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# A New Method under Hesitant Fuzzy Context about Green Supplier Selection

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***Abstract***

*In recent years, to protect the environment by the people all over the world has become a significant point. Green environment and sustainable development are always topic in human life. How to select an optimal green supplier is an important aspect of it. Combing with preference of the decision maker, an optimal green supplier may be obtained. Owing to the difficulty of acquiring the decision maker's preferences accurately, hesitant fuzzy set is introduced to help us construct decision making matrix. Combining attribute weight and position weight, we propose a new operator and score function to evaluate the performance of several suppliers, which is a critical step of the selection of green supplier. Finally, a case study is demonstrated to verify the reliability and applicability of the proposed method.*

***Keywords:*** *green suppler, hesitant fuzzy set, OWA, attribute weight, score function*

## 1. Introduction

With the rapid development of economy and technology, environmental sustainable development has become a new key point all over the world. Supplier selection is a key operational task for developing supply chain partnerships in sustainable development. In this topic, selection of green supplier has been granted a new meaning. Different with traditional supplier, green supplier emphasize environmental and economic tradeoffs. The tradeoffs are typically according to a variety of operational and strategic sustainable metrics. Many researches are developed to study how to select better green supplier, such as multiple attribute decision making method [1-4]. Here, environmental protection issues are considered in many researches [14-17]. Lee *et al.* (2009) applied FAHP integrated with the Delphi method for green supplier evaluation [16]. Humphreys *et al.* (2006) proposed the fuzzy based system for green supplier selection based on the quantitative and qualitative environmental criteria. The major benefit of this system is its computational parsimony [18-19].

Because there are subjective and objective assessments given by decision maker, uncertain and imprecise are the basic characteristics of decision maker's preferences. In other words, to express the decision maker's preferences accurately is difficult. Owing to this, Zadeh defined the basic concept of fuzzy sets based on the theory of fuzzy mathematics. The main characteristic of fuzzy sets is that: the membership function assigns to each element $x$ in a universe of discourse $X$ a membership in interval [0-1] and the non-membership degree equals one minus the membership degree. Since fuzzy set was proposed, it has been a popular method to solve imperfect and vague of decision maker's preferences [5-6]. In 2009, Torra and Narukawa found sometimes it was difficult to give the membership or non-membership into one value and which may be caused by a doubt among a set of different values. Then, they defined hesitant fuzzy sets to handle this problem, which permits the membership into a set presented as several possible values between 0 and 1[7]. Meanwhile, Torra (2010) developed some simple aggregation operators of hesitant fuzzy sets to aggregate the information of

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different decision makers or attributes [8].

In this paper, we introduce a new method named multiple attribute decision making method to evaluate the performance of green supplier. The acquirement of preference of the decision maker or experts is the key step to accomplish this assessment. Thus, hesitant fuzzy set is introduced in this paper, which has been widely used in copying with this imperfect problem[9-11].An operator combining general attribute weights and attribute position weights is proposed to obtain a scientific and reasonable aggregated results of attributes. Score function is also introduced to acquire the decision making result. Finally, a numerical example is demonstrated to verify applicable and reasonable of this evaluation method in hesitant fuzzy environment.

The main contributions of this paper include the following: (1) the construction of the evaluation process of green supplier; (2) the design of WAWA for attribute combination to evaluate the performance of green supplier; (3) the design of a new score function with variance; (4) the application of coordination evaluation based on the proposed method. The rest of this paper is organized as follows. In Section 2, we review some concepts of hesitant fuzzy sets. Section 3 proposed the new method. A case study is demonstrated in Section 4. Finally, Section 5 concludes this paper.

## 2. The Relevant Concepts and Model

In this section, we introduce hesitant fuzzy multiple decision making methods to evaluate performance of green supplier. Based on the basic concepts of hesitant fuzzy aggregated operator and score function, we define new attribute weight and position weight to form novel operator.

Torra proposed the concept of hesitant fuzzy set which is in terms of a function when applied to a fixed set returns a sunset of [0-1]. Then, in order to easily understood, Xia and Xu [12] express the hesitant fuzzy set by mathematical symbol.

Definition 1[13]. Let $X$ be a universe of discourse, then a hesitant fuzzy set $H$ over $X$ is defined as

$$H = \{< x, h_H(x) > | x \in X\}, \tag{1}$$

Where $h_H(x)$ is a set of some values in [0-1], symbolizing the possible membership degrees of the element $x$ to $H$. For convenience, we call $h = h_H(x)$ a hesitant fuzzy element and $H$ the set of all hesitant fuzzy elements.

Definition 2 [13] Let $h$, $h_1$ and $h_2$ be three hesitant fuzzy numbers, their basic operations are defined as

$$(1)\ h^c = \cup_{\gamma \in h}\{1-\gamma\};$$

$$(2)\ h_1 \cup h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\};$$

$$(3)\ h_1 \cap h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}.$$

Here, hc represents the complement of the hesitant fuzzy number h. In addition, Torra [13] defined that the envelopment of a hesitant fuzzy element is an IFV.

Definition 3[13] Let h, be a hesitant fuzzy number, the IFV Aenv(h) as the envelope of h, where Aenv(h) can be defined as (h-, 1- h+), with h- = min{γ | γ∈ h} and h+ = max{γ | γ∈ h}.

(1) $A_{env}(h^c) = (A_{env}(h))^c$;

(2) $A_{env}(h_1 \cup h_2) = A_{env}(h_1) \cup A_{env}(h_2)$;

(3) $A_{env}(h_1 \cap h_2) = A_{env}(h_1) \cap A_{env}(h_2)$;

Then, an aggregation principle for hesitant fuzzy elements is proposed by Torra and Narukawa [7] and Torra [8].

Definition 4 Let H = {h1, h2, …, hn} be a set of n hesitant fuzzy elements, $\Theta$ be a function on H, $\Theta$ : [0, 1]N→ [0, 1]. $\Theta$ can be defined as follow:

$$\Theta_E = \bigcup_{\gamma \in \{h_1 \times h_2 \times \ldots \times h_n\}} \{\Theta(\gamma)\} \tag{2}$$

Definition 5[8] Let h, h1 and h2 be three hesitant fuzzy numbers, their new basic operations are defined by Xia and Xu as follows:

(1) $h^{\lambda} = \bigcup_{\gamma \in h} \{\gamma^{\lambda}\}$ ;

(2) $\lambda h = \bigcup_{\gamma \in h} \{1-(1-\gamma)^{\lambda}\}$ ;

(3) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$ ;

(4) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$ .

Generally speaking, the length of the membership of $h_1$ denoted by $l(h_1(x_i))$ does not mostly equal to that of $h_2$ denoted by $l(h_2(x_i))$ . Many studies have been conducted to deal with this problem. Xu and Xia suggested that we should extend the shorter one depending on the decision maker's risk preferences until both of them have the same length. Optimists expect desirable results and should add the maximum value, while pessimists anticipate unfavorable outcomes and should add the minimal value. In addition, there are some researchers considering that the decision maker's preference is risk-neutral. Therefore, based on the decision maker's all risk preference, a new method is proposed to overcome the drawback. An extension value $\bar{h} = \eta h^{+} + (1-\eta) h^{-}$ ($0 \le \eta \le 1$) is introduced to obtain the final decision results. The parameter $\eta$ is used to reflect the decision maker's risk preference and is more accurately than others.

As mentioned above, according to different value of parameter $\eta$ , there are three special situations:

(1) if $\eta = 1$ , it indicates that the decision maker's risk preference is risk-seeking;

(2) if $\eta = 0$ ,it indicates that the decision maker's risk preference is risk-averse;

(3) if $\eta = \frac{1}{2}$ , it indicates that the decision maker's risk preference is risk-neutral.

## 3. An Evaluation Model of Collaborative Performance Based on Hesitant Fuzzy Sets

### 3.1. Some Basic Operators Based on Hesitant Fuzzy Sets

Based on Definitions 2 to 5, Xia and Xu proposed a series of aggregation operators according to hesitant fuzzy sets.

Definition 6 [12] Let hj (j = 1, 2, …, n) be a collection of HFSs. A hesitant fuzzy weighted averaging (HFWA) operator is a mapping Hn→ H such that

$$\text{HFWA}(h_1, h_2, \ldots, h_n) = \overset{n}{\underset{j=1}{\oplus}} (w_j h_j) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \{1 - \prod_{j=1}^{n} (1-\gamma_j)^{w_j}\} , \tag{3}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $h_j$($j$ = 1, 2, …, $n$) with $0 \le w_j \le 1$ ($j$ = 1, 2, ..., $n$) and $\sum_{i=1}^{n} w_j = 1$ .

When $w = (1/n, 1/n, \ldots, 1/n)^T$, the HFWA operator reduces to the hesitant fuzzy averaging (HFA) operator:

$$\text{HFA}(h_1, h_2,\ldots, h_n) = \overset{n}{\underset{j=1}{\oplus}}\left(\frac{1}{n}h_j\right) = \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\{1-\prod_{j=1}^{n}(1-\gamma_j)^{1/n}\}. \tag{4}$$

Definition 7 [12] Let $h_j$ ($j$ = 1, 2, …, $n$) be a collection of HFSs. A hesitant fuzzy weighted geometric (HFWG) operator is a mapping $H^n\rightarrow H$ such that

$$\text{HFWG}(h_1, h_2, \ldots, h_n) = \overset{n}{\underset{j=1}{\oplus}}(h_j^{w_j}) = \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\{\prod_{j=1}^{n}\gamma_j^{w_j}\}, \tag{5}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $h_j$($j$ = 1, 2, …, $n$) with $0 \le w_j \le 1$ ($j$ = 1, 2, ..., $n$) and $\sum_{i=1}^{n} w_j = 1$.

When $w = (1/n, 1/n, \ldots, 1/n)^T$, the HFWG operator reduces to the hesitant fuzzy geometric (HFG) operator:

$$\text{HFG}(h_1, h_2, \ldots, h_n) = \overset{n}{\underset{j=1}{\oplus}}(h_j^{1/n}) = \cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\{\prod_{j=1}^{n}\gamma_j^{1/n}\}. \tag{6}$$

Definition 8 [12] Let $h_j$ ($j$ = 1, 2, …, $n$) be a collection of HFSs. A hesitant fuzzy ordered weighted averaging (HFOWA) operator is a mapping $H^n\rightarrow H$ such that

$$\text{HFOWA}(h_1, h_2, \ldots, h_n) = \overset{n}{\underset{j=1}{\oplus}}(\omega_j h_{\sigma(j)}) = \cup_{\gamma_{(1)}\in h_{(1)},\gamma_{(2)}\in h_{(2)},\cdots,\gamma_{(n)}\in h_{(n)}}\{1-\prod_{j=1}^{n}(1-\gamma_{(j)})^{\omega_j}\}, \tag{7}$$

Where ($\sigma(1)$, $\sigma(2)$, …, $\sigma(n)$) is a permutation of (1, 2, …, $n$), such that $h_{\sigma(j)} \ge h_{\sigma(j-1)}$ for all j = 2, 3, …, $n$ and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the position weight vector with $0 \le \omega_j \le 1$ ($j$ = 1, 2, ..., $n$) and $\sum_{i=1}^{n}\omega_j = 1$.

When $\omega = (1/n, 1/n, \ldots, 1/n)^T$, the HFOWA operator reduces to the hesitant fuzzy ordered averaging (HFOA) operator:

$$\text{HFOA}(h_1, h_2, \ldots, h_n) = \overset{n}{\underset{j=1}{\oplus}}\left(\frac{1}{n}h_{\sigma(j)}\right) = \cup_{\gamma_{(1)}\in h_{(1)},\gamma_{(2)}\in h_{(2)},\cdots,\gamma_{(n)}\in h_{(n)}}\{1-\prod_{j=1}^{n}(1-\gamma_{(j)})^{1/n}\}. \tag{8}$$

Definition 9 [12] Let $h_j$($j$ = 1, 2, …, $n$) be a collection of HFSs. A hesitant fuzzy ordered weighted geometric (HFWG) operator is a mapping $H^n\rightarrow H$ such that

$$\text{HFOWG}(h_1, h_2, \ldots, h_n) = \overset{n}{\underset{j=1}{\oplus}}(h_{\sigma(j)}^{\omega_j}) = \cup_{\gamma_{(1)}\in h_{(1)},\gamma_{(2)}\in h_{(2)},\cdots,\gamma_{(n)}\in h_{(n)}}\{\prod_{j=1}^{n}\gamma_{\sigma(j)}^{\omega_j}\}, \tag{9}$$

where ($\sigma(1)$, $\sigma(2)$, …, $\sigma(n)$) is a permutation of (1, 2, …, $n$), such that $h_{\sigma(j)} \ge h_{\sigma(j-1)}$ for all j = 2, 3, …, $n$ and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the position weight vector with $0 \le \omega_j \le 1$ ($j$ = 1, 2, ..., $n$) and $\sum_{i=1}^{n}\omega_j = 1$.

When $\omega = (1/n, 1/n, \ldots, 1/n)^T$, the HFOWG operator reduces to the hesitant fuzzy geometric (HFOG) operator:

$$\text{HFOG}(h_1, h_2, \ldots, h_n) = \overset{n}{\underset{j=1}{\oplus}}(h_{\sigma(j)}^{1/n}) = \cup_{\gamma_{(1)}\in h_{(1)},\gamma_{(2)}\in h_{(2)},\cdots,\gamma_{(n)}\in h_{(n)}}\{\prod_{j=1}^{n}\gamma_{\sigma(j)}^{1/n}\}. \tag{10}$$

### 3.2. A New Aggregation-Associated Operator

The determination of the reasonable attribute weights plays an important role in evaluation model. Then, we will define a new aggregation-associated weight vector combining attribute weight vector and position weight vector, which not only considers the importance of hesitant fuzzy numbers and their ordered positions, but also reflect the decision maker’s preference.

Firstly, based on decision makers’ preference, we define a new aggregation-associated weight vector as follows:

$$\lambda_j = \theta\, w_j + (1 - \theta)\, \omega_j, \tag{11}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ is the weight vector of $h_j (j = 1, 2, \ldots, n)$ with $0 \leq \lambda_j \leq 1$ $(j = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} \lambda_j = 1$. The parameter $\theta$ denotes the attitude of decision maker between attribute weights and position weights.

According to Wang (2012), when a decision maker can provide subjective preferences about attribute weights, we assume that the preferences are expressed by linear inequality constraints as the following forms, for $i \neq j$:

Form1. A weak ranking: $\{\lambda_i \geq \lambda_j\}$;

Form2. A strict ranking: $\{\lambda_i - \lambda_j \geq \alpha_i\}(\alpha_i > 0)$;

Form3. A ranking of differences: $\{\lambda_i - \lambda_j \geq \lambda_k - \lambda_l\}$, for $j \neq k \neq l$;

Form4. A ranking with multiples: $\{\lambda_i \geq \alpha_i \lambda_j\}(0 \leq \alpha_i \leq 1)$;

Form5. An interval form: $\{\alpha_i \leq \lambda_i \leq \alpha_i + \varepsilon_i\}(0 \leq \alpha_i \leq \alpha_i + \varepsilon_i \leq 1)$.

Secondly, based on the new weight vector, we define a new serious of operators according to hesitant fuzzy sets as follows:

Definition 10 Let $h_j$ $(j = 1, 2, \ldots, n)$ be a collection of HFSs. A hesitant fuzzy aggregation-associated weighted averaging (HFAWA) operator is a mapping $H^n \rightarrow H$ such that

$$\mathrm{HFAWA}(h_1, h_2, \ldots, h_n) = \mathop{\oplus}_{j=1}^{n} (\lambda_j h_j) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \{1 - \prod_{j=1}^{n} (1 - \gamma_j)^{\lambda_j}\}, \tag{12}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ is the weight vector of $h_j (j = 1, 2, \ldots, n)$ with $0 \leq \lambda_j \leq 1$ $(j = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} \lambda_j = 1$.

When $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$, the HFAWA operator reduces to the hesitant fuzzy aggregation-associated averaging (HFAA) operator:

$$\mathrm{HFAA}(h_1, h_2, \ldots, h_n) = \mathop{\oplus}_{j=1}^{n} (\frac{1}{n} h_j) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \{1 - \prod_{j=1}^{n} (1 - \gamma_j)^{1/n}\}. \tag{13}$$

Definition 11 Let $h_j$ $(j = 1, 2, \ldots, n)$ be a collection of HFSs. A hesitant fuzzy aggregation-associated weighted geometric (HFAWG) operator is a mapping $H^n \rightarrow H$ such that

$$\mathrm{HFAWG}(h_1, h_2, \ldots, h_n) = \mathop{\oplus}_{j=1}^{n} (h_j^{\lambda_j}) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \{\prod_{j=1}^{n} \gamma_j^{\lambda_j}\}, \tag{14}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ is the weight vector of $h_j (j = 1, 2, \ldots, n)$ with $0 \leq \lambda_j \leq 1$ $(j = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} \lambda_j = 1$.

When $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$, the HFAWG operator reduces to the hesitant fuzzy aggregation-associated geometric (HFAG) operator:

$$\mathrm{HFAG}(h_1, h_2, \ldots, h_n) = \mathop{\oplus}_{j=1}^{n} (h_j^{1/n}) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \{\prod_{j=1}^{n} \gamma_j^{1/n}\}. \tag{15}$$

### 3.3. Score Function

After aggregating hesitant fuzzy sets, a key step is to compare different assessments by score function.

Definition 12 [12] Let $h$ be a hesitant fuzzy number. The score function of $h$ can be obtained as follow:

$$S(h) = \frac{1}{l_h} \sum_{\gamma \in h} \gamma \ , \quad (16)$$

Where $l_h$ denotes the number of the elements in $h$.

Definition 13[12] for two hesitant fuzzy number $h_1$ and $h_2$, we have if $S(h_1) > S(h_2)$, then $h_1$ is better than or preferred to $h_2$, denoted by $h_1 \succ h_2$; if $S(h_1) = S(h_2)$, then $h_1$ is indifferent to $h_2$, denoted by $h_1 \sqcup h_2$; if $S(h_1) < S(h_2)$, then $h_1$ is worse than or less preferred to $h_2$, denoted by $h_1 \prec h_2$.

In hesitant fuzzy evaluation model, score function is useful to compare different alternative. However, this score function may have some troubles in some situations. For example, there are three hesitant fuzzy numbers $h_1 = \{0.2, 0.8\}$, $h_2 = \{0.5\}$, $h_3 = \{0.3, 0.5, 0.7\}$, which have the same score 0.5. However, their scores cannot judge which one is the optimal choice. Therefore, we introduce the idea of variance to define a new score function to measure the deviation of hesitant fuzzy elements.

Definition 14 Let $h$ be a HFN. The score function of $h$ is defined as follow:

$$S(h) = \min_{\gamma_i \in h}\{\gamma_i\} + \left(\sum_{i=1}^{l_h} \gamma_i / l_h - \min_{\gamma_i \in h}\{\gamma_i\}\right) \cdot [1 - \sigma(\gamma)] \ , \quad (17)$$

where $\sigma(\gamma) = \sqrt{\dfrac{\sum_{i=1}^{l_h} (\gamma_i - \sum_{i=1}^{l_h} \gamma_i / l_h)^2}{l_h}}$ such that $0 < \gamma_i \le 1$, and $l_h$ denotes the number of the elements in $h$.

The score function with variance can reflect the meaning of hesitant fuzzy number better. Then, we will use the new score function to compare different alternative in the assessment model of green supply coordination mechanism.

### 3.4 Process of Evaluation Model

In this section, we will propose a procedure to form this evaluation model based on mentioned methods, where attribute values take the form of hesitant fuzzy numbers. The procedure includes the following steps:

**Step1**. For a evaluation problem, we construct a decision matrix $H = [\tilde{h}_{ij}]_{m \cdot n}$, where all the arguments $\tilde{h}_{ij}$ ($i = 1, 2, ..., m$; $j = 1, 2, ..., n$) are hesitant fuzzy numbers, given by the decision maker. As for every alternative $A_i$ ($i = 1, 2, ..., m$), the decision maker is invited to express evaluation or preference according to each attribute $C_j$ ($j = 1, 2, ..., n$) by a hesitant fuzzy number $h_{ij}$ ($i = 1, 2, ..., m$; $j = 1, 2, ..., n$). Then, based on the risk preference of decision maker, a normal decision making matrix can be obtained as follows:

$$H_{m \times n} = \begin{pmatrix} \tilde{h}_{11} & \tilde{h}_{12} & \cdots & \tilde{h}_{1n} \\ \tilde{h}_{21} & \tilde{h}_{22} & \cdots & \tilde{h}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{h}_{m1} & \tilde{h}_{m2} & \cdots & \tilde{h}_{mn} \end{pmatrix} \quad (18)$$

**Step2.** The decision maker specifies the attribute weights of the $n$ attributes denoted as $w = (w_1, w_2, \ldots, w_n)^T$ with $0 \le w_j \le 1$ ($j = 1, 2, ..., n$) and $\sum_{i=1}^{n} w_j = 1$, and the position weights of the $n$ positions denoted as $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ with $0 \le \omega_j \le 1$ ($j$ = 1, 2, ..., $n$) and $\sum_{i=1}^{n} \omega_j = 1$. Then, based on parameter $\theta$, aggregation-associated weights can be acquired.

**Step3.**The hesitant fuzzy aggregation-associated weighted averaging (HFAWA) operator denoted as Eq.(12) or the hesitant fuzzy aggregation-associated weighted geometric (HFAWG) operator denoted as Eq.(14) are introduced to aggregate the hesitant fuzzy assessments. Then, we can obtain the aggregation results of each alternative.

**Step4.** The new score function proposed in Definition12 is used to compare each alternative in decision making matrix. The scores of the aggregated hesitant fuzzy numbers can be calculated.

**Step5.** Through different scores of alternative, the rank-order can be obtained using Definition 13. Then, we can select optimal alternative by the largest score.

**Step6.** End.

## 4. Illustrative Example

The evaluation model of green supplier selection will be demonstrated in this section with the proposed method in this paper under hesitant fuzzy environment.

In order to verify the proposed evaluation methods in the selection of green supplier, we invite an expert as the decision maker to implement process of decision making. Four companies are selected by the decision maker as the alternatives including $A_1$, $A_2$, $A_3$, $A_4$ from China. Then, the decision maker identifies four attributes denoted as $C_1$, $C_2$, $C_3$, $C_4$ demonstrated in Table 1.

**Table 1. Description of the Four Attributes**

| Attributes | Explanation |
|---|---|
| $C_1$ | Economic benefits |
| $C_2$ | brand benefits |
| $C_3$ | environmental benefits |
| $C_4$ | Enterprise benefits |

The decision maker gives preference of each alternative on each attribute, respectively. Therefore, a hesitant fuzzy decision matrix $H = [\tilde{h}_{ij}]_{4 \times 4}$ can be illustrated in Table 2.

**Table 2. Original Hesitant Fuzzy Decision Matrix**

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $C_1$ | {0.1,0.3} | {0.2} | {0.3,0.5} | {0.4,0.6,0.8} |
| $C_2$ | {0.3,0.5} | {0.1,0.3,0.5} | {0.2,0.5} | {0.3,0.6} |
| $C_3$ | {0.2,0.3,0.4} | {0.1,0.2,0.3} | {0.1,0.3,0.4} | {0.4, 0.7} |
| $C_4$ | {0.2,0.4,0.5,0.6} | {0.4, 0.7} | {0.2,0.6,0.7.0.8} | {0.5,0.6,0.7} |

We suppose the decision maker is risk-neutral by interviewing with him. Then, it can be obtained that $\eta$ = 1/2. Therefore, the normal decision matrix can be acquired in Table 3.

**Table 3. Normal Hesitant Fuzzy Decision Matrix**

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|---|
| $C_1$ | {0.1,0.2,0.2,0.3} | {0.2,0.2,0.2,0.2} | {0.3,0.4,0.4,0.5} | {0.4,0.6,0.6,0.8} |
| $C_2$ | {0.3,0.4,0.4,0.5} | {0.1,0.3,0.3,0.5} | {0.2,0.35,0.35,0.5} | {0.3,0.45,0.45,0.6} |
| $C_3$ | {0.2,0.3,0.3,0.4} | {0.1,0.2,0.2,0.3} | {0.1,0.3,0.27,0.4} | {0.4, 0.55,0.7} |
| $C_4$ | {0.2,0.4,0.5,0.6} | {0.4, 0.55,0.7} | {0.2,0.6,0.7.0.8} | {0.5,0.6,0.6,0.7} |

Then, the decision maker gives the attribute weights of these four attributes denoted as $w = (0.3, 0.2, 0.15, 0.35)^T$. The position weights is obtained by some other experts, which can be as a standard denoted as $\omega = (0.4, 0.3, 0.2, 0.1)^T$. The decision maker holds that $\theta = 0.5$.

Based on Definition 10, the hesitant fuzzy assessments can be aggregated through the hesitant fuzzy aggregation-associated weighted averaging (HFAWA) operator in Table 4. The hesitant fuzzy aggregation-associated weighted geometric (HFAWG) operator also can aggregate hesitant fuzzy preference in Table 5.

**Table 4. Aggregated Hesitant Fuzzy Preference Based on HFAWA**

| | |
|---|---|
| $A_1$ | {0.3892, 0.3921, 0.3958, 0.4168, 0.4179, 0.4325, 0.4355, 0.4485, 0.4678, 0.4799, 0.4874, 0.4866, 0.5102, 0.5188, 0.5341, 0.5405, 0.5814, 0.5919, 0.6228, 0.6334, 0.6476, 0.6585, 0.6684, 0.6693, 0.6756, 0.6761, 0.6845, 0.6958,} |
| $A_2$ | {0.3954, 0.4152, 0.4258, 0.4351, 0.4456, 0.4579, 0.4867, 0.4911 0.5123, 0.5334, 0.5451,0.5574, 0.5661, 0.5768, 0.5788, 0.5825,0.5924,0.6148, 0.6154, 0.6346, 0.6372, 0.6488, 0.6535, 0.6759} |
| $A_3$ | {0.4286, 0.4392, 0.4439, 0.4521, 0.4755, 0.4866, 0.4921, 0.5189, 0.5245, 0.5345, 0.5481, 0.5538, 0.5679, 0.5781, 0.5828, 0.5946, 0.6152, 0.6229, 0.6335, 0.6448, 0.6568, 0.6688, 0.6728, 0.6815, 0.6959, 0.7111, 0.7358, 0.7545} |
| $A_4$ | {0.4548, 0.4733, 0.4741, 0.4929, 0.5311, 0.5457, 0.5820, 0.5922, 0.6045, 0.6152, 0.6326, 0.6438, 0.6524, 0.6576, 0.6602, 0.6858, 0.6897, 0.6911, 0.7185, 0.7197, 0.7211, 0.7324, 0.7425, 0.7689, 0.7785, 0.7841, 0.7912,} |

**Table 5. Aggregated Hesitant Fuzzy Preference Based on HFAWG**

| | |
|---|---|
| $A_1$ | {0.3981, 0.4082, 0.4125, 0.4167, 0.4235, 0.4328, 0.4413, 0.4476, 0.4519, 0.4786, 0.4881, 0.4916, 0.5192, 0.5235, 0.5348, 0.5495, 0.5725, 0.5928, 0.6134, 0.6358, 0.6586, 0.6768, 0.6895, 0.6924, 0.6986, 0.7012, 0.7125, 0.7224,} |
| $A_2$ | {0.4162, 0.4285, 0.4387, 0.4489, 0.4579, 0.4652, 0.4985, 0.5031 0.5193, 0.5457, 0.5568,0.5778, 0.5887, 0.5898, 0.5912, 0.5945,0.5968,0.6068, 0.6314, 0.6525, 0.6623, 0.6817, 0.6956, 0.7059} |
| $A_3$ | {0.4575, 0.4598, 0.4672, 0.4775, 0.4824, 0.4971, 0.4989, 0.5015, 0.5378, 0.5428, 0.5613, 0.5728, 0.5876, 0.5962, 0.5998, 0.6016, 0.6352, 0.6732, 0.6828, 0.6948, 0.7012, 0.7045, 0.7358, 0.7418, 0.7576, 0.7623, 0.7812, 0.7925} |
| $A_4$ | {0.4652 0.4845, 0.4922, 0.4997, 0.5245, 0.5568, 0.5745, 0.5862, 0.5925, 0.6148, 0.6254, 0.6321, 0.6428, 0.6546, 0.6625, 0.6724, 0.6912, 0.7025, 0.7214, 0.7321, 0.7389, 0.7416, 0.7485, 0.7615, 0.7779, 0.7825, 0.7987, 0.8126, 0.8249, 0.8378, 0.8482, 0.8627} |

 

Based on the new score function, it can be obtained the scores of each alternative showed in Table 6 and Table 7.

**Table 6. Scores and Rank Based on HFAWA**

| | score | rank |
|---|---|---|
| $A_1$ | 0.4117 | 4 |
| $A_2$ | 0.4963 | 3 |
| $A_3$ | 0.5221 | 2 |
| $A_4$ | 0.6445 | 1 |

**Table 7 Scores and Rank Based on HFAWG**

| | score | rank |
|---|---|---|
| $A_1$ | 0.4356 | 4 |
| $A_2$ | 0.5128 | 3 |
| $A_3$ | 0.5513 | 2 |
| $A_4$ | 0.6774 | 1 |

Then, the rank of this assessment problem is demonstrated as $A_4 \succ A_3 \succ A_2 \succ A_1$ from Table 6 and Table 7. It is easy to know that $A_4$ is the optimal green supplier. Although *HIFEWA* operator and *HIFEWG* operator can obtain a same rank of all alternatives, the scores are different.

## 5. Conclusion

Selecting the best supplier is a vital component of the business relationship and it is one of the most critical issues in the competitive environment. Because of the inherent vagueness of human preferences as well as the objects being fuzzy and uncertain, the attributes involved in decision making problems are not always expressed in real numbers, and some are better suited to be denoted by fuzzy values, such as hesitant fuzzy values. Therefore, we define the new operators of hesitant fuzzy sets combining attribute weights and position weights. Meanwhile, considering of variance, we propose a new score function to aggregate the assessments of different alternative on each attribute. Finally, an illustrative example is demonstrated to verify the reliability and applicability of the proposed method.

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