

A Study on Adaptive Direction Teaching-Learning-Based Optimization Algorithm

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Abstract

In the real life learning process, the teacher communicates with the students for a better learning outcome. The teaching-learning-based optimization (TLBO) algorithm simulates this procedure and shows its great performance in solving the constrained and unconstrained nonlinear optimization problem. This paper presents an adaptive direction strategy (ADS) to improve the searching ability for the TLBO algorithm. The improved algorithm is tested through searching the optimal points for a few typical testing functions. The testing result shows that the improved TLBO algorithm could obtain better optimal solutions in shorter time. Compared to the normal TLBO algorithm, the stability and effectiveness of the improved algorithm are increased greatly.

Keywords: Heuristic algorithm; Teaching-learning-based optimization algorithm; Adaptive direction strategy; Nonlinear optimization

1. Introduction

Many real life problems could be modeled as nonlinear mathematical models and typically the problem requires the optimal solution for the model. The mathematical model for a global optimization problem may be denoted as equation (1):

$$\frac{\max}{\min} f(X = x_1, x_2, \dots, x_n) \quad \text{subject to } X \in \Omega \quad (1)$$

The optimization model includes objective function, searching space and searching process. The nonlinear model is widely used in researching and engineering areas. However, the nonlinear models built on the real problem usually have lots of variables and complex objective functions [1-2]. In order to solve these problems, in the past few years, the heuristic algorithms are developed rapidly. These algorithms include genetic algorithm (GA), particle swarm optimization (PSO), colony algorithm (ACA), artificial bee colony (ABC) algorithm and firework explosion optimization (FEO) algorithm. They are widely adopted to solve the optimization problem [3-5]. The actual test shows that the heuristic algorithm works effectively in certain circumstance. However, these algorithms have one common shortage, where certain parameters need to be particularly identified. In the meantime, the parameters also greatly affect the efficiency of the algorithm, or may even decide whether the algorithm could work or not. This shortage limits the application of these algorithms.

To overcome the shortage for the difficulty in identifying the parameters, in 2012 R.V.Rao proposed the TLBO algorithm based on the population heuristic algorithm. This algorithm has less parameters to be identified and performs well in solving the optimization problem for the real continuous functions [6-8]. The TLBO origins from the traditional teaching-learning phenomenon. The students have two ways to learn new

knowledge, one is through learning from the teachers, the other one is through discussing and communicating with other students. This is the main idea for the TLBO algorithm. In the TLBO, the best individual in the population is treated as the supervisor. It contains lots of knowledge which the other individuals requires. As a result, it is obvious that the teaching ability of the teacher decides the evolution speed of the whole population. Because of the less undetermined parameters required, and its high efficiency in solving the complex nonlinear optimization problem, this algorithm has been developed greatly ever since it was proposed [9-11]. Many researchers brought improvement to this algorithm and it has been widely applied for optimization problem of single objects and multiple objects. Nevertheless, despite of the advantage of the TLBO, it still has a few shortages which need to be solved. This paper adopts the ADS and proposed an improved TLBO algorithm so to improve the effectiveness and stability of the TLBO.

The following parts are organized as follows: the next part is the introduction of the basic TLBO; the third part is the detailed description of the ADS-TLBO including the principle of the improvement and the computation procedure; the fourth part is the proof of effectiveness based on numerical testing; and the last part is the conclusion for the paper.

2. Basic TLBO

TLBO is a new optimization algorithm based on population based algorithm. It mainly simulates the two phases of traditional teaching process: the first phase is the teacher phase, in which the students gain knowledge from the teachers; the second phase is the learner phase in which the students gain knowledge through communication with other students [12]. As a population based algorithm, all the learners as a whole is treated as a population. The different parameters contained in the solutions to the objective functions correspond to the different courses for different students. The fitness value to the objective function is treated as the scores of different students in certain course. The best solution in the population is treated as the teacher of the population. The detailed procedure of the basic TLBO algorithm is shown below.

2.1. Teacher Phase

Teacher phase simulates the real life teaching procedure. In order to obtain more knowledge, the students actively learn from the teacher. In this phase, the teacher is the core of the study, he will try the best to improve the average knowledge level of the population [13]. Define M_i to be the average value of the whole population in the i th iteration, and X_j be the individual of the population. Define $X_{Best, i}$ as the best individual in the population, or the teacher. The teacher improves the solution of the whole population by increasing the average value of the solution of the whole population. The detailed procedure for finding the solution is illustrated below:

First, identify the learning ability of each student, or $Difference - Mean_j$ as shown in equation (2),

$$Difference - Mean_j = r_j (X_{Best, i} - T_F M_i) \quad (2)$$

in which r_j is a random number between (0,1). The r_j identifies the learning degree. T_F is the teaching affecting factor, it denotes the how much does the teaching influencing the average value of the group. The value of T_F is either 1 or 2. The equation to identify the T_F is shown as equation (3),

$$T_F = round [1 + rand (0,1)(2 - 1)] \quad (3)$$

Upon the $\text{Difference} - \text{Mean}_j$, the updating rule for the teacher phase is shown as equation (4),

$$X_{new,j} = X_{old,j} + \text{Difference} - \text{Mean}_j \quad (4)$$

where $X_{old,j}$ is the j th original individual in the population, $X_{new,j}$ is the corresponding updated individual. If $X_{new,j}$ has a better fitness value than $X_{old,j}$, then $X_{new,j}$ will replace $X_{old,j}$. Repeat the above process for the entire population, and the population is updated once.

2.2. Learner Phase

The learning phase simulates the real life communication procedure. The individuals improve their fitness value by communicating with each other thus to improve the average fitness of the entire population [14]. For each individual in the population, if the other individual's solution is better than itself, the one could combine the both solutions thus to improve its own solution. In the learner phase, the updating procedure is stated as below:

For each individual X_i , randomly choose another individual X_j ($j \neq i$) and compare the fitness value. If X_i is better than X_j , then update the value following equation (5),

$$X_{new,i} = X_i + r(X_i - X_j) \quad (5)$$

Otherwise, update the value following equation (6)

$$X_{new,i} = X_i + r(X_j - X_i) \quad (6)$$

Similar to the teacher phase, if $X_{new,i}$ is better than X_i , then replace X_i by $X_{new,i}$. Repeat the procedure for the entire group to update the population once more.

The above two processes are repeated to update the population until the stopping criteria is met. This is the most important process for the TLBO algorithm. However, because of the random selection of the teacher and learner phase, the learning process may be unstable. The quality of the solution is only maintained by increasing the population size of or increasing the computation time. Aiming at this shortage, through careful study and researching, the ADS is proposed for improving the performance of the TLBO algorithm.

3. ADS

TLBO algorithm is a heuristic learning algorithm based on the teaching-learning phenomenon. The entire population's fitness values are increased purely by the communication between the teacher and the population. The improving process is all of random. In the real life, the learning procedure of the student is not only influenced by the teachers and the students, but also closely related to their own learning ability. Meanwhile, due to the active communication with the students, the knowledge level of the teacher may also be greatly increased [15,16]. Considering these facts, the ADS is proposed so to increase the stability and the effectiveness of the TLBO algorithm.

3.1. The Application of the ADS in the Teaching Phase

In the basic TLBO's teacher phase, the only thing of interest is the improvement of the average fitness value of the entire population but not the improvement of the teacher. This disagrees with the real life teaching-learning exercise. Base on this factor, a new process is introduced in the teacher phase to improve the fitness value of the teacher. The details of the process is illustrated as below.

First find the influence factor of the communication, λ as shown in Equation (7)

$$\lambda_i = \frac{f(X_{best})}{f(X_{best}) + f(X_i)} \quad (7)$$

where $f(X)$ is the fitness value of the individual X . X_{best} is the teacher of the current generation. X_i is the i th individual of the population. The teacher is updated by Equation (8) and (9).

$$X_{best,i}^{(1)} = \lambda X_{best} + (1 - \lambda) X_i \quad (8)$$

$$X_{best,i}^{(2)} = X_{best} + (X_{best,i}^{(1)} - X_{best,i}^{(1)}) \quad (9)$$

If the obtained individual has a better fitness value than X_{best} , replace X_{best} by the obtained individual.

The relationship between $X_{best,i}^{(2)}$, $X_{best,i}^{(1)}$ and X_{best} is shown in Figure 1.

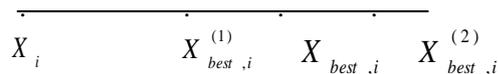
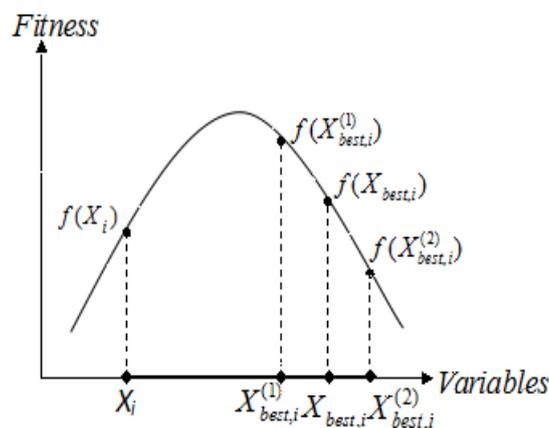


Figure 1. The Location of the Updated Teacher

Suppose the variables mentioned above locate near one of the optimal value, and the fitness value function of the updated teacher is shown in Figure 2, then it is likely to obtain a better teacher from $X_{best,i}^{(1)}$ and $X_{best,i}^{(2)}$. In the meantime, because of the different location of the different individuals, the teacher could be updated and improved continuously while following the updating rule. If the variables are located near different optimal values, similar to the single optimal value case, it is still possible to obtain a better teacher.



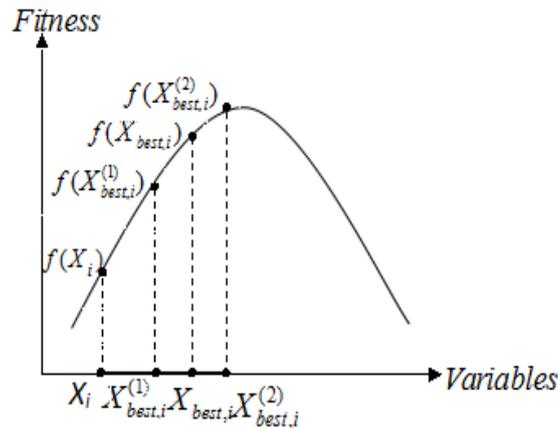


Figure 2. The Relationship Between the Updated Teacher and the Original Teacher

3.2 The Application of ADS in the Learner Phase

This process is based on the fact that the stronger ability of learning for a student is, the better he will learn in the communication process. The idea for improving the learner phase is obtained from mapping the students learning ability to the fitness value. The updating of the individual of the population is based on its fitness value, but not the random process in the basic TLBO algorithm. The usage of ADS in the teacher phase and the learner phase are quite similar.

For each individual X_i , randomly select the other individual X_j ($i \neq j$), then update of the population based on their fitness value.

First calculate the influencing factor λ , as shown in Equation (10)

$$\lambda = \frac{f(X_i)}{f(X_i) + f(X_j)} \quad (10)$$

where, $f(X)$ is the fitness value of X . The updating of an individual is divided into two cases, if X_i is a better solution than X_j , the updating rule is shown in Equation (11) and (12)

$$X_{new,i}^{(1)} = \lambda X_i + (1 - \lambda) X_j \quad (11)$$

$$X_{new,i}^{(2)} = X_i + (X_i - X_{new,i}^{(1)}) \quad (12)$$

Otherwise, follow Equation (13) and (14)

$$X_{new,i}^{(1)} = \lambda X_i + (1 - \lambda) X_j \quad (13)$$

$$X_{new,i}^{(2)} = X_j + (X_j - X_{new,i}^{(1)}) \quad (14)$$

If the obtained solution is better than X_i , then replace X_i with the new value. Similar to teacher phase, the possibility of obtaining a better solution is greatly improved.

3.3. The Evolution Process of the Improved TLBO

The evolution process of the improved TLBO is illustrated as below. First, create an optimization model, and initiate the corresponding parameters including the population size, the gen and the bounds of the variables ($L \leq X \leq U$). Then randomly generate the initial population followed by updating the population through teacher and learner phase. Different from the basic TLBO, a new process is introduced to improve the fitness value of the teacher. The ADS is adopted in both the teacher phase and the learner phase. Repeat the updating process until population satisfies the ending condition. The evolution process of the improved TLBO is shown in Figure 3.

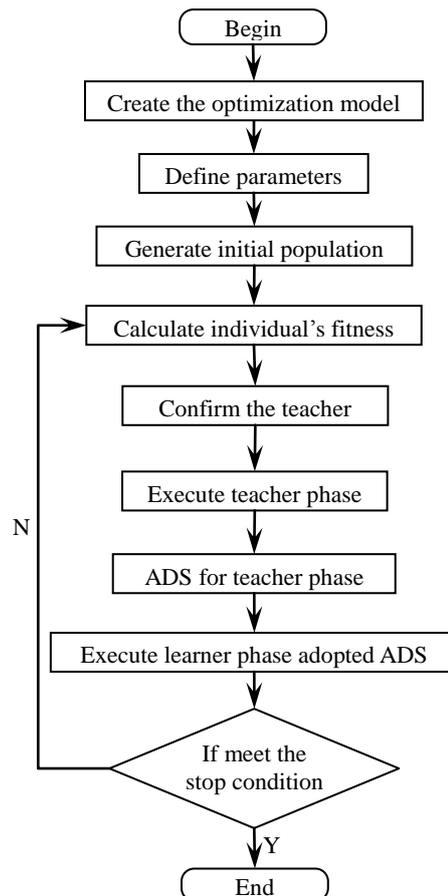


Figure 3. The FlowChart of the ADS-TLBO Algorithm

It can be seen that in the above computation process, there is no procedure handling the constrains. However, in the real life, there are a lot of constrains need to be handled. To broaden the application of the TLBO algorithm, the improved algorithm adopts the empirical rules for solving the constrained optimization problem with heuristic algorithm. The methods are listed as:

- a. In the selection process, the feasible solution is better than infeasible solution.
- b. If both of the solutions are feasible solution, choose the one with larger fitness value.
- c. If both of the solutions are infeasible solution, choose the one which violates less constrains.

These empirical rules apply well for the teacher phase and the learner phase, which helps to solve the constrained optimization problem using TLBO.

4. Testing and Analysis

In this part, the performance of the TLBO is tested by 5 different benchmark testing function. The optimization value is found by ADS-TLBO and Basic-TLBO. Finally, the testing results of ADS-TLBO and Basic-TLBO are listed and carefully compared.

Function 1:

$$\min f_1 = \prod_{j=1}^2 \left\{ \sum_{i=1}^5 i \cos [(i+1)x_j + i] \right\} + 0.5(x_1 + 1.42513)^2 + (x_2 + 0.80032)^2 \quad (15)$$

$$-10 \leq x_1, x_2 \leq 10$$

This is a function with multiple optimal values, 760 local optimal values and one global optimal value. The optimal solution is $x^* = (-1.42513, -0.80032)$, with objective function value $f(x^*) = -186.7309$. The searching process is easily trapped in the local optimal solution -186.3400.

Function 2:

$$\max f_2 = \left(\frac{a}{b + x_1^2 + x_2^2} \right)^2 + (x_1^2 + x_2^2)^2 \quad (16)$$

$$a = 3, b = 0.05, -5.12 \leq x_1, x_2 \leq 5.12$$

In this function, the optimal solution is surrounded by worst solutions. There are also 4 local optimal solution located at the searching region. It is very difficult to find the global optimal solution. The global optimal solution for this function is $x^* = (0,0)$, with maximum value $f(x^*) = 3600$.

Function 3:

$$\min f_3(x) = - \sum_{i=1}^2 x_i \sin \sqrt{|x_i|} \quad (17)$$

$$-500 \leq x_1, x_2 \leq 500$$

This function is symmetrical and dividable. The global optimal solution is located at the searching boundary which is quite far away from suboptimal solution. The optimal solution is $x^* = (420.9687, 420.9687)$, with minimum value $f(x^*) = -837.9658$.

Function 4:

$$\begin{aligned}
 \min f(x) &= 5 \sum_{i=1}^4 x_i - 5 \sum_{i=1}^4 x_i^2 - \sum_{i=5}^{13} x_i \\
 S.T. \quad g_1(x) &= 2x_1 + 2x_2 + x_{10} + x_{11} - 10 \leq 0; \\
 g_2(x) &= 2x_1 + 2x_3 + x_{10} + x_{12} - 10 \leq 0; \\
 g_3(x) &= 2x_1 + 2x_3 + x_{11} + x_{12} - 10 \leq 0; \\
 g_4(x) &= -8x_1 + x_{10} \leq 0; \quad g_5(x) = -8x_2 + x_{11} \leq 0; \\
 g_6(x) &= -8x_3 + x_{12} \leq 0; \quad g_7(x) = -2x_4 - x_5 + x_{10} \leq 0; \\
 g_8(x) &= -2x_6 - x_7 + x_{11} \leq 0; \quad g_9(x) = -2x_8 - x_9 + x_{12} \leq 0; \\
 0 \leq x_i &\leq 1 \quad (i = 1, 2, \dots, 9, 13) \quad 0 \leq x_i \leq 100 \quad (i = 10, 11, 12)
 \end{aligned} \tag{18}$$

This is a second order minimization problem, the objective function consists of 13 variables. The variables are constrained by 9 linear inequality constraints. The feasible region consists of only 0.0003% of the searching region. In the optimal solution point, there are 6 inequality function limits the searching. The optimal solution is $x^* = (1,1,1,1,1,1,1,1,3,3,3,1)$, with objective function value $f(x^*) = -15$.

Function 5:

$$\begin{aligned}
 \max f(x) &= \frac{100 - (x_1 - 5)^2 - (x_2 - 5)^2 - (x_3 - 5)^2}{100} \\
 S.T. \quad g_1(x) &= (x_1 - p)^2 + (x_2 - q)^2 + (x_3 - r)^2 - 0.0625 \leq 0 \\
 0 \leq x_i &\leq 10 \quad (i = 1, 2, 3) \quad p, q, r = 1, 2, \dots, 9
 \end{aligned} \tag{19}$$

This is a second order maximum problem. The objective function consists of three variables. The feasible searching region consists of 93 independent spheres. The condition for point (x_1, x_2, x_3) to be a feasible solution is satisfying any of the constraints corresponding to p, q, r . The optimal solution is $x^* = (5,5,5)$ with $f(x^*) = 1$.

To fully test the performance of the improved algorithm, the optimization is done for different functions using ADS-TLBO and TLBO. The parameters are chosen to be the same for both algorithms. The population size is chosen to be 50, the bounds of the variable and the constraints are introduced in detail above. The maximum evolution time is set to be 300. To test the effectiveness and the robustness of the ADS-TLBO, two different ending conditions are set. The first one is that the difference between the current optimal solution and the known optimal solution is less than a given value, which is 10^{-6} in this paper. The other is that the iteration reaches the maximum evolution time. The evolution is terminated soon as one of the conditions is met. The testing result of the algorithm is shown in Table 1.

Table 1. The Testing Result of the Testing Function

Function	Method	Best	Mean	Worst	Ave times/s	Ave Gen /times
f1	B-TLBO	-186.7309	-185.4104	-181.7540	2.2002	297.83
	ADS-TLBO	-186.7309	-186.7119	-186.3405	1.0081	138.36

f2	B-TLBO	3600.0000	-3596.9210	3577.3249	0.8542	247.67
	ADS-TLBO	3600.0000	3600.0000	3600.0000	0.0907	16.09
f3	B-TLBO	-837.9658	-837.9206	-837.6545	1.2109	267.50
	ADS-TLBO	-837.9658	-837.9658	-837.9658	0.1586	21.91
f4	B-TLBO	-14.9862	-7.2767	-2.6204	3.3628	292.32
	ADS-TLBO	-15.0000	-14.9862	-12.0726	2.7439	205.79
f5	B-TLBO	-1.0000	-0.9955	-0.9668	3.3253	289.15
	ADS-TLBO	-1.0000	-1.0000	-0.9999	0.6920	54.35

From Table 1, it can be seen that in solving the constrained and unconstrained optimization problem, ADS-TLBO algorithm has higher efficiency. It could reach the desired objective function value in a shorter time and less evolution time. It can also be seen that ADS-TLBO algorithm is more stable. In the condition of the same population size and maximum evolution time, ADS-TLBO algorithm reaches the optimal solution more often than the Basic-TLBO. ADS-TLBO shows its great performance in solving these problems.

5. Conclusion

This paper presents an improved TLBO algorithm for solving nonlinear optimization problem. The improved TLBO adopts ADS in both teacher and learner phase. The main idea for improvement is to relate the learning and communication process with the fitness value of an individual. It avoids the random process of the Basic-TLBO and guarantees the stability of the algorithm. The introducing of self-learning process of the teacher also greatly improves the searching efficiency. The proposed improved methods greatly increase the searching and updating ability of the algorithm. Lastly, through the testing result with the benchmark testing function, it can be seen that the improved algorithm has satisfactory result for finding the optimal value of different types of function. It shows that the improved ADS-TLBO is feasible and effective.

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References

- [1] K. Deep and M. Thakur, "A new mutation operator for real coded genetic algorithms", *Applied Mathematics and Computation*, vol. 193, (2007), pp. 211-230.
- [2] D. Goldberg, "Genetic algorithms in search, optimization, and machine learning", Addison-Wesley, New York, (1989).
- [3] T. Back, "Evolutionary algorithms in theory and practice", Oxford University Press, (1996).
- [4] V. Kennedy and R. Eberhart, "Particle swarm optimization", *Proceedings of the IEEE International Conference on Neural Networks*, (1995), pp. 1942-48.
- [5] M. Clerc, *Particle swarm optimization*, ISTE Publishing Company, (2006).
- [6] D. Karaboga, "An idea based on honey bee swarm for numerical optimization", Technical report-TR06, Erciyes University, Engineering Faculty, Computer Engineering Department, (2005).
- [7] B. Basturk and D. Karaboga, "An artificial bee colony (ABC) algorithm for numeric function optimization", *IEEE Swarm Intelligence Symposium*, (2006).
- [8] D. Karaboga and B. Basturk, "On the performance of artificial bee colony (ABC) algorithm", *Applied Soft Computing*, vol. 8, (2008), pp. 687-697.
- [9] M. Dorigo, and T. Stutzle, "Ant colony optimization", MIT Press, (2004).

- [10] C. Blum, "Ant colony optimization: introduction and recent trends", *Physics of Life Reviews*, vol. 2, (2005), pp. 353-373.
- [11] K. S. Lee and Z. W. Geem, "A new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice", *Computer Methods in Applied Mechanics and Engineering*, vol. 194, (2004), pp. 3902-3933.
- [12] A. Ahrari and A. A. Atai, "Grenade explosion method-a novel tool for optimization of multimodal functions, *Applied Soft Computing*", vol. 10, no. 4, (2010), pp. 1132-1140.
- [13] R. V. Rao, V. J. Savsani and D. P. Vakharia, "Teaching-learning-based optimization: a novel optimization method for continuous non-linear large scale problems *Information Sciences*", vol. 183, no. 1, (2011), pp. 1-15.
- [14] R. V. Rao and V. Patel, "An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems", *International Journal Ind. Eng. Comput.*, vol. 3, no. 4, (2012), pp. 535-560.
- [15] T. Niknam, R. A.-Aberghooee and M. R. Narimani, "A new multi objective optimization approach based on TLBO for location of automatic voltage regulators in distribution system", *Journal of Engineering Application of Artificial Intelligence*, vol. 6, (2012), pp. 255-264.
- [16] M. R. Nayak, C. K. Nayak and P. K. Rout, "Application of Multi-Objective Teaching Learning Based on Optimization Algorithm to Optimal Power Flow Problem", *Procedia Technology*, vol. 25, (2012), pp. 1577-1588.