② Focus on expression patterns of experts for different category preferences of alternatives, for example, regarding the situations where different decision makers have comparison information on alternatives [13], have different preference structure information [14], or have incomplete preference information on the result of group decision-making [15], the category judgments of different decision makers are similar to some extent [16], and multiple decision makers provide a certain level of preference for alternatives [17], *etc.*, scholars have provided corresponding group decision-making methods. Secondly, regarding the study on weight determination of multiple stakeholders in group decision making: ① Adopt the simplest and most direct method of subjective weight determination [18-19] which requires the decision makers know each other well or the interest relationship is relatively clear; ② Adopt the method of objective weight determination [20-21]. The weight of the decision-making subject is determined through the preference information provided by the decision-maker. It should be mentioned that the weight determined by the method of objective weight determination is the dynamic weight of the decision makers which is often discussed in the academic circle; ③ Adopt the method of subjective and objective integrated weight determination [22-23] which is quite complex and requires the integration of subjective and objective information to correct the weight of the decision maker. In various similar decision-making theories and methods, many studies only discuss a single problem or individual analysis model or method. More often, values are assigned to the attribute weight or decision-maker weight beforehand in current studies. However, since the expertise, cognitive competence and practical experience of different decision makers are different in the circumstance of multiple decision makers, simply defining the attribute weight or decision-maker weight as known parameters in advance will greatly affect the result of group decision-making. It is questionable whether the result consists with the subjective category judgment of the decision-maker and conforms to objective requirements of group decision making.

This paper extracts a type of new group decision-making problem according to management practices: multiple decision makers have category preference for some

alternatives and the issues of benefit restriction and unknown weights exist in these subjects, thus it boils down to the group decision-making problem of multi-stakeholders on alternatives with category preference. Optimize the attribute weight and the weight of each decision-maker by relevant principles of case-based learning method and fuzzy set under the category preference information respectively. Find out the master slave relation of multiple decision makers from group decision-making problems in the circumstance with interest relationship existing originally by meeting the subjective category preference information of different decision makers and objective facts of group decision-making in succession, and figure out a satisfactory method by which multiple decision makers can compromise, objectively reflect the relative importance of each attribute in each subject circumstance and the decision-making level of each decision-maker. Finally, verify the method with a specific case of decision making in urban planning. The study results have a certain theoretical value and practical significance for complicated multi-attribute group decision making.

2. Problem Description

2.1. Problem Description and Thinking

Generally, the typical, multi-person and multi-attribute decision-making issue is that different decision makers $e_p, p = 1, 2, \dots, k$ rank the alternatives in accordance with a certain rule based on the given index $\omega_j, j = 1, 2, \dots, m$ and the decision alternative $z_i, i = 1, 2, \dots, n$. The multi-attribute classification decision is to classify alternatives into specific categories and the alternatives in the same category are of similar nature. In some decision-making cases, due to the great number of alternatives to be evaluated and the limitation of specialty or research interest, different decision makers often provide relatively definite category judgment information for some alternatives which can be called as judgment information for groups with category preference. These limited samples with relatively definite category information can be studied in line with the analysis method of case study, thus realizing ranking or classification of all alternatives based on conclusions of sample study. This method has a less strict requirement for the information extraction of decision markers and is easier for decision makers to submit and receive relevant information in their own specialties or their focus areas locally. Meanwhile, in the circumstance of multiple stakeholders, the levels of decision makers are not clear and the relation is vague, therefore the weight determination of multiple decision makers should fully reflect the decision-making levels of different decision makers which should not be treated in the same way.

On this basis, this paper describes the multi-attribute group decision-making on alternatives with category preference in the multi-stakeholders circumstance as below: n alternatives form the alternative set $Z = \{z_i | i = 1, 2, \dots, n\}$; m attributes form the attribute set $W = \{\omega_j | j = 1, 2, \dots, m\}$; k decision makers participate in decision-making and form the decision-making group $E = \{e_p | p = 1, 2, \dots, k\}$. When the decision-maker e_p makes a judgment, the weight vector of each attribute that he/she considers is $w^p = (\omega_1^p, \omega_2^p, \dots, \omega_m^p)$.

For $\forall j \in \{1, 2, \dots, m\}$, $\omega_j^p \geq 0$ and $\sum_{j=1}^m \omega_j^p = 1$; λ_p is the weight of No. p decision-maker, and the boundary of subject is unclear, that is, the weight vector λ_p of each decision maker is unknown. For $\forall p \in \{1, 2, \dots, k\}$, $\lambda_p \geq 0$, $\sum_{p=1}^k \lambda_p = 1$; r_{ij}^p is the attribute judgment value given by No. p decision-maker for No. i alternative under No. j index, without loss of

generality. Assume that the attributes in Z are all very large (benefit type). Figure 1, shows the decision-making process of such problems.

Such problems have the following difficulties: (1) The preference information on alternatives from different decision makers is category preference information. A way should be found to balance the decision information and category preference information given by decision makers for all alternatives to determine whether the attribute weights consist with their subjective preferences or not; (2) Interest conflicts exist in difference decision makers, and the master slave relation is uncertain and quite vague. The method to use the uncertainty and vagueness of multiple decision makers should be developed to make the group judgment more comply with the requirements of objective facts.

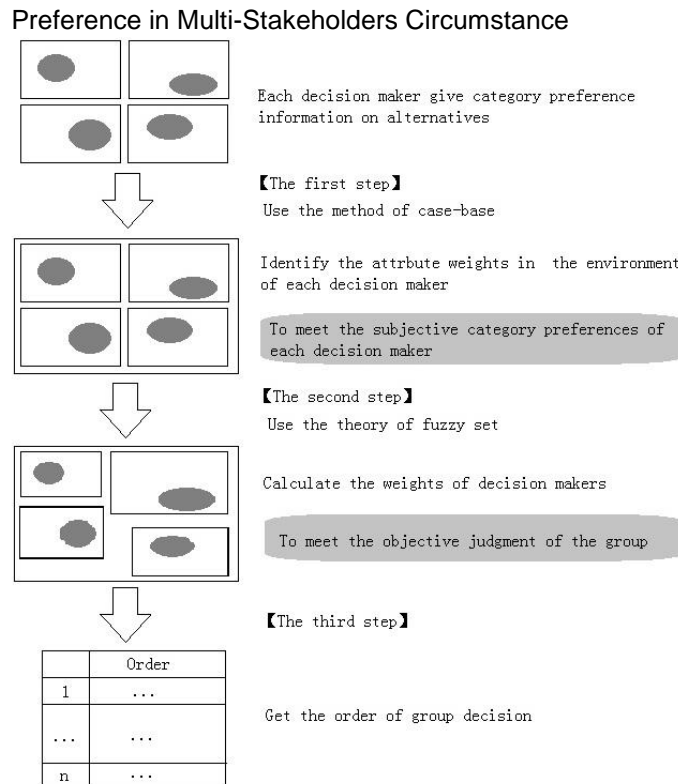


Figure 1. Group Decision-Making Process on Alternatives with Category

2.2. Methods and Steps

In order to realize the final ranking or classification of group decisions, the attribute weight in specific decision-maker circumstance and the weight of each decision-maker should be determined respectively. Since both weights are unknown, the method of case-based learning is first used in this paper to determine attribute weight vectors in different subject circumstances.

2.2.1. Learning Model of Attribute Weight Based on Category Preference Information: The grey target decision making has the principle of “non-uniqueness” in the grey system theory, reflecting the idea that the target is approachable, the plan can be improved and the approach can be optimized. The grey target decision making has been fully applied and embodied in the decision-making theory. This paper calculates the overall off-target distance of the alternative relative to positive and negative ideal points by the method of grey target decision making, and provides the case-based learning based

on the category preference information, thus to obtain attribute weight vectors in different subject circumstances.

(1) Correction of positive and negative target centers and calculation of overall off-target distance.

In grey target decision making, the selection of reference point is very critical. When positive and negative target centers are used for decision-making analysis, they should be set according to the features of the decision-making circumstance. In case of group decision-making, possible alternatives in the process of target center setting should be considered to set the positive and negative target centers of the determined group.

Definition 1: Assume that alternative z_i^p is not in the decision alternative set of decision maker e^p , but may affect the existing evaluation plan or may occur in later comprehensive decisions, so the alternative is the potential alternative of the decision maker e^p .

After the analysis of potential alternative set according to Definition 1, the decision information of other decision makers should be considered fully during the setting of positive and negative target centers to consider the decision information provided by all decision makers comprehensively. The improvement is specifically as below:

Step 1: Set positive and negative target centers under the evaluation set of decision maker e^p . Assume that the positive target center and negative target center of the benefit type index set r_j^p based on the existing alternative set of decision maker e^p is $r_{0j}^{p+} = \max\{r_{ij}^p \mid 1 \leq i \leq n, 1 \leq j \leq m\}$ and $r_{0j}^{p-} = \min\{r_{ij}^p \mid 1 \leq i \leq n, 1 \leq j \leq m\}$ respectively.

Step 2: Find out the potential positive and negative target centers under the group evaluation set. According to the evaluation of all decision makers, analyze whether an individual decision maker is possible to have better or worse potential alternative set or not, and meanwhile, determine the potential positive and negative target centers under the evaluation set of each decision maker based on the suggestions given by all decision makers.

Assume that the positive target center and negative target center of the benefit type index set a_j^{p*} based on the potential alternative set of the individual decision maker e^p is r_j^{p+*} and r_j^{p-*} respectively, so $r_j^{p+*} = \max\{r_{ij}^{p*} \mid 1 \leq i \leq n^*, 1 \leq j \leq m\}$ and $r_j^{p-*} = \min\{r_{ij}^{p*} \mid 1 \leq i \leq n^*, 1 \leq j \leq m\}$.

The decision alternative set of a single decision makers and alternative sets of all decision makers form all possible decision alternatives. If there is no potential alternative set for a single decision maker e^p , the positive target center r_j^+ and negative target center r_j^- of the decision maker e^p are r_{0j}^{p+} and r_{0j}^{p-} respectively; if there is potential alternative set for a single decision maker e^p , the corrected positive target center and negative target center should be $r_j^+ = \max(r_{0j}^{p+}, r_j^{p+*})$ and $r_j^- = \min(r_{0j}^{p-}, r_j^{p-*})$ respectively.

Definition 2: Assuming $r_j^{p+} = \max\{r_{ij}^p \mid 1 \leq i \leq n, 1 \leq j \leq m\}$, $r_j^{p+} = \{r_1^{p+}, r_2^{p+}, \dots, r_m^{p+}\}$ is called as the optimal effect vector of grey target decision making, representing the positive target center of the decision matrix of subject p under attribute j . Assuming $r_j^{p-} = \min\{r_{ij}^p \mid 1 \leq i \leq n, 1 \leq j \leq m\}$, $r_j^{p-} = \{r_1^{p-}, r_2^{p-}, \dots, r_m^{p-}\}$ is called as the worst effect vector of grey target decision making, representing the negative target center of the decision matrix of subject p under attribute j .

The corresponding positive off-target distance, negative off-target distance and positive and negative off-target distances are as below:

Definition 3: The positive off-target distance of the decision-making subject p for alternative z_i is:

$$\varepsilon_i^{p+} = \omega_1 d(r_{i1}^p, r_1^{p+}) + \omega_2 d(r_{i2}^p, r_2^{p+}) + \dots + \omega_m d(r_{im}^p, r_m^{p+}) = \sum_{j=1}^m \omega_j d(r_{ij}^p, r_j^{p+}) \quad (1)$$

The negative off-target distance of the decision-making subject p for alternative z_i is:

$$\varepsilon_i^{p-} = \omega_1 d(r_{i1}^p, r_1^{p-}) + \omega_2 d(r_{i2}^p, r_2^{p-}) + \dots + \omega_m d(r_{im}^p, r_m^{p-}) = \sum_{j=1}^m \omega_j d(r_{ij}^p, r_j^{p-}) \quad (2)$$

The distance between positive target center and negative target center of the decision-making subject p for alternative z_i is:

$$\varepsilon_i^{p0} = \omega_1 d(r_1^{p+}, r_1^{p-}) + \omega_2 d(r_2^{p+}, r_2^{p-}) + \dots + \omega_m d(r_m^{p+}, r_m^{p-}) = \sum_{j=1}^m \omega_j d(r_j^{p+} - r_j^{p-}) \quad (3)$$

In reference [24], the projection of the connecting line between positive target center and negative target center of decision alternative is defined as the overall off-target distance, therefore:

Definition 4: The projection of the connecting line between positive target center and negative target center of decision alternative z_i is defined as the overall off-target distance of the decision-making subject p for the alternative z_i , as below:

$$\varepsilon_i^{p*} = \frac{(\varepsilon_i^{p+})^2 + (\varepsilon_i^{p0})^2 - (\varepsilon_i^{p-})^2}{2\varepsilon_i^{p0}} \quad (4)$$

The overall off-target distance here shows the quality of the effect vector: The smaller the overall off-target distance of z_i , the better the decision alternative will be; the larger the overall off-target distance of z_i , the worse the decision alternative will be.

According to reference [24], the distance ε_i^0 between positive target center and negative target center of each alternative is a constant, so the formula above can be changed into:

$$\varepsilon_i^p = (\varepsilon_i^{p+})^2 + (\varepsilon_i^{p0})^2 - (\varepsilon_i^{p-})^2 \quad (5)$$

(2) Determination method of attribute weight based on case study.

For No. i decision alternative under the judgment of decision-making subject p , it is required to adjust the weight ω_j^p ($j = 1, 2, \dots, m$) to minimize the overall off-target distance, expressed as below:

$$\begin{aligned} \min \varepsilon^p &= \min(\varepsilon_1^p, \varepsilon_2^p, \dots, \varepsilon_n^p) \\ \text{s.t.} \sum_{j=1}^m \omega_j^p &= 1, \omega_j^p \geq 0, j = 1, 2, \dots, m \end{aligned}$$

In consideration of the fair competition between all alternatives, the following expression can be obtained according to Expression (5):

$$\min \beta^p = \min \sum_{i=1}^n \varepsilon_i^p = \min \sum_{i=1}^n \sum_{j=1}^m \omega_j^2 d(r_{ij}^p, r_j^{p+}) + \sum_{i=1}^n \sum_{j=1}^m \omega_j^2 d(r_{ij}^{p+}, r_j^{p-}) - \sum_{i=1}^n \sum_{j=1}^m \omega_j^2 d(r_{ij}^p, r_j^{p-})$$

Definition 5: If a decision maker e_p believes l ($l < n$) alternatives in alternative set $Z = \{z_i | i = 1, 2, \dots, n\}$ are similarly excellent and should be regarded as a equivalent

category, these alternatives of the equivalent category deemed by the decision maker e_p is recorded as $Z^{p\#} = \{z_i^{p\#} | i = 1, 2, \dots, l\} (Z^{p\#} \subset Z)$, so its relation can be expressed as $z_1^{p\#} \approx z_2^{p\#} \approx \dots \approx z_l^{p\#}$, namely, the overall off-target distance of any two alternatives in alternative set $Z^{p\#}$ is equivalent, and the relation can be expressed as $\varepsilon_1^{p\#} \approx \varepsilon_2^{p\#} \approx \dots \approx \varepsilon_l^{p\#}$.

For any two alternatives $z_i^{p\#}$ and $z_{i+1}^{p\#}$ therein, to make $\varepsilon_i^{p\#} \approx \varepsilon_{i+1}^{p\#}$, $\frac{\varepsilon_i^{p\#}}{\varepsilon_{i+1}^{p\#}} \approx 1$ is required.

Introduce the fluctuating variable $0 \leq \rho \leq 1$. So $\frac{\varepsilon_i^{p\#}}{\varepsilon_{i+1}^{p\#}}$ can fluctuate in a certain range as

below: $\rho \leq \frac{\varepsilon_i^{p\#}}{\varepsilon_{i+1}^{p\#}} \leq \frac{1}{\rho}$, on this basis, the following expression can be obtained:

$$\left\{ \begin{array}{l} \rho \leq \frac{\varepsilon_i^{p\#}}{\varepsilon_{i+1}^{p\#}} \leq \frac{1}{\rho} \\ i = 1, 2, \dots, l-1, z_i^{p\#} \in Z^{p\#}, Z^{p\#} \subset Z \end{array} \right.$$

Regarding the value of ρ , ρ reflects the equivalence of the overall off-target distance of any two alternatives in alternative set $Z^{p\#}$, namely the similarity precision of any two alternatives in alternative set $Z^{p\#}$: when ρ increases, the similarity precision of any two alternatives in $Z^{p\#}$ becomes higher, that is, they are more similar; when ρ decreases, the similarity precision of any two alternatives in $Z^{p\#}$ becomes lower, that is, they are less similar; especially, when $\rho = 1$, there is no difference for two alternatives in $Z^{p\#}$ in terms of excellence, that is, they are exactly the same in terms of excellence.

If the category preference given by decision-maker p includes the category preference mentioned in Definition 5, a linear programming model (M-1) can be established:

$$\begin{array}{l} \min \beta^p = \min \sum_{i=1}^n \varepsilon_i^p \\ s.t. \left\{ \begin{array}{l} \rho \leq \frac{\varepsilon_i^{p\#}}{\varepsilon_{i+1}^{p\#}} \leq \frac{1}{\rho} \\ i = 1, 2, \dots, l-1, z_i^{p\#} \in Z^{p\#}, Z^{p\#} \subset Z \end{array} \right. \end{array}$$

According to Expression (1) – (5), the above objective programming model can be converted into (M-2):

$$\begin{array}{l} \min \beta = \min \sum_{i=1}^n \sum_{j=1}^m \omega_j^2 d(r_{ij}^p, r_j^{p+}) + \sum_{i=1}^n \sum_{j=1}^m \omega_j^2 d(r_{ij}^{p+}, r_j^{p-}) - \sum_{i=1}^n \sum_{j=1}^m \omega_j^2 d(r_{ij}^p, r_j^{p-}) \\ s.t. \left\{ \begin{array}{l} \rho \leq \frac{\sum_{j=1}^m \omega_j^2 d(r_{ij}^{p\#}, r_j^{p+}) + \sum_{j=1}^m \omega_j^2 d(r_j^{p+}, r_j^{p-}) - \sum_{j=1}^m \omega_j^2 d(r_{ij}^{p\#}, r_j^{qj-})}{\sum_{j=1}^m \omega_j^2 d(r_{(i+1)j}^{p\#}, r_j^{p+}) + \sum_{j=1}^m \omega_j^2 d(r_j^{p+}, r_j^{p-}) - \sum_{j=1}^m \omega_j^2 d(r_{(i+1)j}^{p\#}, r_j^{p-})} \leq \frac{1}{\rho} \\ i = 1, 2, \dots, l-1, z_i^{p\#} \in Z^{p\#}, Z^{p\#} \subset Z, \sum_{i=1}^m \omega_j^p = 1, \omega_j^p \geq 0, j = 1, 2, \dots, m, i = 1, 2, \dots, n \end{array} \right. \end{array}$$

Theorem 1: Model M-2 must have the optimal solution.

Demonstration: The feasible region of Model M-2 is bounded convex hull, and the objective functions are continuous and have the maximum ($\omega_j < 1$). As the objective functions are continuous functions with the maximum, the feasible region with bounded

convex hull must have the optimal solution, and therefore Model M-2 must have the optimum solution.

Solve the above model through software lingo11.0, and the attribute weight vectors in different subject circumstances can be obtained: $w^p = (\omega_1^p, \omega_2^p, \dots, \omega_m^p)$

2.2.2. Multi-Stakeholders Weight Optimization Method: In the multi-stakeholders circumstance, different decision makers often consider lots of factors from the perspective of their own interests in the process of decision making. Since their expertise, cognitive competence and practical experience, *etc.* are different and the relationship therein is complicated, the weight determination of decision-making subjects, in the circumstance of multiple decision-maker, should fully reflect the decision-making levels and interests of decision makers which should not be treated in the same way. For these reasons, the weight determination is also of high uncertainty and vagueness. Based on the features of such fuzzy information, a method of multiple decision-maker weight assignment based on the fuzzy set theory is provided below.

In fact, the calculation process of the decision-maker weight can be seen as a special form of reference [25]. According to reference [25], when fuzzy information is used, all information during the decision making should be fully used. With all available information, form a fuzzy but relative consistent opinion by using triangular fuzzy numbers. In this paper, the method of scoring is used for assignment of each alternative, and later the minimum, mean and maximum from the scores are used as three measurements of triangular fuzzy numbers. In this study, as the attribute weight in each subject circumstance has been obtained through the steps in 2.2.1, it is more reasonable and scientific to represent the scoring of each alternative through the overall off-target distance of each alternative in each subject circumstance. Therefore, the application in reference [25] is improved in this paper: take the minimum overall off-target distance, maximum overall off-target distance and mean overall off-target distance in each subject circumstance as three measurements of triangular fuzzy numbers. This method uses the calculation result based on the subject category preference information in 2.2.1 effectively and shows more respect for the subjective evaluation of the decision maker than the method of direct scoring. And it is easier for multiple decision makers to receive the final result.

Convert the overall off-target distance ε_i^p of alternative z_i for decision maker e_p into the triangular fuzzy number:

$$\tilde{\varepsilon}_i^p = (\varepsilon_1^p, \varepsilon_2^p, \varepsilon_3^p) = \{[\min(\varepsilon_1^p, \varepsilon_2^p, \dots, \varepsilon_n^p)], [\sum_{i=1}^m \varepsilon_i^p / n], [\max(\varepsilon_1^p, \varepsilon_2^p, \dots, \varepsilon_n^p)]\}$$

Where ε_1^p is the overall off-target distance of the worst alternative thought by the decision maker e_p which represents pessimistic estimate; ε_3^p is the overall off-target distance of the optimal alternative thought by the decision maker e_p which represents optimistic estimate; ε_2^p is the mean overall off-target distance of all alternatives thought by the decision maker e_p . According to relevant demonstrations in reference [25], the objective weight assignment of uncertain multi-subject should be as below:

$$\lambda = (B^{-1}I) / I^T B^{-1}I \tag{6}$$

Where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)^T$ is the weight of each subject, $I = (1, 1, \dots, 1)^T$, and the matrix B is as below:

$$B = (b_{ij})_{k \times k} = \begin{bmatrix} (k-1)(\sum_{l=1}^3 \varepsilon_{1l}^2) & -\sum_{l=1}^3 \varepsilon_{1l} \varepsilon_{2l} & \dots & -\sum_{l=1}^3 \varepsilon_{1l} \varepsilon_{kl} \\ -\sum_{l=1}^3 \varepsilon_{2l} \varepsilon_{1l} & (k-1)(\sum_{l=1}^3 \varepsilon_{2l}^2) & \dots & -\sum_{l=1}^3 \varepsilon_{2l} \varepsilon_{kl} \\ \dots & \dots & \dots & \dots \\ -\sum_{l=1}^3 \varepsilon_{kl} \varepsilon_{1l} & -\sum_{l=1}^3 \varepsilon_{kl} \varepsilon_{2l} & \dots & (k-1)(\sum_{l=1}^3 \varepsilon_{kl}^2) \end{bmatrix}$$

Therefore, the weights of different decision makers can be obtained. Since the data source of the method is the category judgment information of each subject, the weights of decision makers obtained have the following three features: ① They can be regarded as the weight solutions received or compromised by each subject on the basis of the category judgment information of the decision maker, thus avoiding many unfavorable factors of subjective weighting; ② In multi-stakeholders circumstance, they can be considered as the comprehensive reflection of interests of different subjects and have objectivity and acceptability; ③ They can be seen as the basis of correctness or reliability for various decision maker category judgment information, thus avoiding some inherent judgment biases that may exist in group decision making (for example, the weight calculation result may be contrary to previous weight judgment result which has been embodied in the method comparison in the case part of this paper).

2.2.3. Group Preference Aggregation in Multi-Stakeholders Circumstance: Take the overall off-target distance of each alternative as the objective function of such problem of group decision-making in multiple decision maker circumstance, and apply the method of linear weighting, then the overall off-target distance of each alternative z_i in case of group decision-making can be obtained:

$$\varepsilon(z_i) = \sum_{p=1}^k \lambda_p \varepsilon_i^p = \sum_{p=1}^k \lambda_p ((\varepsilon_i^{p+})^2 + (\varepsilon_i^{p0})^2 - (\varepsilon_i^{p-})^2) \quad (7)$$

As the weight vector $W^p = (\omega_1^p, \omega_2^p, \dots, \omega_m^p)$ under each decision maker and the weight λ_p of the decision-making expert are all known, with Model M-2 and Expression (6), the total order relation of the alternative set can be established by using Expression (7).

Therefore, the steps of the method in this paper can be summarized as follows:

Step 1: Correct the positive and negative target centers of the group according to the method in 2.2.1 (1), and calculate the distance of each alternative to the positive and negative target centers of the group. Calculate the attribute weight in each decision maker circumstance according to 2.2.1 (2).

Step 2: Determine the weight of each decision maker according to 2.2.2, and judge the category preference information credibility of the subject based on the result.

Step 3: Calculate the overall off-target distance of each alternative in the circumstance of group decision-making according to 2.2.3, and figure out the total order relation of the alternative according to the overall off-target distance.

3. Case

Urban planning issue is one of typical issues in public management affairs. The evaluation indexes of urban planning generally include: environment level w_1 , economic development level w_2 , cultural construction level w_3 , ROI level w_4 and traffic system index level w_5 . Generally, the government agencies e^1 , experts in urban planning e^2 , constructor representatives e^3 and the public e^4 participated in the investigation and

practice on behalf of multiple decision makers before the planning. The four decision makers represent the interests of respective party, with unclear boundary. The four decision makers offered evaluation information (already normalized, as shown in Table 1) on relevant indexes in their field of view for alternative $z_1 - z_5$, respectively, and meanwhile, different decision-making subjects provided their own category preference information, as shown in the grey part of Table 1. Determine the weight according to the alternatives in this paper, and rank the alternatives.

Table 1. Attribute Values and Category Preference Information Provided by Multiple Subjects

program	w ₁	w ₂	w ₃	w ₄	w ₅	program	w ₁	w ₂	w ₃	w ₄	w ₅
Decision maker e^1 gave a property value and category preference						Decision maker e^2 gave a property value and category preference					
z_1^1	0.9675	0.3130	0.3575	0.9581	0.2177	z_1^2	1	0.5340	0.4894	0.9881	0.2348
z_2^1	0.6836	1	1	0.3336	1	z_2^2	0.6762	0.9156	0.8775	0.4352	1
z_3^1	0.4887	0.4957	0.3944	0.6905	0.3864	z_3^2	0.4602	0.1614	0.1861	0.7500	0.1579
z_4^1	0.6741	0.3457	0.2053	1	0	z_4^2	0.8971	0.3362	0.2151	1	0
z_5^1	0.0872	0.3652	0.0443	0.3600	0.4737	z_5^2	0.0450	0.0913	0.0679	0.2157	0.1447
z_6^1	0	0	0.3752	0	0.4617	z_6^2	0.1224	0	0.5784	0.1003	0.3247
Decision maker e^3 gave a property value and category preference						Decision maker e^4 gave a property value and category preference					
z_1^3	1	0.6989	0.5820	0.6827	0.6534	z_1^4	1	0.4251	0.4675	0.7889	0.8466
z_2^3	0.5807	0.1858	0.2260	0.4306	0.1232	z_2^4	0.6758	1	1	0.3026	1
z_3^3	0.4087	0.4946	0.3748	0.5704	0.3879	z_3^4	0.5789	0.6857	0.5784	0.385	0.5794
z_4^3	0.1785	0.0783	0.1551	0.4372	0.2105	z_4^4	0.1299	0.2989	0.0384	0.6633	0.2919
z_5^3	0.1946	0.0293	0	0.3313	0.1962	z_5^4	0.2441	0.3761	0.2999	0.3871	0.4821
z_6^3	0	0	0.3875	0	0.4789	z_6^4	0.1427	0.0293	0.1388	0.4065	0.4103

Step 1: Correct the positive and negative target centers of the group, $r_{0j}^{(1-4)+} = \{1,1,1,1,1\}$, $r_{0j}^{(1-4)-} = \{0,0,0,0,0\}$, and calculate the distance from the attribute value to the positive and negative target centers of each alternative under different attributes.

Use the method in 2.2.1 (2), take $\rho = 0.95$, and solve the attribute weight vectors on alternatives with category preference information for all subjects, respectively as below:

Decision-making subject e^1 : $\omega^1 = (0.0549, 0.3567, 0.1108, 0.3157, 0.1619)$;

Decision-making subject e^2 : $\omega^2 = (0.2543, 0.0990, 0.0956, 0.0970, 0.4541)$;

Decision-making subject e^3 : $\omega^3 = (0.4355, 0.0903, 0.0421, 0.0660, 0.3661)$;

Decision-making subject e^4 : $\omega^4 = (0.1215, 0.1503, 0.1074, 0.2288, 0.3920)$;

Step 2: Calculate the vector of overall off-target distance ε_i^p of each alternative z_i in the circumstance of each decision-making subject e_p by using the method in 2.2.2, thus to convert the overall off-target distance ε_i^p of the decision maker e_p on alternative z_i into corresponding triangular fuzzy number, as below:

$\tilde{\varepsilon}_i^1 = (0.1347, 0.2843, 0.1347)$

$\tilde{\varepsilon}_i^2 = (0.0564, 0.3765, 0.5259)$

$\tilde{\varepsilon}_i^3 = (0.1021, 0.4264, 0.5462)$

$\tilde{\varepsilon}_i^4 = (0.0826, 0.2292, 0.3324)$

Matrix B:

$$B = (b_{ij})_{4 \times 4} = \begin{bmatrix} 2.4793 & -0.8523 & -0.81121 & -0.89407 \\ -0.8523 & 2.6599 & -0.84575 & -0.91821 \\ -0.81121 & -0.84575 & 2.4386 & -0.88532 \\ -0.89407 & -0.91821 & -0.88532 & 2.9876 \end{bmatrix},$$

$$B^{-1} = (b_{ij})_{4 \times 4} = \begin{bmatrix} 8.7027 & 8.1060 & 8.4671 & 7.6047 \\ 8.1060 & 8.1047 & 8.1714 & 7.3381 \\ 8.4671 & 8.1714 & 8.8438 & 7.6660 \\ 7.6047 & 7.3381 & 7.6660 & 7.1375 \end{bmatrix}$$

According to Expression (6), the weight vector of the decision maker can be obtained: $\lambda = (0.2579, 0.2489, 0.2600, 0.2332)$. Namely, the weight of government agencies e^1 is 0.2579; the weight of experts in urban planning e^2 is 0.2489; the weight of constructor representatives e^3 is 0.2600; and the weight of the public e^4 is 0.2332.

Step 3: Calculate the overall off-target distance of each alternative in multiple decision maker circumstance according to 2.2.3, and obtain the optimum ranking of each alternative in circumstance of each decision maker and the ranking of the group as shown in Table 2.

Table 2. Utility Values and Ranking of Alternatives in the Field of View of Each Decision Maker, Final Group Utility Values and Ranking of Alternatives

z_i^p	Decision maker 1		Decision maker 2		Decision maker 3		Decision maker 4		Group	
	Comprehens-ive off-target distance	Order	Comprehens-ive off-target distance	Order	Comprehens-ive off-target distance	Order	Comprehens-ive off-target distance	Order	Comprehens-ive off-target distance	Order
z_1	0.2399	2	0.3343	2	0.1021	1	0.1075	2	0.1968	2
z_2	0.1347	1	0.0564	1	0.4151	3	0.0826	1	0.1759	1
z_3	0.2404	3	0.4555	4	0.4026	2	0.2300	3	0.3330	3
z_4	0.2406	4	0.4531	3	0.5437	4	0.3311	5	0.3943	4
z_5	0.3459	5	0.5259	6	0.5470	6	0.2900	4	0.4297	5
z_6	0.5038	6	0.4362	5	0.5462	5	0.3324	6	0.4580	6

1. Method Comparison:

This paper has two main innovations: 1. The category preferences of multiple decision makers on alternatives are specially considered to obtain the attribute weight of subjective judgment for different decision makers through calculation; 2. The weight information of multiple subjects are determined by the method of fuzzy mathematics to determine the weight information of multiple decision makers. In this respect, this paper takes references [8-10] for the method comparison of the first innovation and takes intuitive judgment information and reference [22] for the method comparison of the second innovation.

1. Method comparison in attribute weights (For simplicity and intuitiveness, compare the utility values). To compare the method with that in reference [8], take alternatives in the same category indicating “slightly better”. For the ranking results, see Table 3, (reference [8] sequence) and the comparison between calculated utility values of decision maker No.1 (Figure 2). According to the calculation results, the ranking conditions are basically the same, but according to the utility value results, the utility value difference between alternatives in the same category and alternatives in different categories is less significant than that in this paper (for example, the data of decision maker No.1 are compared in Figure 2. From the manifestation, the utility value difference between alternatives in the same category and alternatives in different categories in the method of this paper is greater). To compare the method with that in reference [10], determine the

order relation for alternatives in the same category as \approx , assume other corresponding alternative order relations as $>$ and $<$ in turn, and make decisions based on the correlation of ordinal information. For the ranking results, see

Table 3, (reference [10] sequence) and the comparison between calculated utility values of decision maker No.1 (Figure 3). According to the calculation results, the ranking conditions are basically the same, but according to the utility value results, the similarity of alternatives in the same category is less significant than that in this paper (for example, the data of decision makers No.1 are compared in Figure 3. From the manifestation, alternative 1, alternative 3 and alternative 4 of the same category in the method of this paper are less volatile).

Table 3. Utility Values and Ranking of Alternatives in this Paper and Reference [8] and [10]

The result of document comparison	z_i^p	Decision maker No. 1		Decision maker No. 2		Decision maker No. 3		Decision maker No. 4	
		Utility value	Order	Utility value	Order	Utility value	Order	Utility value	Order
The result in this paper	z_1	0.5422	2	0.5564	2	0.8074	1	0.7479	2
	z_2	0.7725	1	0.8428	1	0.3527	3	0.8010	1
	z_3	0.5280	3	0.2953	4	0.4181	2	0.5507	3
	z_4	0.5000	4	0.3790	3	0.1973	4	0.3310	5
	z_5	0.3304	5	0.1136	6	0.1811	6	0.3959	4
	z_6	0.1163	6	0.2436	5	0.1916	5	0.2905	6
The result in literature [8]	z_1	0.4803	2	0.5510	2	0.7694	1	0.6937	2
	z_2	0.6588	1	0.8327	1	0.3341	3	0.8035	1
	z_3	0.4307	3	0.2986	4	0.4338	2	0.5710	3
	z_4	0.4256	4	0.3721	3	0.2133	4	0.2767	5
	z_5	0.2107	5	0.1194	6	0.1786	6	0.3517	4
	z_6	0.0986	6	0.2575	5	0.1858	5	0.2041	6
The result in literature [10]	z_1	0.5109	2	0.6306	2	0.7317	1	0.6918	2
	z_2	0.8289	1	0.7908	1	0.3003	3	0.8017	1
	z_3	0.4881	3	0.3372	4	0.4386	2	0.5685	3
	z_4	0.4141	4	0.4597	3	0.1943	4	0.2781	5
	z_5	0.2822	5	0.1078	6	0.1404	6	0.3525	4
	z_6	0.1804	6	0.2929	5	0.1838	5	0.2076	6

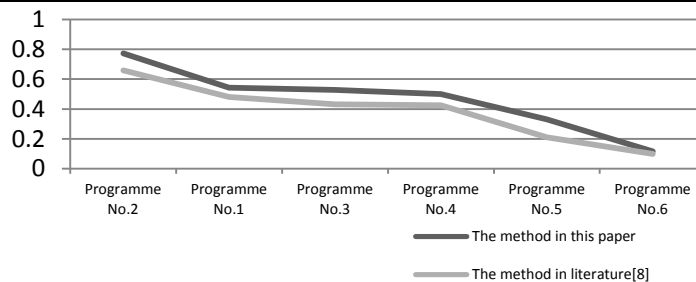


Figure 2. Comparison of Utility Values for Each Alternative between this Paper and Literature[8] (with Decision Maker No. 1 As An Example)

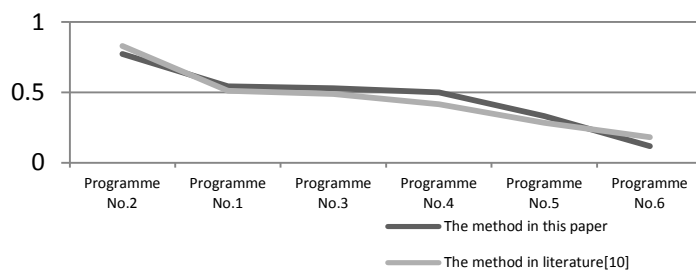


Figure 3. Comparison of Utility Values for Each Alternative between this Paper and Literature[10] (with Decision Maker No.1 as an Example)

2. Method comparison in weights of decision makers.

(1) In this paper, the objective weighting result in the method is different from previous intuitive judgment of people. People thought that government agencies should take up more subject weight during the decision making and that experts in urban planning as the “neutral” party in terms of interests should make objective judgments. However, according to the calculation result, the category preference judgment information of the constructor may be more correct and reasonable, thus the subject weight of the constructor is higher. From the perspective of the respect for the credibility of subjective preference judgment information, we should respect the calculation method on the decision-making subject weight in this paper more.

(2) The main idea in reference [22] is to assign the weight at each evaluation index for each expert subject by using the subject weighting first, and then add the adjusted values of objective weight vectors. For the fairness of comparison, the subjective weight at each evaluation index for different decision-making subjects should be assigned with the same value, namely, the subjective weight vectors at 5 evaluation indexes for 4 decision makers are all $\lambda^* = (0.25, 0.25, 0.25, 0.25)$. Calculate the objective weight vectors at the evaluation indexes for 4 decision makers. Through the coefficient adjustment, the comprehensive weight vectors at 5 evaluation indexes for 4 decision makers are finally obtained:

$$\lambda^{\omega 1} = (0.2064, 0.2948, 0.1964, 0.3024)$$

$$\lambda^{\omega 2} = (0.1996, 0.2721, 0.2324, 0.2959)$$

$$\lambda^{\omega 3} = (0.2784, 0.1916, 0.2788, 0.2512)$$

$$\lambda^{\omega 4} = (0.2574, 0.3011, 0.1236, 0.3179)$$

$$\lambda^{\omega 5} = (0.2667, 0.2012, 0.1789, 0.3532)$$

Thus, the decision makers’ evaluations for 6 alternatives at various indexes are obtained. Assuming that the weights for 5 evaluation indexes are set as $\omega = (0.2, 0.2, 0.2, 0.2, 0.2)$, the total score of each alternative can be obtained (as shown in Table 4). According to the ranking results, two methods are the same, which indicates that the weight assignment of multiple decision makers in the method of this paper is basically reasonable. The weight of the decision maker can be obtained directly through the method in this paper, so, in this respect, it is better than that in reference [22].

Table 4. Evaluation of Each Attribute and Total Score of Each Alternative Calculated Through the Method in Reference [22]

w_j	Program No. 1	Program No. 2	Program No. 3	Program No. 4	Program No. 5	Program No. 6
w_1	0.9933	0.6586	0.4919	0.4779	0.1433	0.0792
w_2	0.4960	0.7878	0.4607	0.2671	0.2158	0.0087
w_3	0.4729	0.7607	0.3952	0.1540	0.1007	0.3582
w_4	0.8793	0.3663	0.5965	0.8234	0.3216	0.1594
w_5	0.5212	0.8431	0.4089	0.1408	0.3608	0.4191
The total result of each program	0.6725	0.6834	0.4706	0.3727	0.2285	0.2049
Order	2	1	3	4	5	6

4. Conclusion

Firstly, the category preference information of the decision maker, representing the “intuitive judgment” of the subject during the decision making, is the judgment information on alternatives of different decision makers in their own professional backgrounds within the field of view. The attribute weight reflects the importance of

different attributes in the formation of decisions. Therefore, the subjective judgment information or intuitive judgment information given by existing decision makers should be fully used and learnt in the determination of attribute weight. Secondly, the weight of the decision maker represents the decision-making power of the subject during the group decision making. Because decision makers have different knowledge backgrounds, practical experience, cognitive levels and preferences, the weight determination of the decision maker should reflect the importance and decision-making level of the decision maker objectively and completely. In the process of group decision-making, both the convergence of basic interests and objectives and conflicts of interests and preferences exist between decision makers. To help decision makers reach consensuses on decision-making problems, judgments should be made based on the full consideration of subjective factors and objective facts. This paper adopts the method of case-based learning to calculate the attribute weight in each decision makers' circumstance, thus to meet subjective preferences of different decision makers; optimizes the weight of each decision maker with the fuzzy set theory, and determines the level boundary between multiple decision makers to meet objective requirements during group decision-making; finally figures out the total order relation of the alternative which not only conforms to the subjective preference information of the decision maker, but also consists with the objective requirement of group decision-making, and seeks for consistent or compromised satisfactory alternatives among multiple subjects of which the boundary was unclear originally, to make group decision-making more objective and reasonable, and effectively integrates subjective judgment preferences of decision makers, which have a certain theoretical value and practical significance.

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