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A Novel Multiple Attribute Decision Making Framework based on Hamacher Operations in Patron Driven Acquisitions Mode of Library Management

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Abstract

As a fuzzy set extension, the hesitant set is effectively used to model situations where it is allowable to determine several possible membership degrees of an element to a set due to the ambiguity between different values. Some new hesitant fuzzy operational rules are introduced based on the Hamacher t-conorm and t-norm, whereby we present a multi attribute decision making method under hesitant fuzzy information. In the daily management of library, book selection problem usually is very complex and unstructured, because variety of uncontrollable and unpredictable factors affects the evaluation and decision-making process at different levels. By integrating the multiple attribute decision making method with (Patron Driven Acquisitions, PDA) mode, a novel multiple attribute decision making framework based on Hamacher operations in PDA mode is proposed to select the best book providing the highest satisfaction for the attributes determined in this paper. Finally, a case study is so demonstrated to verify the reliability and applicability of the proposed framework.

Keywords: *hesitant fuzzy set, Hamacher t-norn and t-conorm, PDA mode, multi attribute decision making framework, aggregation operator*

1. Introduction

Various approaches to multiple attribute decision making require suitable aggregation functions [1–4]. This is particularly important when modeling different kinds of uncertainty using Zadeh's fuzzy set [5] and their higher order extensions, such as the interval-valued fuzzy set [6], intuitionistic fuzzy set [7-8], interval-valued intuitionistic fuzzy set [9]. After the pioneering work by Torra and Narukawa [10-11], the hesitant fuzzy set has been received more and more attention from researchers. Recently, it has been successfully applied to deal with uncertain multi attribute decision making problems. The hesitant fuzzy aggregation operator is one of the core issues. Based on some hesitant fuzzy operational rules, Xia and Xu [12] first proposed hesitant fuzzy weighted averaging (HFWA), hesitant fuzzy weighted geometric (HFWG) operators, generalized hesitant fuzzy weighted averaging (GHFWA), generalized hesitant fuzzy weighted geometric (GHFWG) operators, and applied them in solving decision making problems. It is worthwhile to mention that these existing hesitant fuzzy aggregation operators are based on the algebraic product and algebraic sum operational rules of hesitant fuzzy elements (HFEs), which are a pair of special dual triangular norm (briefly t-norm) and triangular conorm (briefly t-conorm) [3]. Hamacher [13] proposed another kind of t-conorm and t-norm, called the Hamacher t-conorm and t-norm. As a generalization of the algebraic product and sum. In this paper, we extend the Hamacher t-conorm and t-norm to hesitant fuzzy set and propose a family of hesitant fuzzy Hamacher operators that allow

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decision-makers have more choice in multiple criteria decision making problems. It is also very important to research generalized hesitant fuzzy aggregation operators based on the Hamacher t-conorm and t-norm and their application to multi attribute decision making.

PDA (Patron Driven Acquisitions) [14-16], as a new mode of purchasing books for library has been an overarching concern for many librarians. It is adopted to purchase books depending on the actual demand of readers by various university libraries all over the world. In recent years, various kinds of methods have been proposed to solve the acquisition decision problems by manly researchers. PDA is different from traditional purchasing mode, and book selection problem usually is very complex and unstructured, because variety of uncontrollable and unpredictable factors affects the evaluation and decision-making process at different levels. So, it is important and meaningful to study book selection problems in PDA mode.

In order to do so, the paper will be set out as follows: In Section 2, some basic concepts, including hesitant fuzzy sets and the basic hesitant fuzzy operational rules are briefly reviewed. In Section 3, some new hesitant fuzzy operational rules are introduced based on the Hamacher t-conorm and t-norm, whereby we present a multiple attribute decision making method under hesitant fuzzy information. In order to overcome the existing methods' shortcoming to select the most desirable for book for libraries, a novel multiple attribute decision making framework based on Hamacher operations in PDA mode is proposed under the hesitant fuzzy environment, and a numerical example of book selection is given to illustrate the application of the proposed framework in Section 4. We have a brief conclusion in Section 5.

2. Preliminaries

In the following, some basic concepts of hesitant fuzzy set are briefly reviewed.
Torra proposed the concept of hesitant fuzzy set which is in terms of a function when applied to a fixed set returns a sunset of [0, 1]. Then, in order to easily understood, Xia and Xu [17] express the hesitant fuzzy set by mathematical symbol.

Definition 1 [17] Let X be a universe of discourse, then a hesitant fuzzy set H over X is defined as

$$H = \{< x, h_H(x) > | x \in X\}, \tag{1}$$

where $h_H(x)$ is a set of some values in [0,1], symbolizing the possible membership degrees of the element x to H. For convenience, we call $h = h_H(x)$ a hesitant fuzzy element and H the set of all hesitant fuzzy elements.

Definition 2 [10] Let h, h_1 and h_2 be three hesitant fuzzy numbers, their basic operations are defined as

(1) $h^c = \cup_{\gamma \in h}\{1-\gamma\}$;

(2) $h_1 \cup h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\}$;

(3) $h_1 \cap h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}$.

Here, h^c represents the complement of the hesitant fuzzy number h.

Definition 3 [10] Let h, h_1 and h_2 be three hesitant fuzzy numbers, their new basic operations are defined by Xia and Xu as follows:

(1) $h^\lambda = \cup_{\gamma \in h}\{\gamma^\lambda\}$;

(2) $\lambda h = \cup_{\gamma \in h}\{1-(1-\gamma)^\lambda\}$;

(3) $h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$;

(4) $h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$.

The following order relation between HFEs is defined by Xia and Xu:

Definition 4 [17] For a HFE h, $S(h) = \sum_{\gamma \in h} \gamma / \delta(h)$ is called the score function of h, where $\delta(h)$ is the number of elements in h. For two HFEs, $h_1$and h_2, if $S(h_1) > S(h_2)$, then $h_1 > h_2$; if $S(h_1) = S(h_2)$, then $h_1 = h_2$.

3. Multi Attribute Decision Making Method based on Hamacher t-norm and t-conorm

In essence, the addition and multiplication operations in Definition 4 are the algebraic sum and algebraic product operational rules on hesitant fuzzy elements (HFEs), respectively, a special pair of dual t-norm and t-conorm. We can extend these operations to obtain more general operations on HFEs by means of the Hamacher t-norm and t-conorm, which are discussed in this section.

3.1. Hamacher t-norm and t-conorm

An important notion in fuzzy set theory is that of triangular norms and conorms that are used to define a generalized intersection and union of fuzzy sets, which is defined as follows:

Definition 5 [3] A triangular norm is a binary operation T on the unit interval [0,1], *i.e.*, a function T:[0,1]×[0,1]→[0,1], such that for all x, y and $z \in [0,1]$ the following four axioms are satisfied:

(1) $T(1,x)=x$, for all x;

(2) $T(x,y)=T(y,x)$, for all x and y;

(3) $T(x,T(y,z))=T(T(x,y),z)$, for all x, y and z; and

(4) If $x \le x_1$ and $y \le y_1$, then $T(x,y) \le T(x_1,y_1)$.

Definition 6 [3] Given a function $S:[0,1]\times[0,1]\to[0,1]$, it is called a t-conorm when it satisfies the following four constraints:

(1) $S(0,x)=x$, for all x;

(2) $S(x,y)=S(y,x)$, for all x and y;

(3) $S(x,S(y,z))=S(S(x,y),z)$, for all x, y and z; and

(4) If $x \le x_1$ and $y \le y_1$, then $S(x,y) \le S(x_1,y_1)$.

For many t-norms and t-conorms, there are some basic t-norms and t-conorms, namely, minimum T_M and maximum S_M, product T_P and probabilistic sum S_P, Łukasiewicz t-norm TL and Łukasiewicz t-conorm S_L, Einstein product T_E and Einstein product S_E, and drastic product T_D and drastic sum S_D. Hamacher [13] proposed a more generalized t-norm and t-conorm, called the Hamacher t-norm and t-conorm, which are defined as follows:

$$T_r(x,y) = \frac{xy}{r-(r-1)\cdot(x+y-xy)} \quad (2)$$

$$S_r(x,y) = \frac{x+y+(r-2)xy}{1+(r-1)xy} \quad (3)$$

Especially, when r=1, then the Hamacher t-norm and t-conorm reduce to $T_P(x,y)=x\cdot y$, $S_P(x,y)= x + y - x\cdot y$; when r= 2, then Hamachert-norm and t-conorm reduce to Einstein t-norm T_E and t-conorm S_E [18], respectively.

3.2. Hamacher Operation Rules on HFEs

Definition 7 For any given three HFEs, h, h_1, h_2, and $r>0$, based on Hamacher t-norm and t-conorm, some operations, called Hamacher operations, on HFEs h, h_1, and h_2 are defined as follows:

(1) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\frac{\gamma_1 + \gamma_2 + (r-2)\gamma_1\gamma_2}{1+(r-1)\gamma_1\gamma_2}\}$;

(2) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\frac{\gamma_1\gamma_2}{r-(r-1)\cdot(\gamma_1+\gamma_2-\gamma_1\gamma_2)}\}$;

(3) $\lambda h = \bigcup_{\gamma \in h} \{\frac{[1+(r-1)\gamma]^{\lambda} - (1-\gamma)^{\lambda}}{[1+(r-1)\gamma]^{\lambda} + (r-1)(1-\gamma)^{\lambda}}\}, \ \lambda > 0$;

(4) $h^{\lambda} = \bigcup_{\gamma \in h} \{\frac{r\gamma^{\lambda}}{[1+(r-1)(1-\gamma)]^{\lambda} + (r-1)\gamma^{\lambda}}\}, \ \lambda > 0$;

(5) $h^c = \bigcup_{\gamma \in h} \{1-\gamma\}$.

Where h^c denotes the complement of a HFE h.

Theorem 1 For any given three HFEs, h, h_1, h_2, and $r>0$, we have the following conclusions:

(1) $h_1^c \oplus h_2^c = (h_1 \otimes h_2)^c$;

(2) $h_1^c \otimes h_2^c = (h_1 \oplus h_2)^c$;

(3) $\lambda(h^c) = (h^{\lambda})^c, \lambda > 0$;

(4) $(h^c)^{\lambda} = (\lambda h)^c, \lambda > 0$.

Where h^c denotes the complement of a HFE h.

Proof.

(1). $h_1^c \oplus h_2^c = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\frac{(1-\gamma_1)+(1-\gamma_2)-(1-\gamma_1)(1-\gamma_2)-(1-r)(1-\gamma_1)(1-\gamma_2)}{1-(1-r)(1-\gamma_1)(1-\gamma_2)}\}$

$= \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\frac{\gamma_1+\gamma_2+r(1-\gamma_1)(1-\gamma_2)}{\gamma_1+\gamma_2-\gamma_1\gamma_2+r(1-\gamma_1)(1-\gamma_2)}\} = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{1-\frac{\gamma_1\gamma_2}{r+(1-r)(1-\gamma_1)(1-\gamma_2)}\} = (h_1 \otimes h_2)^c$.

(2). $h_1^c \otimes h_2^c = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\frac{(1-\gamma_1)(1-\gamma_2)}{r+(1-r)((1-\gamma_1)+(1-\gamma_2)-(1-\gamma_1)(1-\gamma_2)}\}$

$= \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\frac{(1-\gamma_1-\gamma_2+\gamma_1\gamma_2)}{1-(1-r)\gamma_1\gamma_2}\}$

$= \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{1-\frac{\gamma_1+\gamma_2-\gamma_1\gamma_2-(1-r)\gamma_1\gamma_2}{1-(1-r)\gamma_1\gamma_2}\} = (h_1 \oplus h_2)^c$.

(3). $\lambda(h^c) = \bigcup_{\gamma \in h} \{\frac{(1+(r-1)(1-\gamma))^{\lambda} - \gamma^{\lambda}}{(1+(r-1)(1-\gamma))^{\lambda} - (r-1)\gamma^{\lambda}}\} = \bigcup_{\gamma \in h} \{1-\frac{r\gamma^{\lambda}}{(1+(r-1)(1-\gamma))^{\lambda} - (r-1)\gamma^{\lambda}}\} = (h^{\lambda})^c$.

(4). $(h^c)^{\lambda} = \bigcup_{\gamma \in h} \{\frac{r(1-\gamma)^{\lambda}}{(1+(r-1)\gamma)^{\lambda} - (r-1)(1-\gamma)^{\lambda}}\} = \bigcup_{\gamma \in h} \{1-\frac{(1+(r-1)\gamma)^{\lambda} - (1-\gamma)^{\lambda}}{(1+(r-1)\gamma)^{\lambda} - (r-1)(1-\gamma)^{\lambda}}\} = (\lambda h)^c$.

With the above background, concepts and definitions, in the next section, we propose some hesitant fuzzy Hamacher aggregation functions.

3.3. Some Hesitant Fuzzy Hamacher Aggregation Functions

Definition 8. Let $E = \{h_1, h_2, \ldots, h_n\}$ be a set of n HFEs. A hesitant fuzzy Hamacher weighted averaging (HFHWA) operator is a function $H^n \rightarrow H$ such that

$$\text{HFHWA}(h_1, h_2, \ldots, h_n) = w_1 h_1 \oplus w_2 h_2 \oplus \cdots w_n h_n = \bigoplus_{j=1}^{n} w_j h_j , \tag{4}$$

where parameter $\lambda > 0$, $(w_1, w_2, \ldots, w_n)^T$ is the weight vector of h_j (j=1,2,…,n), satisfying $w_j \in [0,1]$ (j=1,2,…,n) and $\sum_{j=1}^{n} w_j = 1$.

Theorem 2. Let $E = \{h_1, h_2, \ldots, h_n\}$ be a set of n HFEs, then the aggregated value by the HFHWA operator is also a HFE, furthermore,

$$\text{HFHWA}(h_1, h_2, \ldots, h_n) = \bigcup_{\gamma_j \in h_j} \left\{ \frac{\prod_{j=1}^{n} [1+(r-1)\gamma_j]^{w_j} - \prod_{j=1}^{n} (1-\gamma_j)^{w_j}}{\prod_{j=1}^{n} [1+(r-1)\gamma_j]^{w_j} + (r-1)\prod_{j=1}^{n} (1-\gamma_j)^{w_j}} \right\}. \tag{5}$$

Proof. By using mathematical induction on n: From Definition 7, we have:

$$w_j h_j = \bigcup_{\gamma_j \in h_j} \left\{ \frac{[1+(r-1)\gamma_j]^{w_j} - (1-\gamma_j)^{w_j}}{[1+(r-1)\gamma_j]^{w_j} + r(1-\gamma_j)^{w_j}} \right\},\ w_j > 0 .$$

For $n = 2$, we have:

$$\text{HFHWA}(h_1, h_2) = w_1 h_1 \oplus w_2 h_2 =$$

$$\bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \frac{[1+(r-1)\gamma_1]^{w_1} [1+(r-1)\gamma_2]^{w_2} - (1-\gamma_1)^{w_1} (1-\gamma_2)^{w_2}}{[1+(r-1)\gamma_1]^{w_1} [1+(r-1)\gamma_2]^{w_2} - (r-1)(1-\gamma_1)^{w_1} (1-\gamma_2)^{w_2}} \right\}$$

Suppose Theorem 2 holds for $n = k$, that is

$$\text{HFHWA}(h_1, h_2, \ldots, h_k) = w_1 h_1 \oplus w_2 h_2 \oplus \cdots \oplus w_k h_k =$$

$$\bigcup_{\gamma_j \in h_j} \left\{ \frac{\prod_{j=1}^{k} [1+(r-1)\gamma_j]^{w_j} - \prod_{j=1}^{k} (1-\gamma_j)^{w_j}}{\prod_{j=1}^{k} [1+(r-1)\gamma_j]^{w_j} + (r-1)\prod_{j=1}^{k} (1-\gamma_j)^{w_j}} \right\}$$

then for n=k+1

$$\text{HFHWA}(h_1, h_2, \ldots, h_k, h_{k+1}) = w_1 h_1 \oplus w_2 h_2 \oplus \cdots \oplus w_k h_k \oplus w_{k+1} h_{k+1} =$$

$$\bigcup_{\gamma_j \in h_j} \left\{ \frac{\prod_{j=1}^{k} [1+(r-1)\gamma_j]^{w_j} [1+(r-1)\gamma_{k+1}]^{w_{k+1}} - \prod_{j=1}^{k} (1-\gamma_j)^{w_j} [1-(r-1)\gamma_{k+1}]^{w_{k+1}}}{\prod_{j=1}^{k} [1+(r-1)\gamma_j]^{w_j} [1+(r-1)\gamma_{k+1}]^{w_{k+1}} + (r-1)\prod_{j=1}^{k} (1-\gamma_j)^{w_j} [1-(r-1)\gamma_{k+1}]^{w_{k+1}}} \right\},$$

$$= \bigcup_{\gamma_j \in h_j} \left\{ \frac{\prod_{j=1}^{k+1} [1+(r-1)\gamma_j]^{w_j} - \prod_{j=1}^{k+1} (1-\gamma_j)^{w_j}}{\prod_{j=1}^{k+1} [1+(r-1)\gamma_j]^{w_j} + (r-1)\prod_{j=1}^{k+1} (1-\gamma_j)^{w_j}} \right\}$$ *i.e.*, Ep. Theorem 2 holds for n=k+1.

Thus Theorem 2 holds for all n.

Theorem 3 For the HFHWA operator, if $r = 1$, then the HFHWA operator is reduced to the following:

$$\text{HFWA}(h_1, h_2, \ldots, h_n) = w_1 h_1 \oplus w_2 h_2 \oplus \cdots w_n h_n = \bigcup_{\gamma_j \in h_j} \left\{ 1 - \prod_{j=1}^{n} (1-\gamma_j)^{w_j} \right\}, \tag{6}$$

which is called the hesitant fuzzy weighted averaging (HFWA) operator by Xia and Xu [17].

Theorem 4 For the HFHWA operator, if r = 2, then the HFHWA operator is reduced to the following:

$$\text{HFEWA}(h_1, h_2,..., h_n) = w_1 h_1 \oplus w_2 h_2 \oplus \cdots w_n h_n = \cup_{\gamma_j \in h_j} \left\{ \frac{\prod_{j=1}^{n}[1+\gamma_j]^{w_j} - \prod_{j=1}^{n}(1-\gamma_j)^{w_j}}{\prod_{j=1}^{n}[1+\gamma_j]^{w_j} + \prod_{j=1}^{n}(1-\gamma_j)^{w_j}} \right\}, \quad (7)$$

which is called the hesitant fuzzy Einstein weighted averaging (HFEWA) operator by Zhao,*et al.* [19].

Definition 9 (HFHWG operator). Let $E = \{h_1, h_2,..., h_n\}$ be a set of n HFEs. A hesitant fuzzy Hamacher weighted geometric (HFHWG) operator is a function $H^n \rightarrow H$ such that

$$\text{HFHWG}(h_1, h_2,..., h_n) = h_1^{w_1} \otimes h_2^{w_2} \otimes \cdots h_n^{w_n} = \bigotimes_{j=1}^{n} h_j^{w_j}, \quad (8)$$

where parameter $\lambda > 0$, $(w_1, w_2,..., w_n)^T$ is the weight vector of h_j (j=1,2,…,n), satisfying $w_j \in [0,1]$ (j=1,2,…,n) and $\sum_{j=1}^{n} w_j = 1$.

Theorem 5 (HFHWG operator). Let $E = \{h_1, h_2,..., h_n\}$ be a set of n HFEs, then the aggregated value by the HFHWG operator is also a HFE, furthermore,

$$\text{HFHWG}(h_1, h_2,..., h_n) = \cup_{\gamma_j \in h_j} \left\{ \frac{r\prod_{j=1}^{n}(\gamma_j)^{w_j}}{\prod_{j=1}^{n}[1+(r-1)(1-\gamma_j)]^{w_j} + (r-1)\prod_{j=1}^{n}(\gamma_j)^{w_j}} \right\}. \quad (9)$$

Proof. Theorem 5 can be proved by mathematical induction on n. The proof is similar to that of Theorem 2, and is not replicated here.

Theorem 6 For HFHWG operator, if r =1, the HFHWG operator is reduced to the following:

$$\text{HFWG}(h_1, h_2,..., h_n) = h_1^{w_1} \otimes h_2^{w_2} \otimes \cdots h_n^{w_n} = \cup_{\gamma_j \in h_j} \left\{ \prod_{j=1}^{n}(\gamma_j)^{w_j} \right\}, \quad (10)$$

which is called the hesitant fuzzy weighted geometric (HFWG) operator by Xia and Xu [17].

Theorem 7 For HFHWG operator, if r =2, the HFHWG operator is reduced to the following:

$$\text{HFEWG}(h_1, h_2,..., h_n) = h_1^{w_1} \otimes h_2^{w_2} \otimes \cdots h_n^{w_n} = \cup_{\gamma_j \in h_j} \left\{ \frac{2\prod_{j=1}^{n}(\gamma_j)^{w_j}}{\prod_{j=1}^{n}[(2-\gamma_j)]^{w_j} + \prod_{j=1}^{n}(\gamma_j)^{w_j}} \right\}, \quad (11)$$

which is called the hesitant fuzzy Einstein weighted geometric (HFEWG) operator by Zhao,*et al.* [19].

3.4. Multiple Attribute Decision Making Method based on Hamacher t-norm and t-conorm

In this section, the HFHWA (or HFHWG) operator, as one of the proposed hesitant fuzzy Hamacher aggregation functions, is applied to develop a multi attribute decision making procedure for hesitant fuzzy MADA problems, where the following steps are involved.

Step 1. The decision makers give their evaluated values of alternative $a_i \in A$ with respect to attribute $c_j \in C$, which are expressed by hesitant fuzzy elements $h_{ij}(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$.

Step 2. Transform the hesitant fuzzy decision making matrix $D = (h_{ij})_{m\times n}$ into a normalized matrix $R = (r_{ij})_{m\times n}$, according to definition 7.

Step 3. For alternative a_i, aggregate all the hesitant fuzzy values r_{ij} ($j = 1, 2, \ldots, n$) into a global value r_i by means of the hesitant fuzzy Hamacher weighted average (HFHWA) operator or hesitant fuzzy Hamacher weighted geometric (HFHWG) operator.

Step 4. Compute the scores $S(r_i)$ of the global HFE r_i ($i = 1, 2, \ldots, m$) in accordance with Definition 4.

Step 5. Rank all the alternatives a_i ($i = 1, 2, \ldots, m$) and select the best one(s) according to $S(r_i)$ ($i = 1, 2, \ldots, m$).

Step 6. End.

4. Framework of PDA

4.1. Brief Introduction of PDA

PDA (Patron Driven Acquisitions) [14-16], as a new mode of purchasing books for library has been an overarching concern for many librarians. It is adopted to purchase books depending on the actual demand of readers by various university libraries all over the world. Reader who is usually the accepter and terminal during the construction process of literature resources in the conventional purchasing mode has become the initiator and the decision maker about what book should be introduced by the library during the PDA mode. PDA is different from traditional purchasing mode, and Figure.1 shows the difference between the two purchasing modes.

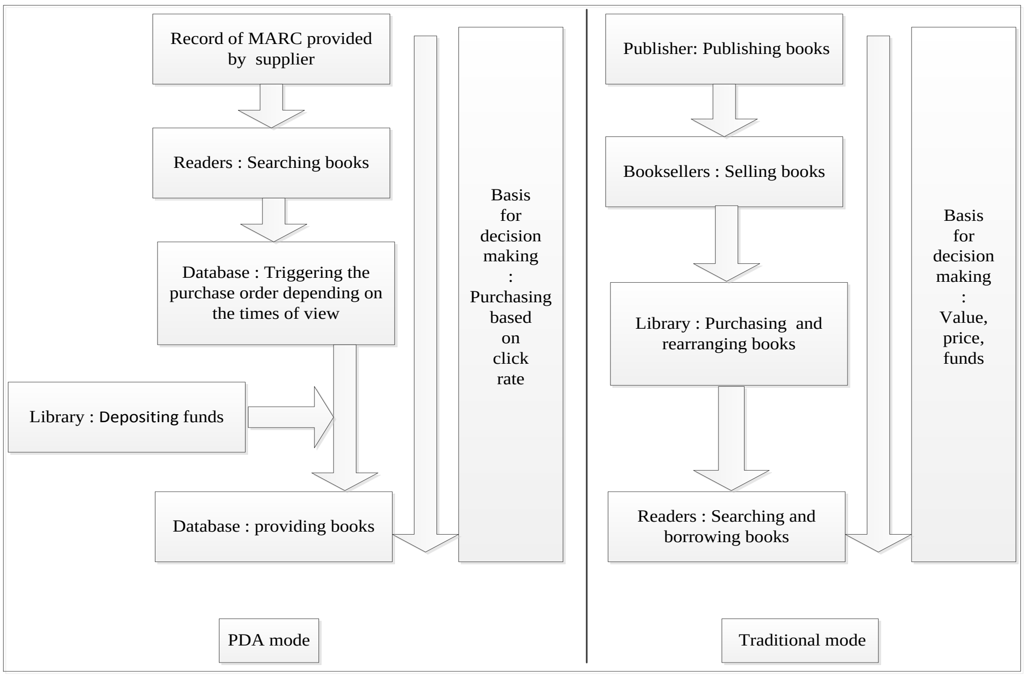

Figure 1. Comparing Between PDA Mode and Traditional Mode

As shown in Figure.1, the basis for decision making is the values and prices of books, and the funds which are used to purchase books in the traditional purchasing mode. It is a challenge for purchaser to make decision about which kinds of books need to buy because of disordered information, and this will enhance the chance of reordering. Dissimilarly, the basis for decision making is the click rate in the PDA mode. But it is unreasonable to make decisions only depending on the click rate, and this will lead to a small quantity about the books which have a small click rate but a great of value.

In recent years, various kinds of methods have been proposed to solve the acquisition decision problems by manly researchers. Huang Yuntao [20] proposed a multiple attribute evaluation method based on AHP to choose the most appropriate book for library. In Huang's method, some factors including the brand, quality, price, performance of product and the service, reputation, scale of manufacturer were considered. Combining calculations of qualitative with quantitative, a mathematical model was developed by Fu Yinzi and Li Xia [21]. And values of book, subject and demand of readers were discussed in their method.

4.2. Framework of PDA Decision Making

In today's competitive business, book selection problem plays a significant role as a strategic feature in library's success. Book selection problem usually is very complex and unstructured, because variety of uncontrollable and unpredictable factors affects the evaluation and decision-making process at different levels. As mentioned above, various decision making approaches have been proposed to tackle the problem as part of a general tendering process, particularly those of multi attribute analysis which use both quantitative and qualitative data. In this section, by integrating the multiple attribute decision making method with PDA mode, we propose a novel multiple attribute decision making framework based on Hamacher operations in PDA mode proposed to select the best book providing the highest satisfaction for the attributes determined, where the procedure of the proposed novel multi attribute decision making framework is shown in Figure. 2, which will be elaborated step by step.

Step 1. The decision makers should identify the most important and significant factors (attributes) *i.e.* $(c_1, c_2, \ldots, c_n)$ in PDA.

As discussed above, many factors have influence on the overall decision results, but which factors must be considered and which factors could not be considered in real decision problems is very important. So the decision makers should identify the most important and significant factors (attributes) in PDA. There are many methods to identify the important and significant factors based on decision makers' experience, designation and qualification, such as Fuzzy Delphi Method (FDM) which was proposed by Ishikawa *et al.* (1993), and it was derived from the traditional Delphi technique and fuzzy set theory.

Step 2. Determine weight vector of attribute *i.e.* $(w_1, w_2, \ldots, w_n)^T$.

Determining the importance of weights by decision makers for book selection attributes is subjective in such a way that decision makers usually select some important attributes and then prioritize them. There are several methods to determine of the attributes weights; including analytic hierarchy process (AHP), entropy analysis, weighted least square method and linear programming for multi dimensions of analysis preference (LINMAP).

Step 3. The decision makers give their evaluated values of alternative book $a_i \in A$ with respect to attribute $c_j \in C$, which are expressed by hesitant fuzzy elements $h_{ij}(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$.

Step 4. Transform the hesitant fuzzy decision making matrix $D = (h_{ij})_{m \times n}$ into a normalized matrix $R = (r_{ij})_{m \times n}$, according to definition 7.

Step 5. For alternative book a_i, aggregate all the hesitant fuzzy values r_{ij} ($j = 1, 2, . . ., n$) into a global value r_i by means of the hesitant fuzzy Hamacher weighted average (HFHWA) operator or hesitant fuzzy Hamacher weighted geometric (HFHWG) operator.

Step 6. Compute the scores $S(r_i)$ of the global HFE r_i ($i = 1, 2, . . ., m$) in accordance with Definition 4.

Step 7. Rank all the alternatives book a_i ($i = 1, 2, . . ., m$) and select the best one(s) according to $S(r_i)$ ($i = 1, 2, . . ., m$).

Step 8. End.

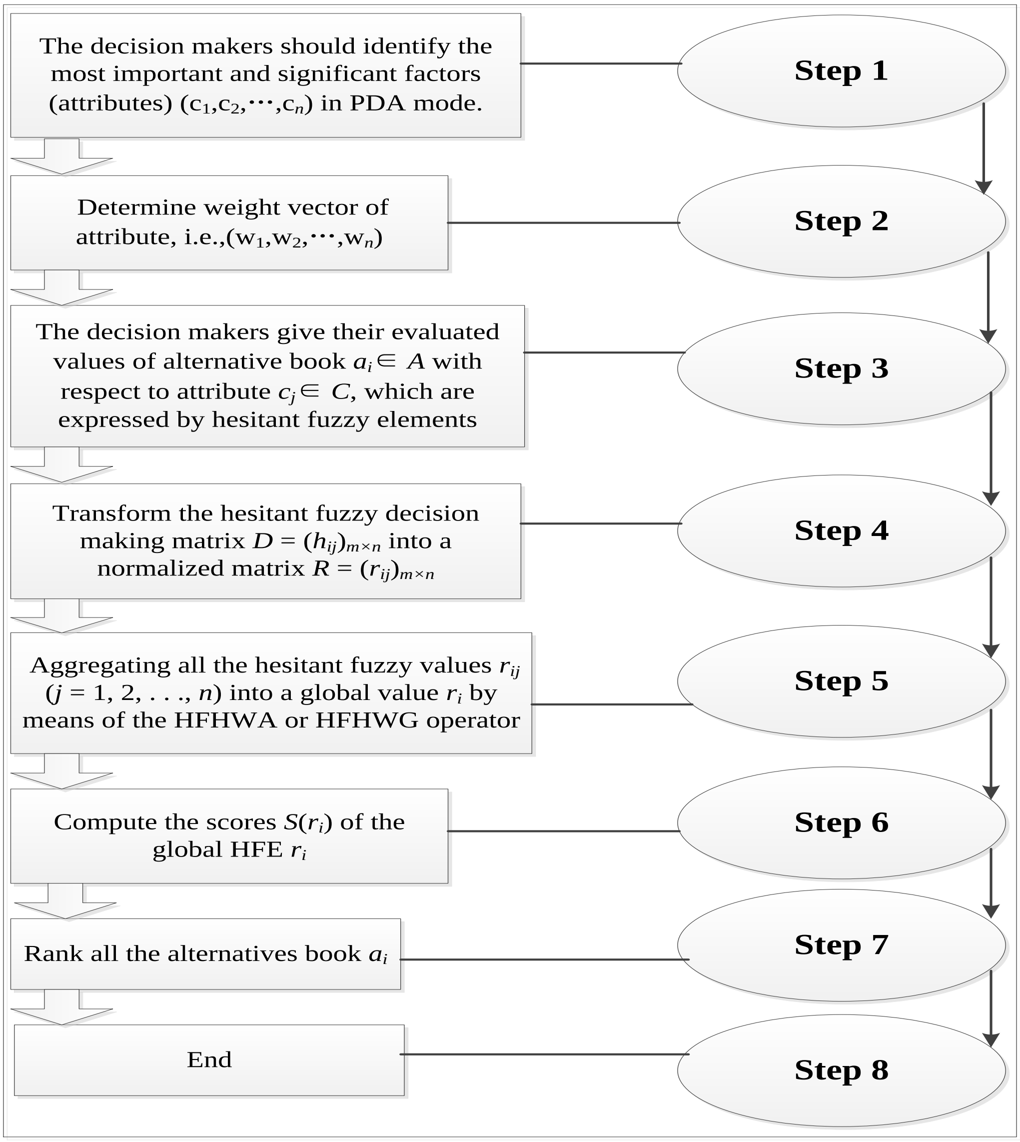

Figure 2. The Procedure of the Proposed Novel Multiple Attribute Decision Making Framework

4.3. Numerical Example

In this section, a book selection problem is used to demonstrate the application and the implementation process of the proposed decision making framework in this paper. The librarian wants to purchase some books for his or her library. Suppose there are five possible books a_i (i = 1, 2, 3, 4, 5), and five attributes: applicability (C_1), Professional (C_2), Authoritative (C_3), Novelty (C_4) and Interestingness (C_5), and the weight vector of the attribute is $w = (0.20, 0.15, 0.20, 0.30,0.15)^T$. To get a more reasonable result, the librarian is required to evaluate the five projects books a_i (i = 1, 2, 3, 4, 5), the hesitant fuzzy decision matrix $D = (h_{ij})_{5\times5}$ is presented in Table 1.

Table 1. Original Hesitant Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5
a_1	({0.9,0.8,0.7})	({0.7,0.8})	({0.1,0.2})	({0.1,0.2})	({0.6,0.8})
a_2	({0.6.0.7})	({0.3,0.4})	({0.4,0.6})	({0.4,0.5,0.6})	({0.5,0.3})
a_3	({0.9,0.7})	({0.5,0.6,0.8})	({0.2,0.3,0.4})	({0.3,0.4})	({0.6,0.5})
a_4	({0.7,0.5,0.6})	({0.8,0.5})	({0.3,0.2})	({0.4,0.2,0.3})	({0.5,0.6})
a_5	({0.7,0.8})	({0.3,0.6})	({0.1,0.3})	({0.4})	({0.1,0.4,0.5})

The five attributes mentioned above are benefit attributes, so they do not need to be normalized and the normalized matrix $R = (r_{ij})_{m\times n}$ is the same as the original hesitant fuzzy decision matrix $D = (h_{ij})_{5\times5}$.

Utilize the HFHWA operator to aggregate the hesitant fuzzy decision matrix ((suppose r=3), then the overall assessments of the five alternatives can be calculated. To save space we just present the result of alternative a_5, which is shown in Table 2.

Table 2. Overall Assessments of Alternative

alternative	overall assessments
r_5	({0.2708, 0.3124, 0.3283, 0.3693, 0.3485, 0.3892, 0.2883, 0.3298, 0.3456, 0.3863, 0.3656, 0.4060, 0.3128, 0.3540, 0.3696, 0.4099, 0.3895, 0.4293, 0.3301, 0.3711, 0.3866, 0.4265, 0.4063, 0.4458, 0.2932, 0.3346, 0.3504, 0.3911, 0.3705, 0.4107, 0.3107, 0.3519, 0.3676, 0.4079, 0.3874, 0.4273, 0.3350, 0.3759, 0.3914, 0.4312, 0.4111, 0.4503, 0.3522, 0.3928, 0.4082, 0.4476, 0.4277, 0.4665, 0.3070, 0.3483, 0.3640, 0.4044, 0.3839, 0.4239, 0.3244, 0.3655, 0.3810, 0.4211, 0.4008, 0.4404, 0.3487, 0.3893, 0.4047, 0.4442, 0.4242, 0.4632, 0.3658, 0.4062, 0.4214, 0.4604, 0.4407, 0.4792, 0.2807, 0.3222, 0.3380, 0.3788, 0.3581, 0.3986, 0.2981, 0.3395, 0.3552, 0.3958, 0.3752, 0.4154, 0.3225, 0.3636, 0.3792, 0.4193, 0.3990, 0.4386, 0.3398, 0.3807, 0.3961, 0.4358, 0.4157, 0.4549, 0.3030, 0.3443, 0.3601, 0.4005, 0.3800, 0.4201, 0.3204, 0.3615, 0.3771, 0.4172, 0.3969, 0.4366, 0.3447, 0.3854, 0.4009, 0.4404, 0.4204, 0.4594, 0.3619, 0.4023, 0.4176, 0.4567, 0.4369, 0.4755, 0.3168, 0.3580, 0.3736, 0.4138, 0.3934, 0.4332, 0.3341, 0.3750, 0.3906, 0.4304, 0.4102, 0.4495, 0.3583, 0.3988, 0.4141, 0.4533, 0.4335, 0.4722, 0.3754, 0.4155, 0.4307, 0.4695, 0.4499, 0.4880})

Calculate the score values $S(r_i)$ (i = 1,2,3,4,5) of the overall values r_i (i = 1,2,3,4,5) in accordance with Definition 4, we can get

$S(r_1)$= 0.5157, $S(r_2)$= 0.4429, $S(r_3)$= 0.4733, $S(r_4)$= 0.4790, $S(r_5)$= 0.3891

According to the score values $S(r_i)$ (i = 1,2,3,4,5), we can get the ranking of alternatives a_i (i =1, 2, 3, 4, 5):$a_1 > a_4 > a_3 > a_2 > a_5$. Therefore, the most desirable alternative is a_1.

In the decision making process, we can choose different aggregation operators according to the practical problems. Different results may be produced, but they reflect different preferences of the decision makers.

5. Conclusion

The existing aggregation operators for HFEs are based on algebraic t-norm and t-conorm. In this paper, we extend the Hamacher operations to aggregate the hesitant fuzzy elements (HFEs). Then the HFHWA (or HFHWG) operator, as one of the proposed hesitant fuzzy Hamacher aggregation functions, is applied to develop a novel multi attribute decision making method to solve hesitant fuzzy MADA problems. Book selection problem usually is very complex and unstructured in the daily management of library, because variety of uncontrollable and unpredictable factors affects the evaluation and decision-making process at different levels. In order to overcome the existing methods' shortcoming to select the most desirable for book for libraries, a novel multiple attribute decision making framework based on Hamacher operations in PDA mode is proposed under the hesitant fuzzy environment. Finally, a numerical example of book selection is given to illustrate the application of the proposed framework. In the future research, we will extend the proposed evaluation framework to deal with interval-valued dual hesitant fuzzy MADM problems, and we will apply them to other domains such as risk management and project management.

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