

Credit Risk Game Simulation for Supply Chain Finance Based on Multi-agent

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Abstract

Supply chain finance (SCF) is an efficient method to solve SMEs' financing problem, credit risk is the core issue of SCF, game simulation is a new perspective in the research of credit risk. From the view of bounded rational, this paper presents "Replicator dynamic" rules that based on "Reinforcement learning" and "Imitation learning". Then constructs two game simulation models: SMEs lending union game simulation model and Banks-SMEs game simulation model, and using multi-agent simulation to simulate. The simulations solve the following problems: analyzing the risk stability of single group and multi-group replicator dynamic systems, respectively; Introducing Brownian motion, changing the deterministic dynamic system to a random dynamic system, then analyzing its risk stability; Influence on the system's risk stability from the change of storage company's decision. The guiding significance of the simulation results is to improve SCF credit risk management of banks.

Keywords: *Supply chain finance, Credit risk, Multi-agent, Game simulation*

1. Introduction

SCF is a financial service that using controllable credit risk of the whole supply chain, instead of the uncontrollable credit risk of SMEs, to solve SMEs' financing problem. Different from the conventional bank credit, SCF reduces the requirements of three financial statements, pays less attention to traditional index (e.g. guaranty style and asset scale), but emphasizes trading background and the core enterprise's actual strength, credit. It means, SCF focus on the credit status of the whole supply chain. Therefore, credit risk is the core problem of SCF.

Berger is the first one who uses SCF in SME financing [1], and drives a group of scholars to research the SCF about operation mode, risk management, financing mode, financing products. He adds that racial policies, financial institution and all the links of the industrial chain should be combined to solve SMEs' lending problem. And aiming at different SMEs, different lending technologies should be formulated to increase the possibility of SME finance [2].

In the aspect of risk management, Buzacott considered demand uncertainty and different loan rates, then discussed retailer's inventory quantity and financing decision [3]. Kerr discussed SCF's cash management based on the combination of trade financing and credit project [4]. Busch summed up the risk congeries characteristic of SCF's external environment and inside subjects, and put forward that management cost can be greatly reduced if using suitable risk control means [5]. Camerinelli analyzed SCF's operation process and pointed out that cash flow can be optimized due to the enhancement of interactions among enterprises, but integrating and managing capital is risky [6]. It can be seen that SCF is often regarded as a mean of financial optimization, and researches

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about its risk management are usually closely connected with supply chain's structure, trading link and financial management.

Recent years, scholars try to apply the game theory to SCF. Yan N. studied game relationship between bank and SME based on profit-maximization objective, and presented the optimal decision [7]. Wan D.H. used game theory to discuss the cost-benefit of SCF's special business mode [8].

As for agent, recently, agent modeling and simulation method is widely used in economic management, and game simulation is a special application of it. Axelrod found that no matter how complex the opponents are, "Tit-for-tat" will eventually win in cooperation [9]. Zhang T. and Sun L.Y. analyzed gamers' individual rationality, then designed an evaluation strategy based on evaluation of the cumulative aspiration tense to simulate the repeated game of several kinds of typical conflict [10]. Zhang F and Xuan H.Y. analyzed the evolution of social cooperation under two disaggregate behavior models, respectively and using imitative learning algorithm to improve strategy [11]. Liu J.J. and Wang J.Y. took the game model of credit market as an example to study evolutionary game based on multi-agent [12]. Liu Z, Zhang X.L. built a multi-agent game simulation framework based on finite automata to study bounded rationality, and realized multi-agent sequential game, mixed game, evolutionary game[13]. Xu Y, Hu B and Qian R used multi-agent simulation modeling method to simulate the dynamic game of group strategic alliance, and the dynamical system's stability was discussed [14].

It is observed that applying the game theory in SCF becomes a new research hotspot, this method clearly describes the interests of SCF's business models. However, these researches mainly use traditional game theory to study SCF and still focus on "completely rationality" level, which means the lack of consideration in the SCF's complexity and participants' bounded rationality. Modeling and simulating techniques based on agent have been applied extensively in the field of economic, and researches about multi-agent game simulation have scored great achievements, but these theories have not yet combined with the risk management of the SCF. Thus it can be deeply studied.

This paper builds credit risk game model for SCF's credit business, then uses multi-group simulation method to simulate the game model, simulates the influence on SCF system from participants' behavior decision-making, so that can analyze credit risk on SCF service.

2. Model Preparation

In this chap, symbols used in this paper are explained in Table 1, while different agents' game model and gain matrix are given by Table 2, Table 3 and Table 4, respectively.

Table 1. Symbolic Hypothesis

Loan amount	L	Profit that SMEs get through loans	R
Bank lending rates	r_0	SMEs' default loss(Operating losses causes from being expelled from the SCF system)	T
Movable property pledge rate	v	SMEs' bonus for long-term good reputation(Including SMEs that get no loans)	Y
Bank supervision cost	C	Fine that storage company pay to bank when it is delinquency	G
Pledge value	L/v	Commission fees that bank pays to storage company	F

All variables are bigger than zero and $Lr_0 - C > 0$.

Table 2. The Game Model and Gain Matrix of SMEs' Credit Union

		SME2	
		Integrity	Default
SME1	Integrity	$R - M, R - M$	$R - M - D, R + L(1 + r_0) - T$
	Default	$R + L(1 + r_0) - T, R - M - D$	$R + L(1 + r_0) - zT, R + L(1 + r_0) - zT$

Table 3. The Game Model and Gain Matrix of Bank and SME

		SME	
		Integrity	Default
Bank	Loan	$Lr_0 - C, R - Lr_0 + Y$	$-L(1 + r_0) - C + S, R + L(1 + r_0) - T$
	No loan	0, Y	0, 0

Table 4. The Game Model and Gain Matrix of Bank, SME and Storage Company

		SME			
		Integrity	Default	Integrity	Default
Bank	Loan	$L'r_0 - C' - F', R' - L'r_0 + Y'$	$-L'(1 + r_0) - C' - F' + L'/v', R' + L'(1 + r_0) - T' - L'/v'$	$L'r_0 - C' - F', R' - L'r_0 + Y'$	$-L'(1 + r_0) - C' - F', R' + L'(1 + r_0) - T' - H'$
	No Loan	F'	F'	F'	$F' + H' - G'$
		Conscientiousness		Muting	
		Storage Company			

3. Multi-Agent Game Simulation Based on Replicator Dynamic

3.1. The Activity Rules Under Replicator Dynamic

In each interaction, participants continually modifies their behavior strategy according to the existing game effect, including “Reinforcement learning” and “Imitation learning”, that is, considering which strategy of the participant can get the highest profit and which strategy of others is more successful. In the evolutionary game system, when the average profit brings by one strategy is higher than that by mixed strategy, the number of participants who use the strategy will increase [15]. Based on the idea, scholars established replicator dynamical model to describe the using proportion and its behavior of a certain strategy.

Therefore, this paper sets the activity rule for the participants of SCF in a replicator dynamical mechanism: “Replicator dynamic” that based on “Reinforcement learning” and “Imitation learning”. The concrete manifestation of participants' behaviors are described by replicator dynamical model.

3.2. Replicator Dynamical Model

Replicator dynamical model is divided into single group model (Study object is one group) and multi-group model (Study object is multi-group).

3.2.1. Single Group Replicator Dynamical Model: The model assumes that each individual in the group shares the same pure strategy set and can only choose one pure strategy s_i at any time. Each participant selects a pure strategy and gets a payment, which changes with group status [16]. Let $S = \{s_1, s_2, \dots, s_k\}$ denotes a pure strategy set for individuals. $x = (x_{s_1}, x_{s_2}, \dots, x_{s_k})$ is the group status at time t, with x_{s_i} denotes the proportion of individuals that select strategy s_i at this time. $f(s_i, x)$ is the expected profit for individuals that select s_i when plays random mating game. $f(x, x)$ is the average profit expectancy of group, $f(x, x) = \sum_k x_{s_i} f(s_i, x)$.

Let $f(s_i, x)$ denotes the growth rate of individual number who select strategy s_i , then single group replicator dynamical model is:

$$\frac{dx_{s_i}(t)}{dt} = [f(s_i, x) - f(x, x)] \cdot x_{s_i}(t) \quad (1)$$

As can be seen from Eq.1, if the average profit brings by strategy s_i is higher than (lower than/equal to) that by mixed strategy, the number of participants who use the strategy will increase (decrease/remain stable).

3.2.2. Multi-Group Replicator Dynamical Model: This model assumes that individuals in different groups have different pure strategy set, group level payment and group evolution speed [17]. Then multi-group replicator dynamical model is:

$$\frac{dx_{s_i}^j(t)}{dt} = [f(s_i^j, x) - f(x^j, x^{-j})] \cdot x_{s_i}^j \quad (2)$$

$j (j = 1, 2, \dots, K)$ is the jth group. $x_{s_i}^j$ is the proportion of individuals in group j who select strategy s_i at time t. x^j is the state of group j at one point. x^{-j} is the state of groups except j at one point. s_i^j is the ith pure strategy of the jth group's strategy set. x is the mixed strategy combination of a mixed group. $f(s_i^j, x)$ is the expected profit of individual in group j who selects s_i^j when the mixed group state is x . $f(x^j, x^{-j})$ is the average profit of the mixed group.

3.3. The Multi-Player Game Simulation of SMEs' Credit Union

3.3.1. Model Building and Stability Analysis: In a SCF system, SMEs form a loan union that guaranteed by core enterprise, obtaining long time and stable business cooperation relationships with bank and core enterprise is the union's goal. This section researches the credit risk of SCF's credit operations through analyzing each member's maintenance or betrayal. This is a single group game, whose game model is based on Table 2.

Assuming that at time t, the proportion of members who betray and maintain is x and $1-x$, $x \in [0, 1]$, respectively. Then in the next period, the two kinds of members' expected profit are as Eq.3 and Eq.4, respectively.

$$f_M = (R - M) \cdot (1 - x) + (R - M - D) \cdot x = R - M - D \cdot x \quad (3)$$

$$f_B = [R + L(1 + r_0) - T] \cdot (1 - x) + [R + L(1 + r_0) - zT] \cdot x = (1 - z) \cdot T \cdot x + R + L(1 + r_0) - T \quad (4)$$

Then the average expected profit of each SME in the union is:

$$f = f_M \cdot (1 - x) + f_B \cdot x = [(1 - z)T + D] \cdot x^2 + [M - D - T + L(1 + r_0)] \cdot x + R - M \quad (5)$$

The replicator dynamical model is as Eq.6, the proportion's change curve of SMEs who betray can draw with the model.

$$\frac{dx}{dt} = x \cdot (f_B - f) = [(z - 1) \cdot T - D] \cdot x^3 + [(2 - z) \cdot T - L(1 + r_0) - M + D] \cdot x^2 + [L(1 + r_0) - T + M] \cdot x \quad (6)$$

Find the equilibrium solution of Eq.6, that is find a solution for $\frac{dx}{dt} = 0$. Obviously one of the equilibrium solutions is $x(t, 0) = 0$. It means at time t, the proportion of betrayed members is zero, which is an ideal equilibrium. Considering under what circumstance the zero solution can maintain stability.

According to the second method of Lyapunov [18], if Eq.6 has a positive definite Lyapunov function $v(x)$, and $\frac{dV}{dt}$ is negative definite, then Eq.6 is stable at the equilibrium. For easier reference, Eq.6 is simplified into Eq.7:

$$\frac{dx}{dt} = a \cdot x^3 + b \cdot x^2 + c \cdot x \quad (7)$$

There into, $a = (z - 1) \cdot T - D$, $b = (2 - z) \cdot T - L(1 + r_0) - M + D$, $c = L(1 + r_0) - T + M$

Select Lyapunov function $V(x) = x^2$, then

$$\frac{dV}{dt} = x^2 \cdot (a \cdot x^3 + b \cdot x^2 + c \cdot x) = x^3 \cdot [a \cdot (x + \frac{b}{2a})^2 + c - \frac{b^2}{4a}] \quad (8)$$

$$\frac{dV}{dt} < 0 \quad (9)$$

$$K = a \cdot (x + \frac{b}{2a})^2 + c - \frac{b^2}{4a} \quad (10)$$

Because $x \in [0, 1]$, then just considering the positive and negative value of Eq.10:

When $a > 0$ and $b > 0$, let $x = 1$, can make K maximize, $K_{max} = a + b + c$.

When $a < 0$ and $b > 0$, let $x = 1$, can make K maximize, $K_{max} = a + b + c$.

When $a > 0$ and $b < 0$, let $\lim x = 0$, can make K close to maximum, $K_{max} = c$.

When $a < 0$ and $b < 0$, let $\lim x = 0$, can make K close to maximum, $K_{max} = c$.

Because

$$a + b + c = (z - 1) \cdot T - D + (2 - z) \cdot T - L(1 + r_0) - M + D + L(1 + r_0) - T + M = 0 \quad (11)$$

Thus solution of inequality $K < 0$ is $\begin{cases} a > 0 \\ b < 0 \end{cases}$ or $\begin{cases} a < 0 \\ b > 0 \end{cases}$, that is

$$\begin{cases} (z - 1) \cdot T - D > 0 \\ (2 - z) \cdot T - L(1 + r_0) - M + D < 0 \\ L(1 + r_0) - T + M < 0 \end{cases} \text{ or } \begin{cases} (z - 1) \cdot T - D < 0 \\ (2 - z) \cdot T - L(1 + r_0) - M + D > 0 \\ L(1 + r_0) - T + M < 0 \end{cases} \quad (12)$$

Therefore, if the system matches Eq.11 and Eq.12, the equilibrium state $(t, 0)$ is stable.

3.3.2. Model Refinement Based on Brownian Motion: In the SME loan union, SME's judgment is influenced by the unpredictable change of individuals' emotional factors, risk preference and expected profit, the decision-making situation of other members, the external environment changes, and so on. Thus the evolution of SME's behaviour decision is uncertain. This paper adds Brownian motion disturbance item to Eq.6 to describe the uncertainties:

$$dx = \{[(z - 1) \cdot T - D] \cdot x^3 + [(2 - z) \cdot T - L(1 + r_0) - M + D] \cdot x^2 + [L(1 + r_0) - T + M] \cdot x\} dt + \delta \cdot dW \quad (13)$$

w is standard Brownian motion, dW is time's Brownian motion random increment, δ is interference strength coefficient. Let $\delta = \sqrt{x(1-x)}$ [14], then $\sqrt{x(1-x)}$ satisfies $x \in [0,1]$; If and only if $x = 1/2$, $\sqrt{x(1-x)}$ reaches the maximum. It means when the amount of members who betray is equal to the amount of members who maintain, stability is most easily to get disturbed. Thus the new replicator dynamical model with Brownian motion random increment is:

$$dx = \{[(z-1) \cdot T - D] \cdot x^3 + [(2-z) \cdot T - L(1+r_0) - M + D] \cdot x^2 + [L(1+r_0) - T + M] \cdot x\} dt + \sqrt{x(1-x)} dW \quad (14)$$

Since dW is a stochastic process, it makes x a stochastic process, as well. Then the system changes from a deterministic dynamic system to a random dynamic system, and the zero solution for Eq.6 cannot satisfy Eq.14. But when the system is under slight disturbance, $\lim_{x(t,0)} = 0$. Therefore, when disturbance is slight, the system can remain or approach stable; But if the fluctuation range of the random increment is very large, the system's stability at zero solution would be drastically affected.

3.3.3. The Simulation and Result Analysis: Then use Simulink in Matlab to simulate the nonlinear stochastic and dynamic system described by Eq. 13.

According to the definition of Brownian motion, the increment per step is independent and obeys a Gaussian distribution: $dW = (W_t - W_s) \sim N(0, t-s)$. Therefore, selects gauss white noise in the Simulink to describe the random increment. Then assign values to parameters of Eq.13, so that can simulate the evolutionary process of x and t . Setting the initial value of x is 0.5.

(1) Parameter assignment case 1

Setting $L = 100, r_0 = 6.56\%, R = 20, T = 120, M = 5, D = 20, z = 1.5$, then the gain matrix is as Table 5.

Table 4. The Gain Matrix of Loan Union with Parameter Assignment Case 1

		SME2	
		Integrity	Default
SME1	Integrity	15,15	-5,6.56
	Default	6.56,-5	-53.44,-53.44

The replicator dynamical model is:

$$dx = (40x^3 - 31.56x^2 - 8.44x) dt + \sqrt{x(1-x)} dW \quad (15)$$

The proportion's change curve of SMEs who betray is as Figure 1(a). Abscissa is t (the number of game), longitudinal coordinate is x .

As can be seen from Figure 1(a) and Figure 1(b), when δ is relatively small, the proportion of SMEs who betray would finally evolve to zero. That means in the long-term game, this loan union is stable, and eventually tends to be integrity. As Figure 1(c) shown, when δ is relatively big, the proportion's change curve of SMEs who betray is in a disorderly fashion and fluctuates wildly, thus cannot maintain stable at any rate.

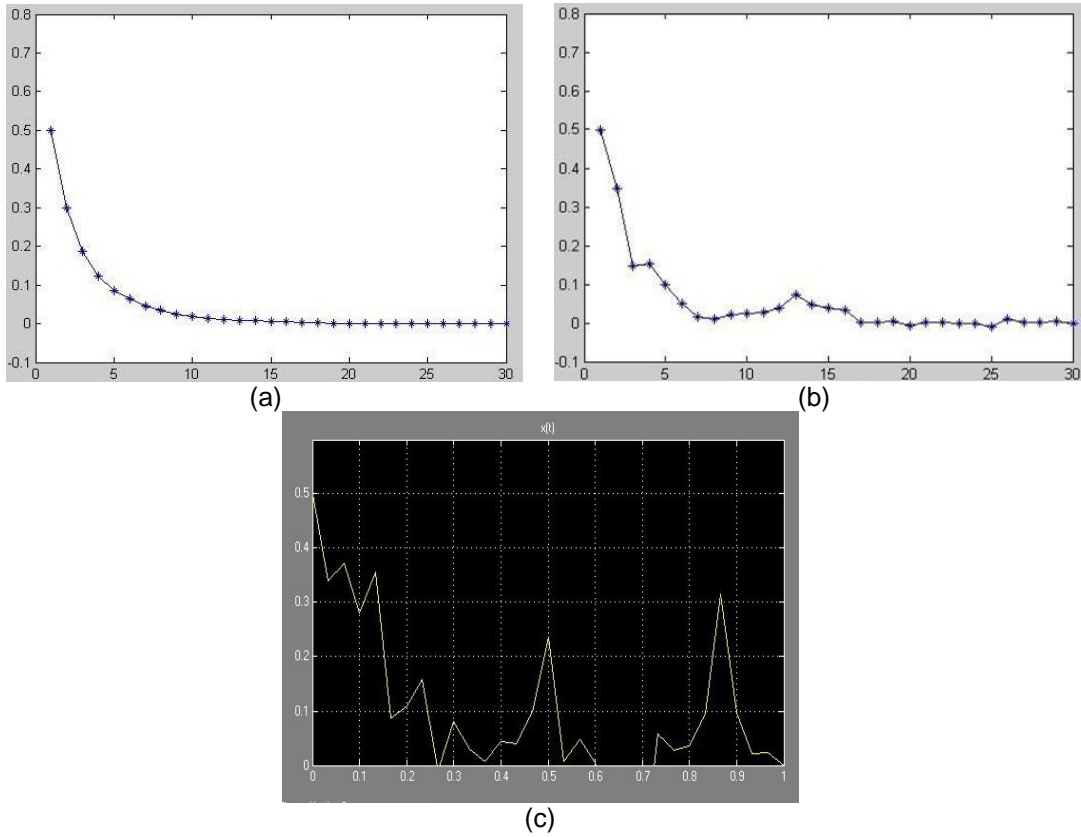


Figure 1. A Zero Means Gauss White Noise, Variance is 0, 10, 100

(2) Parameter assignment case 2

Setting $L = 100, r_0 = 6.56\%, R = 20, T = 90, M = 5, D = 20, z = 1.5$, then the gain matrix is as Table6.

Table 6. The Gain Matrix of Loan Union with Parameter Assignment Case 2

		SME2	
		Integrity	Default
SME1	Integrity	15,15	-5,36.56
	Default	36.56,-5	8.44,8.44

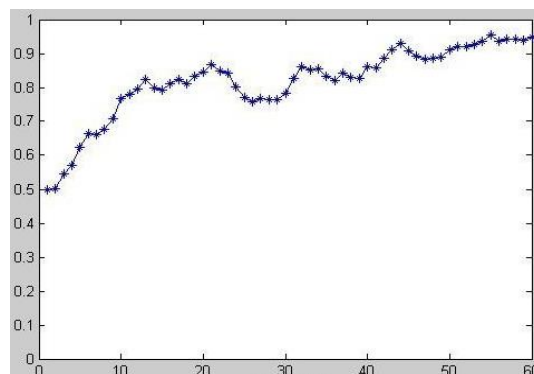


Figure 2. A Zero Mean Gauss White Noise, Variance is 10

The replicator dynamical model is:

$$dx = (25x^3 - 46.56x^2 + 21.56x)dt + \sqrt{x(1-x)}dW \quad (16)$$

The proportion's change curve of SMEs who betray is as Figure 2, the proportion of SMEs who betray would finally tend to 1, that means all SMEs go against the interest of union, the union will dissolve and the credit relation will end.

In conclusion, in case 1, there is a win-win result. That is "reciprocity" game, and finally reaches a steady state that all SMEs select the strategy of "Integrity". While in case 2, the increase of one's profit is at the expense of decreasing others' profit, that is "prisoners dilemma" game, and finally most of SMEs select the strategy of "Default". Thus mutually beneficial cooperation among SMEs cannot have obvious conflict of interest so that the SCF system can maintain a steady trust relationship.

3.4. The Multi-Player Game Simulation of Banks and SMEs

3.4.1. Model Building and Stability Analysis: This section analyzes the connection between the loan probability of banks and the default probability of SMEs, and there are multiple banks and SMEs in this system. This is a multi-group game, whose game model is based on Table 3.

Assuming that at time t, the proportions of banks who select "Loan" and "No loan" are y_1 and $1 - y_1$, $y_1 \in [0, 1]$, respectively. And the proportions of SMEs who "Integrity" and "Default" are y_2 and $1 - y_2$, $y_2 \in [0, 1]$.

The expected profit of banks that select "Loan" is:

$$f_{Loan}^{Bank} = (Lr_0 - C) \cdot (1 - y_2) + [-L(1 + r_0) - C + S] \cdot y_2 \quad (17)$$

The expected profit of banks that select "No loan" is:

$$f_{No}^{Bank} = 0 \quad (18)$$

The average expected profit of banks' mixed strategy is:

$$f^{Bank} = f_{Loan}^{Bank} \cdot y_1 + f_{No}^{Bank} \cdot (1 - y_1) = (-2Lr_0 - L + S) \cdot y_1 y_2 + (Lr_0 - C) \cdot y_1 \quad (19)$$

The expected profit of SMEs that select "Default" is:

$$f_D^{SME} = [R + L(1 + r_0) - T] \cdot y_1 \quad (20)$$

The expected profit of SMEs that select "Integrity" is:

$$f_I^{SME} = (R - Lr_0 + Y) \cdot y_1 + Y \cdot (1 - y_1) = (R - Lr_0) \cdot y_1 + Y \quad (21)$$

The average expected profit of SMEs' mixed strategy is: (22)

Then the replicator dynamical model of account receivable financing between banks and SMEs is:

$$\begin{cases} \frac{dy_1}{dt} = y_1 \cdot (f_{Loan}^{Bank} - f^{Bank}) = (2Lr_0 + L - S) \cdot y_1^2 y_2 - (Lr_0 - C) \cdot y_1^2 - (2Lr_0 + L - S) \cdot y_1 y_2 + (Lr_0 - C) \cdot y_1 \\ \frac{dy_2}{dt} = y_2 \cdot (f_D^{SME} - f^{SME}) = -(2Lr_0 + L - T) \cdot y_1 y_2^2 + Y \cdot y_2^2 + (2Lr_0 + L - T) \cdot y_1 y_2 - Y \cdot y_2 \end{cases} \quad (23)$$

That is,

$$\begin{cases} \frac{dy_1}{dt} = (2Lr_0 + L - S) \cdot y_1^2 y_2 - (Lr_0 - C) \cdot y_1^2 - (2Lr_0 + L - S) \cdot y_1 y_2 + (Lr_0 - C) \cdot y_1 \\ \frac{dy_2}{dt} = -(2Lr_0 + L - T) \cdot y_1 y_2^2 + Y \cdot y_2^2 + (2Lr_0 + L - T) \cdot y_1 y_2 - Y \cdot y_2 \end{cases} \quad (24)$$

Next is stability analysis of the system.

From $\frac{dy_1}{dt} = 0$ and $\frac{dy_2}{dt} = 0$ can get five balance points of this system: $E_1(0,0)$, $E_2(0,1)$, $E_3(1,0)$, $E_4(1,1)$, $E_5(y_1^*, y_2^*)$. Thereinto, $y_1^* = \frac{Y}{2Lr_0 + L - T}$ and $y_2^* = \frac{Lr_0 - C}{2Lr_0 + L - S}$.

According to the stability theory of differential equation, analyzing the local stability of the system's Jacobian matrix can get the steadiness of equilibrium point. When the determinant of the matrix $\det(J) \neq 0$, for any equilibrium point, if $\det(J) > 0$ and trace $tr(J) < 0$, then the equilibrium point is stable.

The Jacobian matrix of this system is: $A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$,

Thereinto, $A_1 = 2(2Lr_0 + L - S) \cdot y_1 y_2 - 2(Lr_0 - C) \cdot y_1 - (2Lr_0 + L - S) \cdot y_2 + Lr_0 - C$

$A_2 = (2Lr_0 + L - S) \cdot y_1^2 - (2Lr_0 + L - S) \cdot y_1$, $A_3 = -(2Lr_0 + L - T) \cdot y_2^2 + (2Lr_0 + L - T) \cdot y_2$

$A_4 = -2(2Lr_0 + L - T) \cdot y_1 y_2 + 2Y \cdot y_2 + (2Lr_0 + L - T) \cdot y_1 - Y$

Determining $\det(J)$ and $tr(J)$ for five equilibrium points, respectively, as shown in Table7.

The ideal stable equilibrium is: banks are willing to provide loans to SMEs, and SMEs keep promise to banks, that is $E_3(1,0)$. Thus:

$$\begin{cases} (-2Lr_0 - L + T + Y)(Lr_0 - C) > 0 \\ Lr_0 + L + C - T - Y < 0 \end{cases} \quad (25)$$

Since $Lr_0 - C > 0$, and $L, r_0, C, T, Y > 0$, Eq.25 is equivalent to Eq.26:

$$2Lr_0 + L - T - Y < 0 \quad (26)$$

Therefore, to make the system achieve the ideal stable equilibrium, the model variables should meet the constraint of Eq.26.

Table 7. The Local Stability of the System's Jacobian Matrix Analysis

equilibrium point	$\det(J)$	$tr(J)$
$E_1(0,0)$	$(-Lr_0 + C) \cdot Y$	$Lr_0 - C - Y$
$E_2(0,1)$	$(-Lr_0 - L - C + S) \cdot Y$	$-Lr_0 - L - C + S + Y$
$E_3(1,0)$	$(2Lr_0 + L - T - Y)(-Lr_0 + C)$	$Lr_0 + L + C - T - Y$
$E_4(1,1)$	$(-2Lr_0 - L + T + Y)(Lr_0 + L + C - S)$	$-Lr_0 + C + T + Y - S$
$E_5(y_1^*, y_2^*)$	$\frac{(2Lr_0 + L - T - Y)(Lr_0 + L + C - S)(Lr_0 - C)}{(2Lr_0 + L - T)(2Lr_0 + L - S)}$	0

3.4.2. The Simulation and Result Analysis: Under Eq.26, fixing $L = 100, r_0 = 6.56\%$, then observing the system's evolution when other variables change. Setting the initial value of both y_1 and y_2 is 0.5, $s = 50, C = 1, T = 300, Y = 20$, then the simulation result based on Eq.24 is as Figure 3.

As Figure 3(a) shown, the system finally achieves an ideal stable equilibrium. Then change the value of s, T and Y , observe the convergence rate of the system to achieves equilibrium: Keep other variables unchanged, set $s = 200$, then the simulation result is as Figure 3(b); Keep other variables unchanged, set $T = 400$, then the simulation result is as Figure 3(c); Keep other variables unchanged, set $Y = 100$, then the simulation result is as Figure 3(d).

Comparing each figure from Figure 3(a to d) respectively, we can know: if banks can make a larger claims when SMEs default, then the proportion of banks that provide SMEs

with loan will be bigger and the convergence rate of y_1 will be faster. It reflects banks' awareness of self-protection. If SMEs' default loss is bigger, then the proportion of SMEs that select "Integrity" will be larger and the convergence rate of y_2 will be faster. It reflects the deterrent to SMEs from punitive measure. If SMEs' bonus for long-term good reputation is bigger, the convergence rate of y_2 will be faster. It is the result of giving a positive motivation to integrity. Besides, increasing the stimulation to banks (such as increase the value of s and c) can increase the convergence rate of y_1 , while the convergence rate of y_2 changes little. But increasing the stimulation to SMEs (such as increase the value of r and γ), can increase the convergence rate of both y_1 and y_2 . It reflects different cooperative willingness between banks and SMEs: As capital providers, banks hope participants can be of integrity. But as capital consumers, SMEs would have the willingness of avoiding its repayment. Therefore, to accelerate the bank-SME credit game system evolve to the ideal equilibrium, banks should focus more on factors that are closely related to SMEs' profit-and-loss.

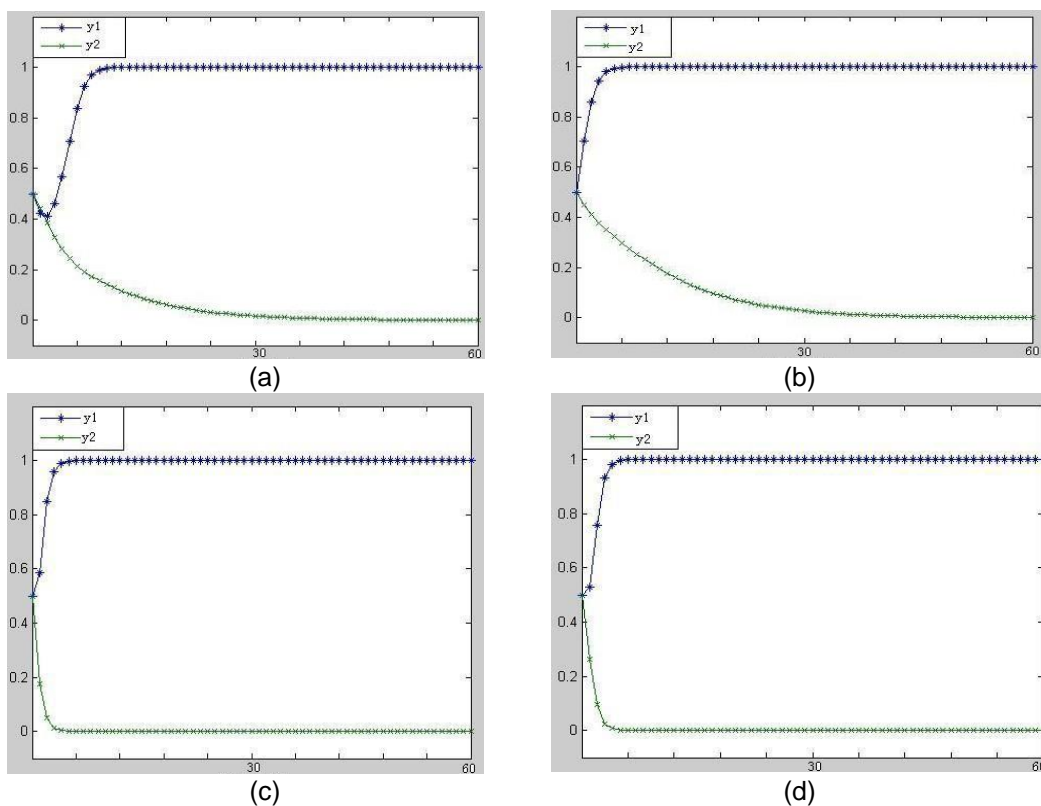


Figure 3. Banks-SMEs Game Simulation (a to d)

3.5. The New Model with "Change" — Bank-SME-Storage Company Model

As bank's assistance, the reasons why storage company becomes a ratter are always complex, for example, SMEs' backhander and the parties' irrational factors, which are difficult to predict. Thus this paper regards storage company's muting as a random event. When storage company selects "Muting", profits of bank and SME would be impacted greatly. Based on this characteristic, this section introduces the mutant ideology, it means the strategy of storage company changes from "Conscientiousness" to "Muting", to observe the influence on the stability of SCF system from storage company's strategy.

The specific simulation ideas are: during the multi-player game simulations of banks, SMEs and storage company, randomly selecting a round to change; Then the profits of

bank and SME would change correspondingly; This change can change the strategy choices of bank and SME, so that impact the system's equilibrium.

The design concept of "Change" is: set up "Random" square to stochastically produce a number θ between 0 and 1; Set up "Compare" square, set a value, take 0.9 for example: it means change probability is 10%, that is the proportion of storage companies that select "Muting" is 10%; When $\theta \geq 0.9$, the system outputs "1" and changes. When $\theta < 0.9$, the system outputs "0" and keep unchangeable. The specific design is shown in Figure 9.

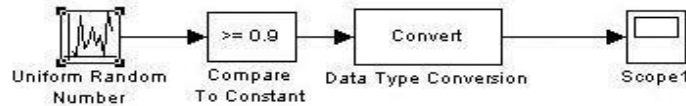


Figure 4. The Design of "Change"

As can be seen from Table 4, when SME selects "Default" and storage company selects "Muting", bank's profit would decrease while SME's profit would increase. Thus it can be inferred that the effect from "Change" is: the proportion of banks that select "Loan" would decrease while the proportion of SMEs that select "Default" would increase. Let $\Delta U_{\text{银}}$ and $\Delta U_{\text{企}}$ denote the effect on $\frac{dy_1}{dt}$ and $\frac{dy_2}{dt}$ from "change", respectively.

Setting $\Delta U_{\text{银}} = -10, \Delta U_{\text{企}} = 8$.

Selecting change probability are 0%, 2%, 10% and 50% to simulate, respectively. Simulations are based on Eq.24. Simulation results are as Figure 5 shown.

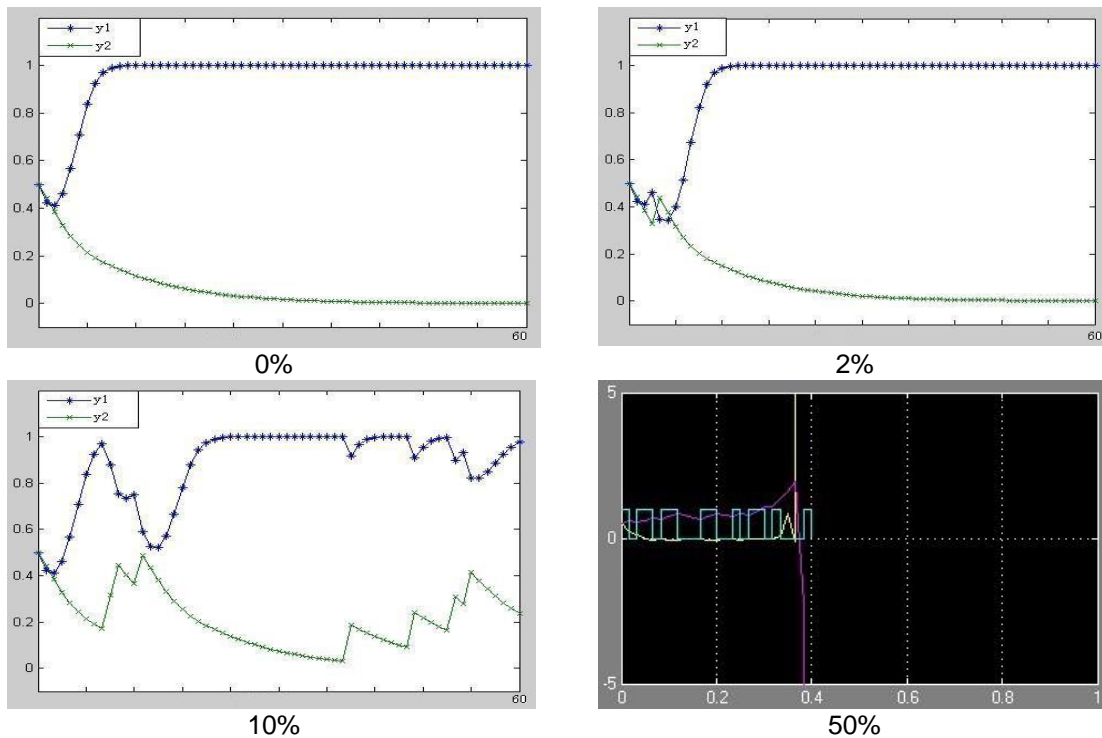


Figure 5. Simulation with Change Probability with 0%, 2%, 10%, 50%

As can be seen from the four figures above, when "Change" is few, the system still can evolve to the equilibrium after slight fluctuations; But when "Change" is more, the system cannot remain stable, especially when change probability is 50%, the system crashes soon. In fact, several attempts suggests that when change probability reaches 15%, the

system will crash. It means this system's bearing capacity for "Change" is not strong. It can give the bank practical guidance. Carefully choose storage companies, because a high incidence of storage company to select "Muting" would have a negative influence on the credit business of SCF service; Although storage company is responsible for post-loan management, it is still important in risk prevention, thus banks should enhance attention.

4. Conclusion

This paper mainly carries through theoretical analysis and simulation experiment. Based on game theory, analyzing participants' behavior strategy and mutual effect. Building credit risk dynamic game simulation model towards the characteristics of SMEs loan union, banks and storage companies: The multi-player game simulation of SMEs' credit union, the multi-player game simulation of banks and SMEs, the three-player game simulation of bank, SME and storage company. Then using multi-agent simulation method to run these models.

During the research process, this paper resolves the following issues:

Towards the characteristics of SCF, summing up the literatures, then setting an activity rule for the participants of SCF: "Replicator dynamic" that based on "Reinforcement learning" and "Imitation learning".

On the basis of the replicator dynamic model, introducing Brownian motion disturbance item so that the system changes from a deterministic dynamic system to a random dynamic system. Then selecting gauss white noise to describe the random increment.

Using the concept of "Change" to describe the impact on SCF system from storage company's behavior, which is stochastic.

Through analyzing simulation results, this paper proposed related suggestions about SCF credit risk management for banks: Banks should select SMEs with good relationship of mutually beneficial cooperation and strong profitability to have SCF service, and select storage company with responsibility to supervise. Meanwhile, enforcing penalty to SMEs that are in default can reduce credit risk while increasing the system's benefit.

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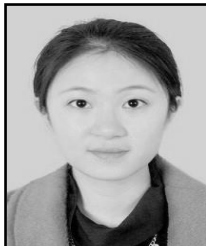
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