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Singular Distributed Parameter System Iterative Learning Control with Forgetting Factor with Time- Delay

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Abstract

In this paper, an iterative learning control topic for a kind of singular distributed parameter system with forgetting with time-delay which initial value can vary in a little space has been discussed. And a high level algorithm is extended and proven, using Green formula and Gronwall inequality, that it is suitable for the given system. Convergence of the proposed approach is analyzed and the uniform boundedness of tracking error is obtained in this paper. Numerical simulation is presented for a parabolic partial differential equations solved using ILC based on Eular difference format. At present, the application research of ILC method to the singular distributed parameter system with forgetting factor with time-delay is less. So this paper expands the scope of research for singular system and iterative learning control. Numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: *Iterative Learning Control; Forgetting Factor; Time-Delay; Distributed Parameter System*

1. Introduction

Iterative Learning Control was first proposed by Arimoto in 1984 [1]. Since then, ILC becomes a very important issue of control field. A lot of achievements have been published [2-4]. Many of the discussed systems described by ordinary differential equations. However, there are so many systems can be modeled by partial differential equations but papers are rate. On the other hand, distributed parameter system can not be approached by ordinary differential equations. So ILC of distributed parameter system is a very important research field.

During the last three decades, the research on stability analysis for time-delay systems has been widely investigated. Time-delay occurs in various physical, industrial and engineering systems such as biological systems, neural networks, networked control systems, multi-agent systems, and so on [5-6]. It is well known that the existence of time-delay is source of proof performance and instability of dynamic systems [7-8].

Iterative learning control(ILC) is a branch of intelligence control which is described by accurate mathematics. It is suitable for dynamic system with repetitive trajectory [9]. The main idea of ILC is to adjust input signal with error which is produced by expected output minus current output, and to produce a new input signal for next iterative cycle and repeat the whole process to make practical output convergence to expected output.

In the realm of industrial and engineering, time-delay as presented [10-11] and nonlinear as presented [12] are always encountered such as robotic hands with flexible

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nexus component, biology procedure and nuclear reactor control, *etc* An example is the control of density, temperature, current and momentum spatial profiles in tokamak plasmas, whose dynamics are ruled by a set of nonlinear parabolic PDEs as presented[13-16].

In order to produce the toroidal plasma current using transformer action, but at present, tokomaks operates in a pulsed mode and each pulse is called a discharge. In practice, the aim is to achieve the best possible approximate matching within a short time during the early flattop phase of the total plasma current pulse. And such matching problem can be formulated as a PDE control problem. ILC could be used to correct the control actuation discharge after discharge as to minimize nonlinear PDE process with ILC method. Xie [17-20] discussed iterative learning control of uncertain distributed parameter system based on a high level P-type algorithm. The paper studied a class Lipschitz nonlinear distributed parameter system with control time delay with forgetting factor. Sufficient conditions are given for the convergence of system by employing special norm. Numerical simulation is presented for a parabolic PDE solved using ILC based on Eular difference format. The numerical example is given to illustrate the effectiveness of the proposed method.

2. Problem Statement

The nonlinear distributed parameter system are following:

$$\begin{cases} \frac{\partial Q(x,t)}{\partial t} = D\Delta Q(x,t) + f(t, Q(x,t), Q(x,t-\theta), u(x,t)) \\ y(x,t) = g\big(t, Q(x,t)\big) + h\big(t, u(x,t)\big) \\ Q(x,t) = \phi(x,t), \;\; t \in [-\theta, 0] \end{cases} \tag{1}$$

Where $(x,t) \in \Omega \times [0,t], Q(x,t) \in R^n, u(x,t) \in R^m, y(x,t) \in R^l, D = \mathrm{diag}\{d_1, \cdots, d_n\}$, p_i is a known constant, $0 < p_i \le d_i < \infty, \phi(x,t)$ is differentiable for t, Δ is Laplace operator, Ω is an open bounded domain, $\partial\Omega$ is smooth. System (1) satisfied

$$\|f(t, Q_1, Q_{\theta 1}(x,t-\theta), u_1) - f(t, Q_2, Q_{\theta 2}(x,t-\theta), u_2)\| \le L_f(\|Q_1 - Q_2\| + \|Q_{\theta 1} - Q_{\theta 2}\| + \|u_1 - u_2\|) \tag{2}$$

$$\|J_g\| \le L_g \tag{3}$$

Where $J_g = \frac{\partial g}{\partial Q}, L_g, L_f$ are constants. The boundary condition of (1) is:

$$\alpha Q(x,t) + \beta \frac{\partial Q(x,t)}{\partial \vec{n}} = 0, (x,t) \in \partial\Omega \times [0,T] \tag{4}$$

where $\alpha = \mathrm{diag}\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$, $\alpha_i \ge 0$; $\beta = \mathrm{diag}\{\beta_1, \beta_2, \cdots, \beta_n\}$, $\beta > 0$, $\frac{\partial Q(x,t)}{\partial \vec{n}}$ is a out norm derivative on $\partial\Omega$.

For the system (1), supposed its expected output is $y_d(x,t)$, its corresponding input is $u_d(x,t)$, and its output equation is

$$y_d(x,t) = g\big(t, Q_d(x,t)\big) + h(t, u_d(x,t)) \tag{5}$$

The high level control algorithm is following[10]

$$\begin{cases} u_k(x,t) = \sum_{i=1}^{N} F_i(t) u_{k-i}(x,t) + G_i(t) e_{k-i}(x,t) \\ \qquad u_m(x,t) = e_m(x,t) = 0 \end{cases} \tag{6}$$

Where $k \in N^+, e_k(x,t) = y_d(x,t) - y_k(x,t)$.

And during the whole iterative course, the initial values assumed as following:

$$\|Q_d(x,t) - Q_k(x,t)\|_{L^2}^2 \le lr^k \tag{7}$$

Where $k = 0,1,2,\cdots, t \in [0,T], r \in (0,1), l > 0$.

According to (3), we have

$$\|g(t,Q_1) - g(t,Q_2)\| \leq L_g\|Q_1 - Q_2\| \tag{8}$$

The iterative learning law is given by the following equation:

$$u_{k+1}(t) = \big(1 - r(k)\big)u_k(t) + r(k)u_0(t) + \Gamma\dot{e}_k(t+\theta) + Le_k(t+\theta) \tag{9}$$

Where $r(k)$ is forgetting factor, $r(k) \in [0,1)$; Γ, L are iterative learning gain matrix, $\Gamma \in R^{m\times q}$, $L \in R^{n\times q}$; $\dot{e}_k(t+\theta)$ is the derivative of iterative error at $t+\theta$ moment.

3. Convergence Proof

Definition 1. Vector function $h(t): [0,T] \to R^n$, its λ norm definition is

$$\|h(t)\|_\lambda = \sup_{t\in[0,T]}\{e^{-\lambda t}\|h(t)\|\},\ \lambda > 0 \tag{10}$$

Lemma1. For vector function $f(t), h(t): [0,T] \to R^n$, if $h(t) = \int_0^\tau f(\tau)\,d\tau$, then

$$\|h(t)\|_\lambda \leq \frac{1-e^{-\lambda t}}{\lambda}\|h(t)\|_\lambda \tag{11}$$

Theorem 1. If System (1) and Equation (9) satisfy $\left\|\big(1-r(k)\big)I - \Gamma f(\cdot,t)\right\| < 1$, $\lim_{k\to\infty} r(k) \to 0$, then $\lim_{k\to\infty} y_d(x,t) - y_k(x,t) \to 0,\ \ t \in [0,T]$.

For convenience, we mark the $\frac{\partial Q(x,t)}{\partial t} = \dot{x}(t), f\big(t, Q(x,t), Q(x,t-\theta), u(x,t)\big) = Ax(t) + Bu(t-\theta),\ g\big(t, Q(x,t)\big) = Cx(t)$, so according to Eq.(9), we have

$$\begin{aligned}\Delta u_{k+1}(t) &= \big(1 - r(k)\big)\Delta u_k(t) + r(k)\Delta u_0(t) - \Gamma\dot{e}_k(t+\theta) - Le_k(t+\theta) = \\ &\big(1 - r(k)\big)\Delta u_k(t) + r(k)\Delta u_0(t) - LC\Delta x_k(t+\theta) - \Gamma C(A\Delta x_k(t+\theta) + B\Delta u_k)) = \\ &\Big(\big(1 - r(k)\big)I - \Gamma CB\Big)\Delta u_k + r(k)\Delta u_0(t) - (\Gamma CA + LC)\Delta x_k(t+\theta)\end{aligned} \tag{12}$$

Where $\Delta u_k = u_d(t) - u_k(t),\ \Delta x_k(t+\theta) = \Delta x_d(t+\theta) - x_k(t+\theta)$.

Marking $G = \Big(\big(1 - r(k)\big)I - \Gamma CB\Big),\ H = \Gamma CA + LC$, then the Eq.(12) is rewritten

$$\Delta u_{k+1}(t) = G\Delta u_k + r(k)\Delta u_0(t) + H\Delta x_k(t+\theta) \tag{13}$$

Taking the norm of Eq.(13), we have

$$\|\Delta u_{k+1}(t)\| = \|G\|\|\Delta u_k\| + \|r(k)\|\|\Delta u_0(t)\| + \|H\|\|\Delta x_k(t+\theta)\| \tag{14}$$

Next we estimate the boundary of $\|\Delta x_k(t+\theta)\|$. According to system(1) and marking, we have $x_k(t) = x_0(t) + \int_0^t [Ax_k(\tau) + Bu_k(\tau-\theta)]\,d\tau$, then taking norm

$$\|\Delta x_k(t)\| = \left\|\int_0^t [Ax_k(\tau) + Bu_k(\tau-\theta)]\,d\tau\right\| \leq a\left\|\int_0^t \Delta x_k(\tau)d\tau\right\| + b\left\|\int_0^t \Delta u_k(\tau-\theta)d\tau\right\| \tag{15}$$

Where a=A; b=B. By Lemma1 we know

$$\begin{cases}\left\|\int_0^t \|\Delta x_k(t)\|dt\right\|_\lambda \leq \frac{1}{\lambda}\|\Delta x_k(t)\|_\lambda \\ \left\|\int_0^t \|\Delta u_k(t)\|dt\right\|_\lambda \leq \frac{1}{\lambda}\|\Delta u_k(t)\|_\lambda\end{cases} \tag{16}$$

The method is to left multiply $e^{-\lambda t}$ to the two sides of Eq.(15), so we have

$$\|\Delta x_k(t)\|_\lambda \leq \frac{a}{\lambda}\|\Delta x_k(\tau)\|_\lambda + b\left\|\int_0^t \Delta u_k(\tau-\theta)d\tau\right\|_\lambda \tag{17}$$

When $\lambda > a$, by Eq.(13), we have

$$(1-\frac{a}{\lambda})\|\Delta x_k(t)\|_\lambda \le b\left\|\int_0^t \Delta u_k(\tau-\theta)d\tau\right\|_\lambda \Rightarrow \|\Delta x_k(t+\theta)\|_\lambda \le \frac{\lambda b}{\lambda-a}\left\|\int_0^{t+\theta} \Delta u_k(\tau-\theta)d\tau\right\|_\lambda \quad (18)$$

Next we will consider

$$J=\int_0^{t+\theta}\Delta u_k(\tau-\theta)d\tau=\int_0^{t}\Delta u_k(\tau-\theta)d\tau+\int_t^{t+\theta}\Delta u_k(\tau-\theta)d\tau=J_1+J_2 \quad (19)$$

$$J_1=\int_0^{t}\Delta u_k(\tau-\theta)d\tau=\int_{-\theta}^{t-\theta}\Delta u_k(s)ds \quad (20)$$

When $0 \le t \le \theta$, the $J_1=0$, when $\theta < t \le T$, the

$$J_1=\int_{-\theta}^{0}\Delta u_k(s)ds+\int_0^{t-\theta}\Delta u_k(s)ds=\int_0^{t-\theta}\Delta u_k(s)ds \le \left\|\int_0^{t-\theta}\Delta u_k(s)ds\right\| \le \int_0^t\|\Delta u_k(s)\|ds \quad (21)$$

So the $J_1 \le \int_0^t\|\Delta u_k(s)\|ds$ is always satisfied.

$$J_2=\int_t^{t+\theta}\Delta u_k(\tau-\theta)d\tau=\int_{t-\theta}^{t}\Delta u_k(s)ds=\int_{t-\theta}^{0}\Delta u_k(s)ds+\int_0^{t}\Delta u_k(s)ds=\int_0^{t}\Delta u_k(s)ds \le \int_0^t\|\Delta u_k(s)\|ds \quad (22)$$

By (17)(21)(22), we have

$$J=2\int_0^t\|\Delta u_k(s)\|ds \quad (23)$$

Substituting the Eq.(21) in the Eq.(18). We have

$$\|\Delta x_k(t+\theta)\|_\lambda \le \frac{\lambda b}{\lambda-a}\left\|\int_0^{t+\theta}\Delta u_k(\tau-\theta)d\tau\right\|_\lambda \le \frac{2\lambda b}{\lambda-a}\left\|\int_0^t\|\Delta u_k(s)\|ds\right\|_\lambda \le \frac{2\lambda b}{\lambda-a}\frac{1}{\lambda}\|\Delta u_k(t)\|_\lambda = \frac{2b}{\lambda-a}\|\Delta u_k(t)\|_\lambda \quad (24)$$

Using $e^{-\lambda t}(\lambda > a)$ multiply both sides of (14), taking the norm about λ, then substituting the Eq.(24) in the Eq.(14), we have

$$\|\Delta u_{k+1}(t)\|=\rho\|\Delta u_k(t)\|+\|r(k)\|u_0 \quad (25)$$

Where $\rho=\|G\|+\frac{2b}{\lambda-a}\|H\|; u_0= \|\Delta u_0(t)\|$.

Choosing a large enough $\lambda > a$, so $\|\Delta u_k(t)\|_\lambda \le \frac{u_0}{1-\rho}r(k), \lim_{k\to\infty}\|\Delta u_k(t)\|_\lambda \le 0$,

But $\lim_{k\to\infty}\|\Delta u_k(t)\|_\lambda \ge 0$, so

$$\lim_{k\to\infty}\|\Delta u_k(t)\|_\lambda=0 \quad (26)$$

From Eq.(24), we have $\lim_{k\to\infty}\|\Delta x_k(t+\theta)\|_\lambda \le \frac{2b}{\lambda-a}\lim_{k\to\infty}\|\Delta u_k(t)\|_\lambda=0$, so

$$\lim_{k\to\infty}\|\Delta x_k(t+\theta)\|_\lambda=0 \quad (27)$$

We know the out norm of iterative error can be described as following:

$$\|e_k(t+\theta)\|_\lambda=\|C\Delta x_k(t+\theta)\|_\lambda \quad (28)$$

We take the limit of Eq.(27), then we have

$$\lim_{k\to\infty}\|e_k(t+\theta)\|_\lambda=\frac{2bc}{\lambda-a}\lim_{k\to\infty}\|\Delta u_k(t)\|_\lambda \quad (29)$$

Where $c=\|C\|, t\in[-\theta, T-\theta]$, together (26) and (29), we have

$$\lim_{k\to\infty}\|e_k(t+\theta)\|_\lambda=0, t\in[0,T] \quad (30)$$

Because of the $\sup_{t\in[0,T]}\{\|e_k(t)\|\} \le e^{\lambda t}\|e_k(t)\|_\lambda$, so the convergence is proofed.

$$\lim_{k\to\infty} \sup_{t\in[0,T]} \|e_k(t)\| = 0 \quad (31)$$

4. Numerical Simulation

For show the effectiveness of the proposed ILC scheme. We suppose the expected output is

$$y_d(t) = \begin{bmatrix} e^{-2\pi t}\sin(\pi x) \\ 2t^3 \end{bmatrix} \quad (32)$$

Time-delay $\theta = 0.2$, forgetting factor $r(k) = \frac{1}{k^3}$, $\Gamma = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. When the initial state is $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, t \in [0, 0.2]$, so $u_k(1) = [14.383; 0.02], u_k(1) = [9.419; -2.738]$, where $k = 0,1,2,3,\cdots;$ when $t \in (0.2,2]$, $u_0 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. Using Eular forward difference format of parabolic partial differential equation with together proposed ILC algorithm, we have Figure 1-Figure 3.

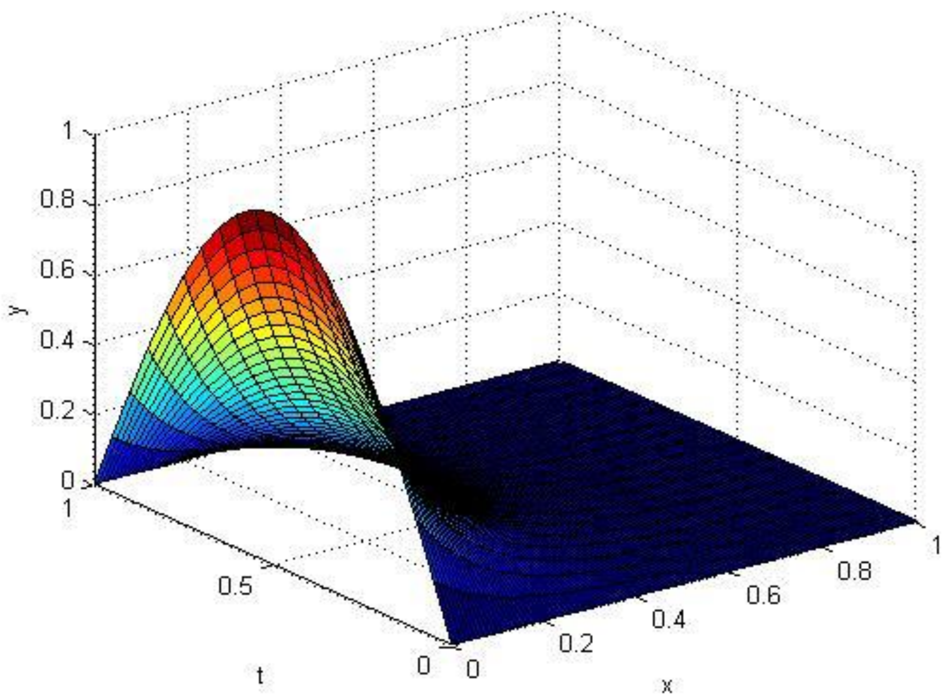

Figure 1. The Expected Output $y_{1d}(x,t) = e^{-2\pi t}\sin(\pi x)$

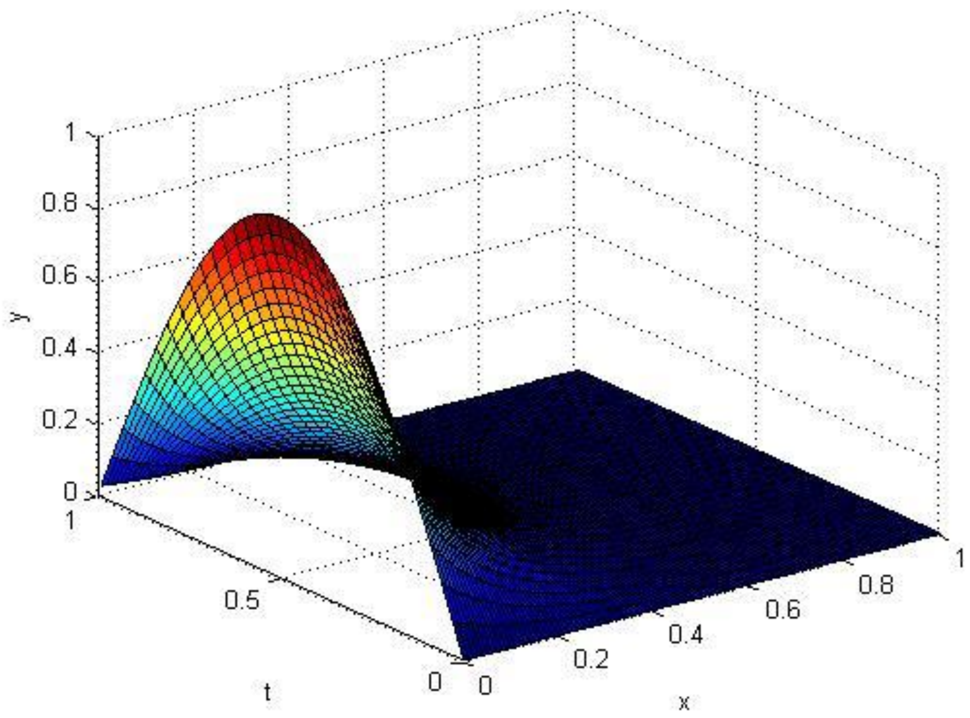

Figure 2. The Output after 20 Times Iterative $y_{1d}(x,t)$

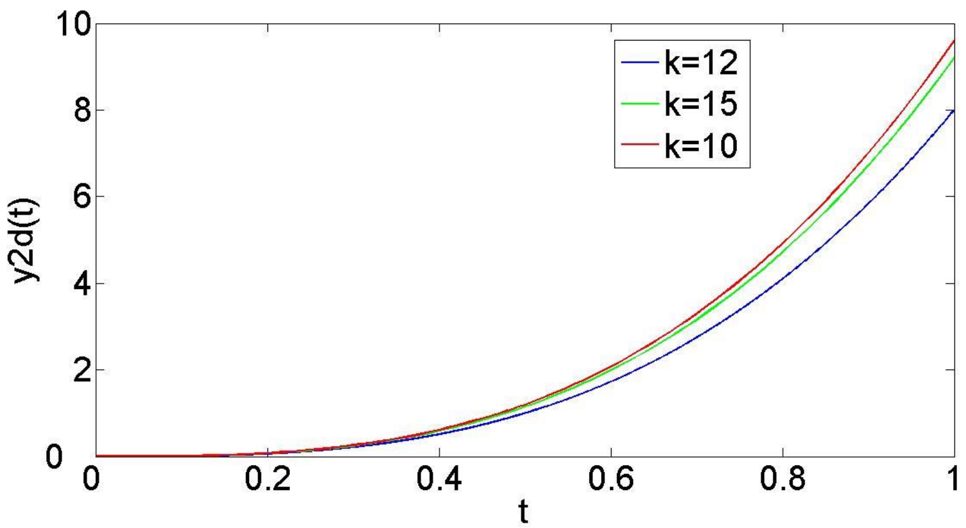

Figure 3. The Expected Output $y_{2d}(x,t) = 2t^3$

Compare with iterative learning control algorithm with time-delay with forgetting factor and ordinary iterative learning algorithm, from Figure4, we can see that choosing suitable forgetting factor can improve the convergence of algorithm. Shown in Figure4.

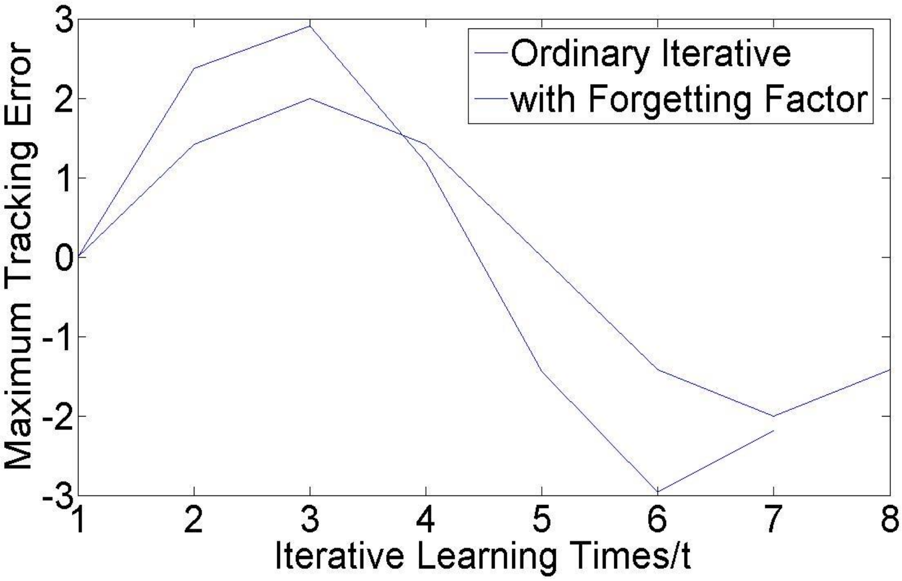

Figure 4. Maximum Tracking Error Curve Comparison

5. Conclusions

In this paper, we have discussed a class of singular distributed parameter with state time-delay with forgetting factor, extending the high level algorithm. Meantime we give the iterative learning control algorithm of trajectory tracking and proven the error of given system convergence to 0. We expand the application of iterative learning control method to nonlinear distributed parameter with state time-delay with forgetting factor. We also use Eular forward difference format to simulate the given example and we get the expected outcome.

About the research, we have performed the results for the temperature and humidity systems. It can be easily extended to more complex systems like one-or two-link inverted pendulum. Although the research has performed some targets that we expect, from some views, It may be incomplete, so we need do more things to complete the iterative learning algorithm for singular distributed parameter.

References

[1] S. Arimoto, S. Kawamura and F. Miyazaki, “Bettering operation of robots by learning”, Journal of Robotic System, vol. 1, **(1984)**, pp. 440-447.
[2] R. Marino and P.Tomei, “An Iterative Learning Control for a Class of Partially Feedback Linearizable Systems”, Automatic Control, IEEE Transactionson, vol. 54, **(2009)**, pp. 1991-1996.
[3] P. Dabkowski, K. Galkowskiy and E. Rogers, “Iterative Learning Control Based on Relaxed 2-D Systems Stability Criteria”, Control Systems Technology, IEEE Transactionson, vol. 21, **(2012)**, pp. 1016-1023.
[4] W. Cao, C. Wang and M. Sun, “Iterative learning control of variable Index gain with initial state study”, Systems Engineering and Electron-ics, vol. 34, **(2012)**, pp. 773-777.
[5] J.-H. Kim, “Note on stability of linear systems with time-varying delay”, Automatica. vol. 47, **(2008)**, pp. 2118-2121.
[6] L. Arockiam and N. Sasikaladevi, “Simulated annealing versus genetic based service selection algorithms”, International Journal of u- and e- Service, Science and Technology, vol. 3, **(2012)**, pp. 35-50.
[7] S. Lakshmanan, T. Senthilkumar and P. Balasubramaniam, “Improved results on robust stability of neutral systems with mixed time-varying delays and nonlinear perturbations”, Appl. Math. Model [J]. vol. 35, **(2011)**, pp. 5355-5368.
[8] J. Liu and J. Zhang, “Note on stability of discrete-time time-varying delay systems”, IET Control Theory Appl., vol. 6, **(2012)**, pp. 335–339.

[9] X. Meng, J. Lam, B. Du and H. Gao, "A delay-partitioning approach to the stability analysis of discrete time systems", Automatica, vol. 46, **(2010)**, pp. 610–614.
[10] P. Park, J. W. Ko and C. K. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays", Automatica, vol. 47, **(2011)**, pp. 235–238.
[11] K. Ramakrishnan and G. Ray, "Robust stability criteria for a class of uncertain discrete-time systems with time-varying delay", Appl. Math. Model, vol. 37, **(2013)**, pp. 1468–1479
[12] J. Sun, G. P. Liu, J. Chen and D. Rees, "Improved delay-range-dependent stability criteria for linear systems with time-varying delays", Automatica, vol. 46, **(2010)**, pp. 466–470
[13] S. L. Xie, "Iterative Learning Control Theory and Application", Science Press, Beijing, **(2005)**, pp. 75-81.
[14] C. Tianzhou and H. Jiangwei, "A practical dynamic frequency scaling scheduling algorithm for general purpose embedded operating system", International Journal of u- and e- Service, Science and Technology, vol. 1, **(2009)**, pp. 53-60.
[15] Q. Rafsel and T. W. John, "Iterative Learning Control for Non-smooth Dynamical Systems", Proceeding of the 46th IEEE Conference on Decision and Control, New Orleans, L,A, USA, no. 46, **(2007)**, pp. 245-251.
[16] Y. Ou " Towards model-based current profile Control at DIII-D, Fusion Engineering and Design, vol. 82, **(2007)**, pp. 1153-1160.
[17] L. Ch and W. Dong, "Research on Resources Integration of Urban Distribution System based on System Dynamics", International Journal of u- and e- Service, Science and Technology, vol. 6, **(2013)**, pp. 97-106.
[18] J. H. Moon, T. Y. Doh and M. J. Chung, "An iterative learning control scheme for manipulators. Proceedings of international conference on intelligent robots and systems", Grenoble, France, vol. 12, **(2009)**, pp. 84-89.
[19] J. F. Hung and J. C. Lin, "The development of the simulation modeling system and modeling ability evaluation. International Journal of u- and e- Service", Science and Technology, **(2013)**, vol. 6, pp. 1-16.
[20] Z. Chentouf and M.l A. Duwais, "fuzzy and cognitive attraction based solution for the software architectural change decision problem", evaluation. International Journal of u- and e- Service, Science and Technology, vol. 1, **(2014)**, pp. 91-100.

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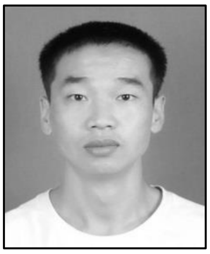

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