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Genetic Analysis of Generalized S−Transform

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Abstract

This text starts with the short time Fourier transform and continuous wavelet transform to deduce the generalized S-transformation. From the point of generation views, we analyzed a relative relationship between generalized S-transformation and the short time Fourier transform, and the other relative relationship between generalized S transform and continuous wavelet transform. The article gives the definition of "the gene mutation of formula" and "the genetic restructuring of formula", and introduces the deriving process of the two core concept. Theoretical analyses show that generalized S-transformation inherited the desirable characteristics in short time Fourier transform which use the window function to select suitable signal. Through genome sequencing of specific parameters, generalized S-transformation has a stronger adaptation that the time-frequency window could make real-time adjusting of frequency. Moreover, generalized S-transformation breaks out limitation that the wavelet function has to content the admissible conditions. From the point of gene mutation, we give the definition of "the gene mutation of formula". Based on the structure form of wavelet functions, we define the generalized S-transformation with a wider domain of definition. Generalized S-transformation inherited the desirable characteristics of the short time Fourier transform and continuous wavelet transform. It has great utility and flexibility in analyzing non-stationary signals.

Keywords: *the short time Fourier transform; wavelet transform; generalized S-transformation*

1. Introduction

The traditional Fourier transform could map the signal from the time dimension to one-dimension frequency domain. It can't show the relation of frequency with time. At this point, it is not suitable to process the non-stationary signal. At present, the most common way to analyzing non-stationary signals are the short time Fourier transform, continuous wavelet transform and so on. Stockwell first proposed the S transform, which inherited the desirable characteristics of the short time Fourier transform and continuous wavelet transform[1][5]. So it has comprehensive application in analyzing non-stationary signals.

Analyzing non-stationary is an important research area in modern signal processing. It is widely used in different domains such as radar, communication, seismic detection and so on. Literature [6] proposed a hybrid DS / FH spread spectrum signals based on the S

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transform, it has laid the groundwork for future development. Literature [7] discussed the recognition of signals from colored noise background in generalized S-transformation. Literature [8] makes the S transform be widely applied in underwater acoustic domain. Literature [9] solves the power quality analysis problem in S-transform. Literature [10] uses the generalized S-transformation to process the magnetotelluric sounding data. Through the domestic and international current situation, the generalized S-transformation has contributed to progress in several fields. So, it is necessary to research the origin of generalized S-transformation and give a exact definition for this.

2. The Gene Analysis of Generalized S Transform

Stockwell first proposed the S transform, which take inspiration from the short time Fourier transform and continuous wavelet transform.

The article gives the definition of "the gene mutation of formula" and "the genetic restructuring of formula", which make the generalized S transform absorb the advantages of the short time Fourier transform and continuous wavelet transform, and there is not enough of them. It has genetically modified in local parameter, which make the generalized S-transformation more practical in time-frequency analyzing [5].

2.1. The Genetic Restructuring of Formula Based on Short Time Fourier Transform

The short time Fourier Transform is one of the widely used means in analyzing non-stationary signals. The short time Fourier Transform used the idea of windowing method and translated parameter within the framework of traditional Fourier transform.

The generalized S-transformation could combine frequency with scaling factor which could control the window width. So the window functions could make real-time adjusting of frequency. We can get a high time-frequency resolution. In this article, we call "the genetic restructuring of formula", it makes the formula retain the original structure, and only to change some local parameters with little modification. The generalized S-transformation could also mean that it is the genetic restructuring of formula based on the short time Fourier transform.

Next, the theoretical analysis will be deduced in detail. We are going to further interpret the relationship between short time Fourier transform and generalized S-transformation.

The short time Fourier transform is to do x(t) dot window functions which under translation in time and frequency modulation.[2]

$$STFT(\tau, f) = \int_{-\infty}^{+\infty} x(t)\omega(t-\tau)e^{i2\pi ft}dt \tag{1}$$

The short time Fourier transform has windowing method, which make it could analyze local properties of the signal.

According to uncertainty principle, the time width and bandwidth product could not be the narrowest at the same time. The method is equivalent to the time-bandwidth product could not infinitesimal [3]. The time-frequency area of Gaussian window could be minimized. The window function can be defined by:

$$\omega(t) = \frac{1}{\delta\sqrt{2\pi}}e^{-\frac{t^2}{2\delta^2}} \tag{2}$$

From the property of Gauss function, δ is the scaling factor which could control the window width in the window function. In order to make the width has flexibility to handle different frequency components in non-stationary signals. δ can be defined by:

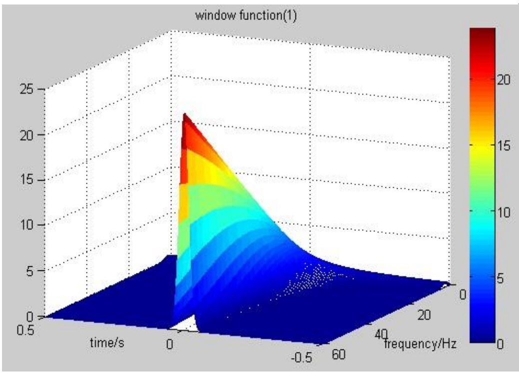
$$\delta(f) = \frac{1}{\gamma |f|^m} \tag{3}$$

Among them, γ and m are the some constant greater than zero [12].

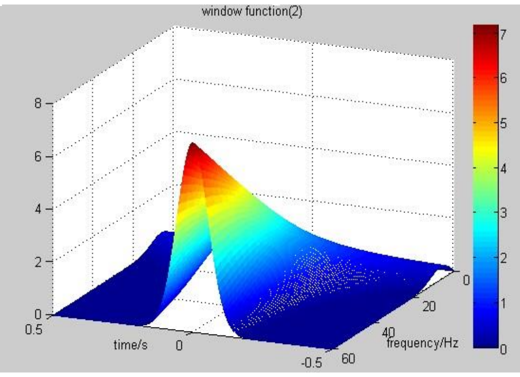
Restructuring window function has the flexibility to handle different frequency components.

$$\omega(t, f) = \frac{\gamma |f|^m}{\sqrt{2\pi}} e^{-\frac{t^2 \gamma^2 f^{2m}}{2}} \tag{4}$$

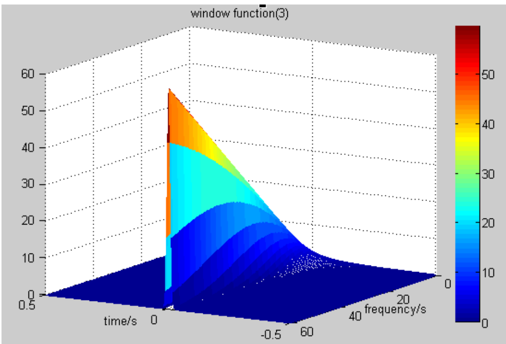
By changing parameter γ and m, we can change the specific feature of the window function to meet the different emphasis in non-stationary signals.



a（1） γ =1， m=1



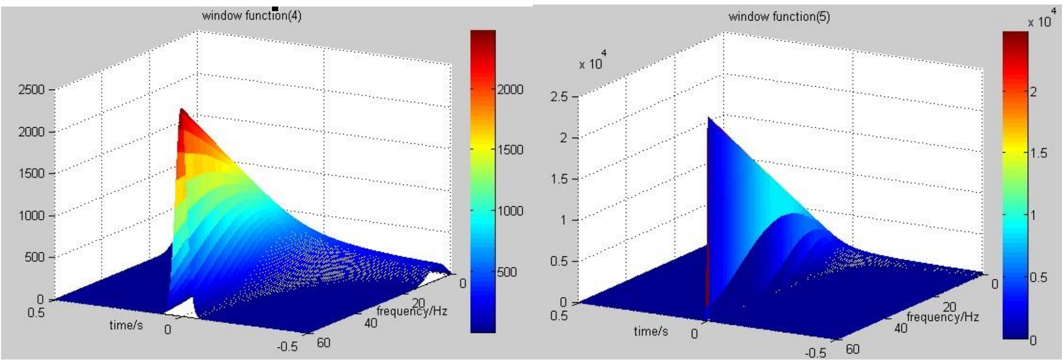
a（2） γ =0.3， m=1



a（3） γ =2.5， m=1

Figure 1. The Influence of Parameter γ and *m*

By comparison, we arrive at the following conclusions: when the parameter m is fixed, if γ >1, the time window width will decrease with the increasing of γ and the amplitude of main peak will increase with the increasing of γ , the time window width have a accelerating inverse relationship with frequency in the condition of the same parameters. If γ <1, the time window width will increase with the decreasing of γ and the amplitude of main peak will decrease with the increasing of γ , the time window width have a slowing inverse relationship with frequency in the condition of the same parameters. In both case, the ridge of window function will have a linear increasing with frequency.

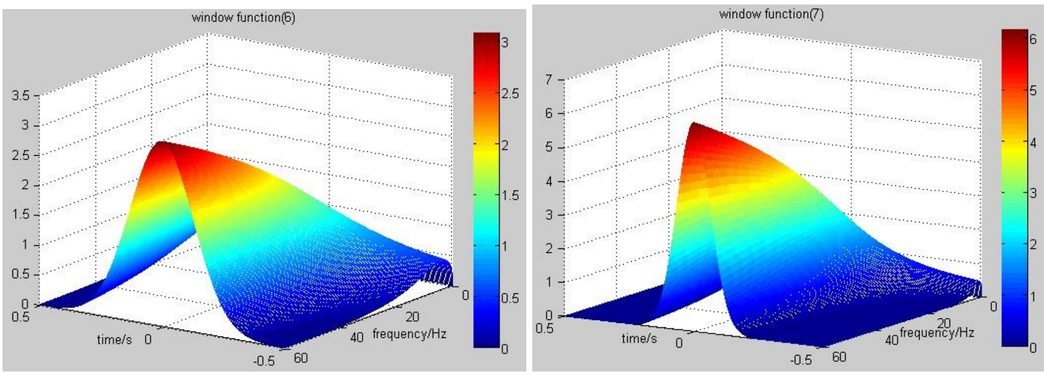


b（1） γ =0.1, m=1.5 b（2） γ =1, m=1.5

Figure 2. The Influence of Parameter γ and *m*

Through comparing the two simulation diagrams of Figure b, we know that the ridge of window function will have a nonlinear increasing with frequency. And γ will change the speed of time window becoming sharper, the amplitude of main peak and the time window width.

By analyzing Figure b(2) and Figure a(1), we conclude that the parameter m also have important roles to change the time window width and the amplitude of main peak.



c（1） γ =1, m=0.5 c（2） γ =2, m=0.5

Figure 3. The Influence of Parameter γ and *m*

Through comparing the two simulation diagrams of Figure c, we conclude that the ridge of window function will have another nonlinear increasing with frequency. And γ will also change the speed of time window becoming sharper, the amplitude of main peak and the time window width.

With the above simulation diagrams and analysis, we can find the change rule of window function in different numerical value with γ and m. γ could change the speed of time window becoming sharper; γ and m could influent the amplitude of main peak and the time window width, but the roles is not the same. As can be seen from the diagram, the time window width has an inverse relationship with frequency and the amplitude of main peak is varied in direct proportion. In the low frequency band, the width of window function is wider and the amplitude of main peak is lower, so it could have a higher resolution for frequency and the resolution of time is poor. In the high frequency band, the width of window function is relatively narrow and the amplitude of main peak is higher, so it could have a higher resolution for time and the resolution of frequency is poor.

In practical application, we can base on the special need in analyzing non-stationary signals. We should to change the appropriate parameters γ and m to meet the practical application [4]. These characteristics make the generalized S transform very flexible and exercisable in the practical application.

Below, we take (4) into the definition of short time Fourier transform. We call it "the genetic restructuring of formula", it makes the formula retain the original structure, and only to change some local parameters with little modification. The method make local parameterized further optimized. So it could adapt to the diversity of time-frequency resolution in analyzing non-stationary signals.

We re-engineered the short time Fourier Transform with the optimized Gaussian Window , we can get the generalized S-transformation:

$$GST(\tau, f)=\int_{-\infty}^{+\infty} x(t)\frac{\gamma\,|f|^m}{\sqrt{2\pi}}e^{-\frac{\gamma^2|f|^{2m}(\tau-t)^2}{2}}e^{-i2\pi ft}dt \tag{5}$$

$$\gamma > 0, m > 0$$

If $\gamma = 1, m = 1$, we call （5） : the S-Transform.

$$ST(\tau, f) = \int_{-\infty}^{+\infty} x(t)\frac{|f|}{\sqrt{2\pi}}e^{-\frac{f^2(\tau-t)^2}{2}}e^{-i2\pi ft}dt \tag{6}$$

The S-transform is an exception for generalized S-transformation.

Meanwhile, we can get the generalized S inverse transform [5]:

$$x(t) = \int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty} GST(\tau, f)d\tau\right\}e^{i2\pi ft}df \tag{7}$$

2.2. The Gene Mutation of Formula Based on Continuous Wavelet Transform

The continuous wavelet transform is a special time-frequency distribution. It could process the signals with multi-resolution analysis. So it can get whole or detail characteristic at different scales [3]. It is useful tools in the fields of signal mutation detection of non-stationary signals.

In this theory system, the limited parameters and initial conditions will be extended on a larger scale, so we get the generalized S-transformation. The continuous wavelet transform could analyze signals based on translation and multi-scale analysis. The generalized S-transformation not only keeps the advantages of continuous wavelet transform, but also overcome the limitation under the specific definition [11]. In this article, we define the relationship the gene mutation of formula. The generalized S-transformation could be could be understood as "the gene mutation and genetic restructuring of formula based on continuous wavelet transform"

Next, we will give a detail theoretical derivation and drive the point of this relationship.

The continuous wavelet transform is to do x(t) dot wavelet functions under translation in time and dilations.

$$WT_x(\tau, S) = \frac{1}{\sqrt{S}}\int_{-\infty}^{+\infty} x(t)\psi^*(\frac{t-\tau}{S})dt \tag{8}$$

We called $\psi(t)$ basis wavelet function, and it must satisfy the admissible conditions:

If the FFT of $\psi(t)$ is $\Psi(j\Omega)$, then it must satisfy:

$$C_{\psi} = \int_{R^*} \frac{|\Psi(j\Omega)|^2}{|\Omega|} d\Omega < +\infty \tag{9}$$

$$R^* \in (-\infty, 0) \cup (0, +\infty)$$

The admissible condition is equal to:

$$\int_R \psi(t) dt = 0 \tag{10}$$

In other words, the $\psi(t)$ encircled area of horizontal axis is zero. And the graph should fluctuate up and down around the horizontal axis as well the domain is limited.

The simple formula of continuous wavelet transform is:

$$WT_x(\tau, f) = 1\int_{-\infty}^{+\infty} x(t)\psi(\text{t-}\tau, f)dt \tag{11}$$

We do so based on the basic idea of the gene mutation of formula. We make the certain coefficient 1 to be revised to $e^{-i2\pi f\tau}$, and we define the basic wavelet as:

$$\psi(t-\tau, f) = \frac{\gamma|f|^m}{\sqrt{2\pi}} e^{-\frac{\gamma^2 f^{2m}(t-\tau)^2}{2}} e^{-j2\pi f(t-\tau)} \tag{12}$$

The basic wavelet is composed of the translation in time and scaling factor which depends on the frequency. It is not need to satisfy the acceptability conditions. This way makes the defined conditions of partial parameters to be expanded. We call it "the gene mutation of formula".

In conclusion, the generalized S-transformation can be defined as:

$$\begin{aligned} GST(\tau, f) &= e^{-i2\pi f\tau}\int_{-\infty}^{+\infty} x(t)\psi(t-\tau, f)dt \\ &= e^{-i2\pi f\tau}\int_{-\infty}^{+\infty} x(t)\frac{\gamma|f|^m}{\sqrt{2\pi}} e^{-\frac{\gamma^2 f^{2m}(t-\tau)^2}{2}} e^{-i2\pi f(t-\tau)} dt \end{aligned} \tag{13}$$

It is clear that (13) is equal to (5).

In the same way, we can get the S transform:

$$\begin{aligned} GST(\tau, f) &= e^{-i2\pi f\tau}\int_{-\infty}^{+\infty} x(t)\psi(t-\tau, f)dt \\ &= e^{-i2\pi f\tau}\int_{-\infty}^{+\infty} x(t)\frac{|f|}{\sqrt{2\pi}} e^{-\frac{f^2(t-\tau)^2}{2}} e^{-i2\pi f(t-\tau)} dt \end{aligned} \tag{14}$$

It is clear that (14) is equal to (6).

3. Conclusion

We can clearly see that the generalized S-transformation isn't continuous wavelet transform. But it has same benefits of continuous wavelet transform. The generalized S-transformation absorbs its essence and resists its dark side, which make it has comprehensive applications in the field of analyzing non-stationary signals.

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References

[1] R. G. Stockwell, L. Mansinha and R. P. Lowe, "Localization of the complex spectrum: The S transform", IEEE Trans. on Signal Processing, vol. 44, no. 4, **(1996)**.

[2] X. D. Zhang and Z. Bao, "The non-stationary signal analysis and processing", National Defense Industry Press, Beijing, **(2001)**.

[3] B. Liu, "The non-stationary signal analysis: an introduction", National defence industry press, Beijing, **(2006)**.

[4] X. H. Chen, Z. H. J. He, "Improved S-Transform and Its Application in Seismic Signal Processing", Journal of Data Acquisition & Processin, vol. 20, no. 4, **(2005)**.

[5] C. R. Pinnegar, L. Mansinha, "The S-transform with windows of arbitrary and varying shape", Geophysice, vol. 68, no. 1, **(2003)**.

[6] M. Z. Zhu and H. B. Ji, "Parameter estimation of hybrid DS/FH spread spectrum signals based on the S transform", Journal of Xidian University, vol. 10, no. 06, **(2010)**.

[7] J. H. Gao, W. S. Man and S. M. Chen, "Recognition of signals from colored noise background in generalized S-transformation domain", Chinese. Geophys, vol. 47, no. 5, **(2004)**.

[8] X. Xin and X. M. Zhang, Feature Extraction of Underwater Target Based on S-transform", Elementary Electroacoustics, vol. 10, no. 3, **(2007)**.

[9] P. K. Dash, B. K. Panigrahi and G. Panda, "Power quality analysis using S-transform", IEEE Transactions on Power Delivery, vol. 18, no. 2, **(2003)**.

[10] J. E. Jing, W. B. Wei and H. Y. Chen, "Magnetotelluric sounding data processing based on generalized S transformation", Chinese Journal of Geophys, vol. 55, no. 12, **(2012)**.

[11] S.-C. Pei and P.-W. Wang, "Discrete Inverse S Transform With Least Square Error in Time-Frequency Filters", IEEE Transactions On Signal Processing, vol. 58, no. 7, **(2010)**.

[12] Y. Duan, J. Sun and J. Liu, "Nonstationary Signals-Resolution Analysis Based on Generalized S Transform", The Leading Edge, vol. 22, no. 2, **(2013)**.

[13] M. Shaoliang, Z. Haizhu, L. Shufang and Z. Xiaobing, "The Design of Direct-sequence spread spectrum based of FPGA", Journal of Harbin University of Science and Technology, vol. 6, no. 106, **(2012)**.

[14] J. Tiejun, R. Lijun and Z. Aiming, "Coherent averaging weak signal detection method without synchronization signal", Journal of Harbin University of Science and Technology, **(2002)**, 03, pp. 35-37.

[15] C. Bao-yuan and L. Wei-ming, "The Algorithm Research of Speech End point Detection and Simulated Based on the Wavelet Analysis", Journal of Harbin University of Science and Technology, vol. 14, no. 1, **(2009)**.

