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Application of Twin Support Vector Regression in Subgrade Settlement Prediction

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Abstract

Due to the normal forecasting methods for subgrade settlement using observation data have different applicabilities, and the predicting results has bigger volatility and lower accuracy. In view of the above problems, based on the twin support vector regression tool, the settlement prediction model is established by combining with the measured roadbed settlement data; The related parameters of the prediction model are given and compared with the standard support vector regression machine, the comparison tests show that the twins support vector regression is a new method to predict the settlement of the roadbed, and is superior in forecasting accuracy to the standard support vector regression.

Keywords: *Subgrade Settlement prediction, Combination forecast model, Twin support vector regession, Least Square twin support vector regession*

1. Introduction

The development of subgrade settlement has complex characteristics, such as non-linear, non-stabilization, and including numerous uncertain information, so it is very difficult to forecast the subgrade settlement accurately [1]. The observation data of subgrade settlement is a response to the comprehensive effect of various factors, which contains a wealth of information. Based on the observation data modeling method, it mainly has the empirical formula method, the system analysis and the control theory method (such as the gray system method and the neural network method, *etc*.) [2]. Empirical formula method is according to the theoretical analysis and engineering experience to assume that mathematical model of subgrade settlement, and according to certain mathematical method to determine the model parameters, finally determine the forecasting embankment settlement with the parameters of the model. The assumption of the mathematical model or the theoretical derivation based on a variety of assumptions or based on engineering practice, are too simple to cover up the development of the roadbed settlement of the complex nature of the law. At the same time, the model parameters are determined by the traditional deterministic method is often due to the complexity of the calculation and the impact of human factors, the prediction results with a larger error [3]. Grey system method is based on grey prediction model established by grey generating function and grey differential equation of grey theory, the G (1, 1) model of grey differential equation is first order, without considering the secondary consolidation. At the same time, model parameters due to the influence of geological conditions, environmental factors, change along with the time, and the parameters of the model are fixed, leading to the prediction error, even can't predict; neural network method is using the strong nonlinear mapping ability of neural network, through neural network self-learning Subgrade Settlement Development inherent laws, establish the mathematical model of subgrade settlement. Although the neural network has a strong nonlinear mapping ability,

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but the neural network is poor, and the structure is difficult to determine, the impact of the application of neural network. In short, various prediction models have their own advantages and their respective scope of application, there are some shortcomings [4].

Support vector regression machine is a machine learning method based on statistical theory, according to the principle of the minimum structure risk of Vapnik and the idea of maximum distance, at present, the new research focus in the field of machine learning is solving the practical problems such as small samples, nonlinear, high dimension and local minimum points, and at the same time obtain good generalization ability [5-6]. Twin support vector regression machine is a kind of expansion of the standard support vector regression, the use of two non parallel hyper plane structure of the final regression function, and solve the two small scale quadratic programming problem, and unlike the standard support vector regression machine in solving a large-scale quadratic programming problem. Compared with the standard support vector regression machine, the dual support vector regression machine has shorter operation time and better generalization ability [7-8]. The twin support vector regression machine is introduced into the field of subgrade engineering, attempts to apply the double support vector regression machine to solve the subgrade settlement prediction, Engineering case analysis shows that the combined forecasting model has better prediction accuracy and stability, it has important theory and engineering practical value.

2. Standard Support Vector Regression Machine

Support vector regression machine based on support vector machine for the establishment of an effective tool to solve the regression problem. The regression problem is to find a function $f(x)$ on the R^n according to the given training set T, deduce the corresponding y value of any input x with $y = f(x)$.

The given training set is defined as:

$$T = \{(x_1, y_1), \cdots, (x_l, y_l)\} \subset (X \times Y)^l \tag{1}$$

In the formula, $x_i \in R^n$ is input, $y_i \in R$ is output, $i = 1,2,\cdots,l$, l is total number of training points, X is all input points x_i constitute the input matrix, Y is all the input points x_i corresponding to the output y_i of the output matrix.

The regression function that is supposed to be looking for is:

$$y = f(x) = w^T x + b \tag{2}$$

In the formula, $w \in R^n$, $b \in R$.

In order to get the regression function formula (2), the optimization problem is constructed:

$$\begin{aligned} &\min_{w,b,\xi_i,\xi_i^*} \frac{1}{2} w^T w + C\left(e^T \xi + e^T \xi^*\right) \\ &\text{s.t.} \quad Y - (Xw + eb) \le \varepsilon e + \xi, \xi \ge 0 \\ &\qquad (Xw + eb) - Y \le \varepsilon e + \xi^*, \xi^* \ge 0 \end{aligned} \tag{3}$$

In the formula, ξ and ξ^* is relaxation factor, C is non negative penalty coefficient. The solution of the above optimization problem is usually solved by solving the dual problem, and the dual problem of formula (3) is:

$$\min_{\alpha_i,\alpha_i^*} \ \varepsilon e^T\left(\alpha + \alpha^*\right) - Y\left(\alpha - \alpha^*\right) - \frac{1}{2}\left(\alpha - \alpha^*\right)^T X^T X\left(\alpha - \alpha^*\right)$$
$$\text{s.t. } e^T\left(\alpha_i - \alpha_i^*\right) = 0 \tag{4}$$
$$0 \le \alpha \le Ce, 0 \le \alpha^* \le Ce$$

In the formula, $\alpha \ge 0$, $\alpha^* \ge 0$ is Non negative Lagrange dual variable, and $\alpha\alpha^* = 0$.

Calculate $w = X\left(\alpha - \alpha^*\right)$, select $\alpha_j > 0$ component in the α , based on the calculation $b = y_j - w^T x_j - \varepsilon$, select $\alpha_j^* > 0$ component in the α^* , based on the calculation $b = y_j - w^T x_j - \varepsilon$, finally, we can construct regression function formula (2).

When the kernel function is introduced, it is easy to extend the linear model to the nonlinear model, and the usual kernel function is the RBF kernel function, Its formula is:

$$K\left(x_i, x\right) = \exp\left(-\left\|x - x_i\right\|^2 \Big/ \left(2\sigma^2\right)\right) \tag{5}$$

In the formula, σ is width coefficient of RBF kernel function.

3. Twin support Vector Regression Machine

The idea of double support vector regression machine and double support vector classification and the idea is roughly consistent, it is to find out the two non parallel hyper plane, but also has the essential difference: first, twin support vector classification is to classify, mainly is to find two classification hyperplane, and twin support vector regression machine is to find a regression; second, twin support vector regression confidential find two non parallel hyperplanes, and finally the regression function by the two super plane constituted on average, and loud support of classification is to find two classification hyperplane, just look at the new input point from which the plane close to belong to what kind .

Twin support vector regression machine is also needed to solve the two optimization problems. Firstly, the two optimization problems of the dual support vector regression machine are given:

$$\begin{aligned} &\min \ \frac{1}{2}\left(Y - e\varepsilon_1 - \left(Xw_1 + eb_1\right)\right)^{\mathrm{T}}\left(Y - e\varepsilon_1 - \left(Xw_1 + eb_1\right)\right) + C_1 e^T \xi \\ &\text{s.t. } \ Y - \left(Xw_1 + eb_1\right) \ge e\varepsilon_1 - \xi, \xi \ge 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\min \ \frac{1}{2}\left(Y + e\varepsilon_2 - \left(Xw_2 + eb_2\right)\right)^{\mathrm{T}}\left(Y + e\varepsilon_2 - \left(Xw_2 + eb_2\right)\right) + C_2 e^T \eta \\ &\text{s.t. } \ \left(Xw_2 + eb_2\right) - Y \ge e\varepsilon_2 - \eta, \eta \ge 0 \end{aligned} \tag{7}$$

In the formula, C_1, C_2 are penalty parameter, ε_1 、 ε_2 is ε parameters of insensitive loss function, ξ 、 η is relaxation factor. By solving the dual problem, we can get the solution of the above two optimization problems, and then find the two functions $f_1(x)$ and $f_2(x)$, and the final regression function is composed of the average of the two functions, that is:

$$f(x) = \frac{1}{2}\left(f_1(x) + f_2(x)\right) = \frac{1}{2}\left(w_1^T x + w_2^T x f_2(x)\right) + \frac{1}{2}\left(b_1 + b_2\right) \tag{8}$$

The purpose of the dual support vector regression machine is to find two functions $f_1(x) = w_1^T x + b_1$ and $f_2(x) = w_2^T x + b_2$ to construct the final regression function. First of all, in the objective function of the problem (6), the first represent the distance of the minimum sample point to the Super plane $f_1(x) = 0$, the second is minimizing the error. The constraint conditions are represented by the sample points as far as possible around ε_1 Non sensitive lower bound of the hyper plane $f_1(x) = 0$, that is around $f_1(x) - \varepsilon_1 = 0$. Intuitive geometric interpretation as shown in figure 1. The band between the two hyper planes $f_1(x) = 0$ and $f_1(x) - \varepsilon_1 = 0$ the band is similar to that of the standard support vector regression machine ε_1. There is a similar explanation for the problem (7), the hyper plane $f_2(x) = 0$ determines the upper bound of the upper bound of the ε_2, The band between the two hyper planes $f_2(x) = 0$ and $f_2(x) - \varepsilon_2 = 0$ the band is similar to that of the standard support vector regression machine ε_2.

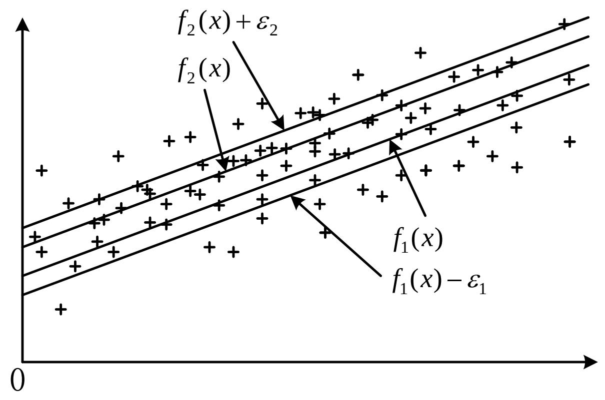

Figure 1. Geometric Interpretation of Least Square Support Vector Regression Machine

For the nonlinear regression problem, the kernel function is introduced to solve the problem, and the two optimization problems of the dual support vector regression machine are:

$$\begin{aligned} \min \quad & \frac{1}{2}\left(Y - e\varepsilon_1 - \left(K(X,X)^T w_1 + eb_1\right)\right)^{\mathrm{T}}\left(Y - e\varepsilon_1 - \left(K(X,X)^T w_1 + eb_1\right)\right) + C_1 e^T \xi \\ \text{s.t.} \quad & Y - \left(K(X,X) w_1 + eb_1\right) \geq e\varepsilon_1 - \xi, \xi \geq 0 \end{aligned} \tag{9}$$

$$\begin{aligned} \min \quad & \frac{1}{2}\left(Y + e\varepsilon_2 - \left(K(X,X)^T w_2 + eb_2\right)\right)^{\mathrm{T}}\left(Y + e\varepsilon_2 - \left(K(X,X)^T w_2 + eb_2\right)\right) + C_2 e^T \eta \\ \text{s.t.} \quad & \left(K(X,X) w_2 + eb_2\right) - Y \geq e\varepsilon_2 - \eta, \eta \geq 0 \end{aligned} \tag{10}$$

In the formula, $K = K\left(X, X^T\right)$ is kernel matrix.

By solving the dual problem of the above problem, the solution of the above problem can be solved, and the regression function can be obtained:

$$f(x) = \frac{1}{2}\left(f_1(x) + f_2(x)\right) = \frac{1}{2}\left(w_1^T K(X,x) + w_2^T K(X,x)\right) + \frac{1}{2}\left(b_1 + b_2\right) \tag{11}$$

4. Prediction of Subgrade Settlement Model with Double Support Vector Regression Machine

4.1. Prediction of Subgrade Settlement

The roadbed settlement is a set of time series recorded by time sequence, and the prediction of subgrade settlement is the time series analysis of the observation data. Through the double support vector regression machine to predict the settlement of subgrade, the first to determine the structure of the training set for training, that is, the training of the input data X and output data Y . According to the phase space reconstruction theory, the time series of the observation data is $\{s(t), t = 1, \cdots, n\}$, and the delay time and embedding dimension are respectively taken as τ and m , the prediction problem of subgrade settlement can be described as:

$$s(t + m\tau) = f\left(s(t), s(t+\tau), \cdots, s(t + (m-1)\tau)\right) \tag{12}$$

Formula (12) showed that the $t + m\tau$ settlement data is related to the t , $t+\tau$, …, $t + (m-1)\tau$ settlement data. The selection of τ is to let to participate in the reconstruction of adjacent data as far as possible not related, to make the embedded phase space point as far as possible to reflect the dynamic characteristics of the system; and the selection of m to be able to fully contained by the state transfer attractor constitute the minimum phase space dimension. Therefore, the prediction process of the subgrade settlement is to reconstruct the phase space of the roadbed settlement system by the time series of the observation data, realize the process $f: R^m \to R$ of mapping the vector $\left(s(t), s(t+\tau), \cdots, s(t + (m-1)\tau)\right)$ of the vector m of the embedding dimension into one dimension space $s(t + m\tau)$.

4.2. Prediction of Roadbed Settlement Based on TSVR

Engineering examples: a highway K525+200 the 3 of observation history 1260 pilgrim's base settlement observation data, the 21 observational data as the research object, the establishment of subgrade settlement prediction model. For the purpose of comparison, two methods are used to deal with the same problem by using the standard support vector regression machine and the double support vector regression machine.

(1) Data preparation and pretreatment.

Determining the delay time and embedding dimension is a necessary condition for the formation of training samples for SVR and TSVR. In this paper, the mutual information method and Cao's method to determine the delay time, embedding dimension, the 21 can constitute a sample data of 18, selection of 14 sample as the SVR and TSVR training samples. After four sample data as the test sample. In order to avoid the change of the large range of data, the data of the smaller range is covered, and the information that is carried by the sample data is lost, and the input and output of the sample is normalized.

$$\bar{s}_i = 0.1 + \frac{0.9 - 0.1}{s_{\max} - s_{\min}}(s_i - s_{\min}) \tag{13}$$

Normalized data has lost its original meaning, and the network output must be restored:

$$s_i = s_{\min} + \frac{s_{\max} - s_{\min}}{0.9 - 0.1}\left(\bar{s}_i - 0.1\right) \tag{14}$$

(2) The selection of model parameters

By formula (9) and (10) we can know that the parameters of the TSVR model include the loss function ($\varepsilon_1, \varepsilon_2$), the penalty factor ($C_1, C_2$), and the width parameter σ of the kernel function in the case of the Gauss kernel function such as the formula (5). Typically, the provisions $C_1 = C_2$, $\varepsilon_1 = \varepsilon_2$, there are 3 parameters, for the selection of 3 parameters can be found through the 50 percent off cross examination. First determine which a parameter usually experience value is selected as $\sigma = 1.5$, the cross examination of the remaining two parameters: the training sample of five aliquots, one group as the test data, the other 4 groups as the training data are calculated; the selection range of the parameters for $\{2^i \mid i \in [-5,5], i \in R\}$, select calculation were minimum variance of a set of parameters as the optimal parameters, after the experiment, optimal parameters are $C_1 = C_2 = 6.7289$, $\varepsilon_1 = \varepsilon_2 = 0.0118$, $\sigma = 1.5$.

(3) Experimental results and analysis

TSVR and SVR in the determination of the relevant parameters are independently run 10 times, to the best one as a result of the prediction of roadbed settlement. Table 1 lists the prediction results and relative error of SVR and TSVR. The prediction results and relative error of each individual prediction model and combined forecasting model are shown in Table 2. Comparison of the single phase prediction model and the combined forecasting model, the results of SVR and TSVR were described and compared with the actual values, the relative error of the results of SVR and TSVR were compared.

Table 1. Four Kinds of Forecast Methods Comparison of Forecast Results

NO.	Measured values (cm)	SVR Model		TSVR model	
		Predicted values (cm)	Relative Error (%)	Predicted values (cm)	Relative Error (%)
1	4.62	4.7536	-2.8917	4.7021	-1.777
2	6.36	6.3481	0.1871	6.3556	0.0691
3	7.32	7.3618	-0.5710	7.3564	-0.4972
4	8.66	8.4165	2.8117	8.4365	2.5808
5	8.85	9.4896	-7.2271	9.2467	-4.4824
6	10.00	10.0988	-0.9880	10.0823	-0.8230
7	11.55	10.9909	4.8406	11.1645	3.3376
8	11.49	11.4487	0.3594	11.4512	0.3376
9	11.94	12.0324	-0.7738	12.0154	-0.6314
10	13.22	13.0568	1.2344	13.1032	0.8835
11	13.20	13.3155	-0.8750	13.2923	-0.6992
12	13.46	13.4270	0.2451	13.4358	0.1797
13	14.00	14.1778	-1.2700	14.1498	-1.0700
14	14.25	14.2656	-0.1094	14.2543	-0.0301
15	14.40	14.3561	0.3048	14.3698	0.2097
16	14.25	14.5344	-1.9957	14.4887	-1.6750
17	14.53	14.6045	-0.5127	14.5886	-0.4033
18	14.71	14.7483	-0.2603	14.7367	-0.1815

From the Table 1 data, we can see that the relative error of the TSVR model is 4.4824%, which is better than the SVR model 7.2271%. The maximum relative error of the STVR model is 1.6750%, while the SVR model is 1.9957%. Visible, TSVR model, whether it is for the training of the data of the sample data, or the number of test samples for the prediction, performance indicators are better than the SVR model. It shows that the TSVR model has a good generalization ability compared with the SVR model.

In Figure 2, curve segment (the 180~1080 days) describe TSVR and SVR models, the training sample data fitting, curve segments (the day 1080~1260) TSVR and SVR models, the forecast results of the test samples are described can be seen for curve fitting and curve prediction period, TSVR model were than the SVR model has better effect. In Figure 3, curve segment (the 180~1080 days) describe the relative error of the SVR model and discuss the mechanism model of the training sample data fitting results, curve segments (the day 1080~1260) describes the SVR model and TSVR model on the test sample prediction results of relative error, relative error curve cleaning show that for TSVR model both curve fitting relative error of predicted relative errors were less than SVR model.

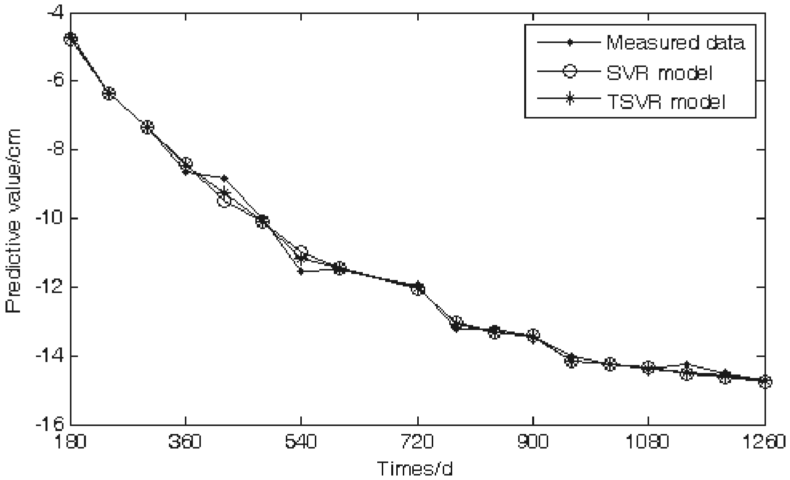

Figure 2. The Predictive Effect of Four Kinds of Forecasting Models

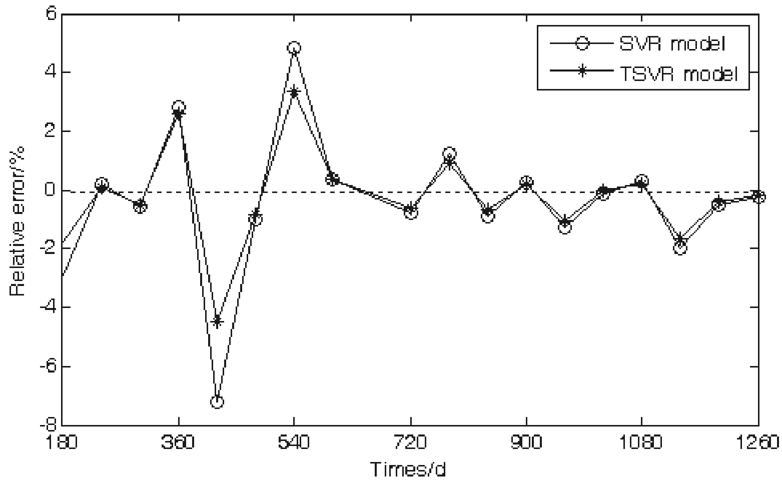

Figure 3. The Relative Error of Four Kinds of Forecasting Model

6. Conclusion

Domestic support vector regression machine is an extension of the standard support vector machine, and its training time is shorter than the standard support vector regression machine, and the accuracy is higher. In this paper the twin support vector regression built the subgrade settlement prediction model, and through an example to verify the twin support vector regression machine in the subgrade settlement prediction aspects than the standard support vector machine regression prediction accuracy higher and reduces the risk prediction, for subgrade settlement prediction provides an effective tool.

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