International Journal of u- and e- Service, Science and Technology
Vol.9, No. 3 (2016), pp.131-148
http://dx.doi.org/10.14257/ijunesst.2016.9.3.14

A Novel Similarity Measure for Generalized Trapezoidal Fuzzy Numbers and its Application to Decision-Making

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Abstract

Similarity measures of fuzzy numbers have been widely applied in various areas. In the last decade, many similarity measures of generalized fuzzy numbers were proposed. However, there are two main limitations in existing similarity measures: 1) they cannot correctly calculate the degree of similarity between two generalized trapezoidal fuzzy numbers in some cases; and 2) the definitions of recently developed similarity measures are complicated and difficult to interpret. In this paper, a novel approach to similarity measurement between generalized trapezoidal fuzzy numbers is proposed. The proposed similarity measure has a simple definition and is easier to understand intuitively. Furthermore, we analyze its properties and compare it with existing similarity measures. The results show that the proposed measure outperforms existing similarity measures. Finally, we apply the proposed similarity measure to develop a fuzzy-logic-based approach for new product go/nogo decision-making at the front end. The proposed fuzzy software quality evaluation method is more flexible and more intelligent than existing methods due to the fact that it considers the degrees of confidence of evaluators' opinions.

Keywords: *Fuzzy numbers; Similarity measure; Decision-making*

1. Introduction

Fuzzy number is a quantity whose value is imprecise, which takes into account the fact that all phenomena in the physical universe have a degree of inherent uncertainty. In many respects, fuzzy numbers describe the physical world more realistically than "ordinary" (single-valued) numbers. Because of the suitability for representing uncertain values, fuzzy numbers especially generalized trapezoidal fuzzy numbers have been widely used in many areas including statistics, computer science, and engineering.

Similarity measures are an important research issue in fuzzy numbers. Over the past decades, many similarity measures of generalized fuzzy numbers have been proposed and applied in some important real problems, such as decision-making, risk analysis, pattern recognition, and database query [10-16]. Clearly, effective similarity measures are critical to the applications of fuzzy numbers to various areas. However, since fuzzy numbers are represented by possibility distributions, they can overlap with each other and, as a result, it is not easy to develop an effective similarity measure. In particular, existing similarity measures have two main drawbacks: 1) they cannot correctly calculate the degree of similarity between two generalized trapezoidal fuzzy numbers in some situations; and 2) the definitions of recently developed similarity measures are rather complicated and difficult to interpret.

In this paper, a novel similarity measure of generalized trapezoidal fuzzy numbers is proposed. Since any generalized trapezoidal fuzzy number can be represented by a five tuple, we measure the similarity between two generalized trapezoidal fuzzy numbers in two steps: first, the similarity between the i-th components of two generalized trapezoidal fuzzy numbers is measured; second, all such information is aggregated to obtain the

ISSN: 2005-4246 IJUNESST

overall similarity between these two fuzzy numbers. The rationale behind the idea is that: the more similar their components are, the more similar two generalized trapezoidal fuzzy numbers are. Consequently, the proposed similarity measure of generalized trapezoidal fuzzy numbers has a simple definition and is easier to understand intuitively. Furthermore, we analyze its properties and compare it with existing similarity measures. The results show that the proposed measure can overcome the drawbacks of existing similarity measures.

The remainder of the paper is organized as follows. Section 2 reviews the definitions of generalized trapezoidal fuzzy numbers and existing similarity measures of fuzzy numbers. Section 3 proposes a novel similarity measure for generalized trapezoidal fuzzy numbers discusses its properties. Section 4 uses numerical examples to compare the proposed similarity measure with existing similarity measures. Conclusions are given in Section 5.

2. Preliminaries

I this section, the concepts of generalized fuzzy numbers are first briefly reviewed. Then, existing similarity measures of generalized fuzzy numbers are introduced.

2.1. Generalized Fuzzy Numbers

Dfinition 1 [12]. A fuzzy subset Ã of the real line is called a generalized fuzzy number if its membership function $\mu\tilde{A}$ has the following properties:

(1) $\mu\tilde{A}$ is a continuous mapping from $\mathbb{R}$ to the closed interval [0, h];

(2) $\mu\tilde{A}(x) = 0$ for $-\infty < x < a$;

(3) $\mu\tilde{A}(x) = l\tilde{A}(x)$ is strictly increasing on [a, b];

(4) $\mu\tilde{A}(x) = h$ for $b \le x \le c$;

(5) $\mu\tilde{A}(x) = r\tilde{A}(x)$ is strictly decreasing on [c, d];

(6) $\mu\tilde{A}(x) = 0$ for all $d \le x < \infty$;

where $0 < h \le 1$, a, b, c, d are real numbers, $l\tilde{A}$ and $r\tilde{A}$ are called the left membership function and the right membership function, respectively.

Definition 2. A generalized fuzzy number is called a generalized trapezoidal fuzzy number, denoted by Ã = (a, b, c, d; h), if its membership function has the following form:

$$\mu_A(x) = \begin{cases} 0 & x < a \\ \dfrac{h(x-a)}{b-a} & a \le x < b \\ h & b \le x \le c \\ \dfrac{h(x-c)}{d-c} & c < x \le d \\ 0 & x > d \end{cases}$$

Figure 1 shows two different generalized trapezoidal fuzzy numbers Ã = (a, b, c, d; h1) and $\tilde{B}$ = (a, b, c, d; h2) which denote two different decision-makers' opinions. Here, h1 and h2 represent the degrees of confidence of the opinions of the decision-makers Ã and $\tilde{B}$.

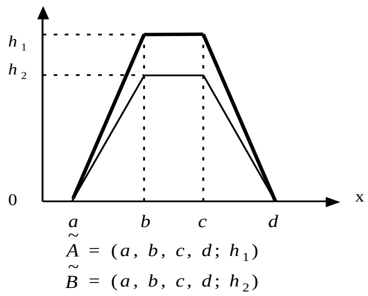

Figure 1. Two Generalized Trapezoidal Fuzzy Numbers $\tilde{A}$ and $\tilde{B}$

2.2. Existing Similarity Measures between Generalized Fuzzy Numbers

Suppose there are two trapezoidal fuzzy numbers Ã = (a1, a2, a3, a4) and $\tilde{B}$ = (b1, b2, b3, b4). In [3], Zwick *et. al.* defined the degree of similarity SZCB(Ã, $\tilde{B}$) between Ã and $\tilde{B}$ as follows:

$$S_{ZCB}(A,B) = \frac{\int_x (\min\{\mu_A(x), \mu_B(x)\})dx}{\int_x (\max\{\mu_A(x), \mu_B(x)\})dx}$$

The larger the value of SZCB(Ã, $\tilde{B}$), the more the similarity between Ã and $\tilde{B}$.

In [4] and [5], Chen defined the degree of similarity SC(Ã, $\tilde{B}$) between two trapezoidal fuzzy numbers Ã = (a1, a2, a3, a4) and $\tilde{B}$ = (b1, b2, b3, b4) as follows:

$$S_C(A,B) = 1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}$$

where $S_C(\tilde{A}, \tilde{B}) \in [0,1]$. If Ã and $\tilde{B}$ are two triangular fuzzy numbers, where Ã = (a1, a2, a3) and $\tilde{B}$ = (b1, b2, b3), then the degree of similarity $S_C(\tilde{A}, \tilde{B})$ between Ã and $\tilde{B}$ can be calculated as follows:

$$S_C(A,B) = 1 - \frac{\sum_{i=1}^{3} |a_i - b_i|}{3}$$

In [6], Hsieh *et. al.*, proposed a similarity measure based on the idea of graded mean integration-representation distance, where the degree of similarity $S_{HC}(\tilde{A}, \tilde{B})$ between two fuzzy numbers Ã and $\tilde{B}$ was defined as follows:

$$S_{HC}(A,B) = \frac{1}{1 + d(A,B)}$$

where d(Ã) = |P(Ã) − P($\tilde{B}$)|. P(Ã) and P($\tilde{B}$) are the graded mean integration representations of Ã and $\tilde{B}$, respectively. If Ã and $\tilde{B}$ are two triangular fuzzy numbers, where Ã = (a1, a2, a3) and $\tilde{B}$ = (b1, b2, b3), then

$$P(A) = \frac{a_1 + 4a_2 + a_3}{6}$$

$$P(B) = \frac{b_1 + 4b_2 + b_3}{6}$$

If Ã and $\tilde{B}$ are trapezoidal fuzzy numbers, where Ã = (a1, a2, a3, a4) and $\tilde{B}$ = (b1, b2, b3, b4), then

$$P(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

$$P(B) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

In [7] and [8], Lee defined the similarity measure $S_L(\tilde{A}, \tilde{B})$ between two trapezoidal fuzzy numbers Ã and $\tilde{B}$ as follows:

$$S_L(A, B) = 1 - \frac{\left\| A - B \right\|_{l_p}}{\left\| U \right\|} \times 4^{-1/p}$$

where U is the universe of discourse,

$$\left\| A - B \right\|_{l_p} = \left(\sum_{i=1}^{4} \left(\left| a_i - b_i \right| \right)^p \right)^{1/p}$$

and

$$\left\| U \right\| = \max(U) - \min(U)$$

In [9] and [10], Chen *et. al.*, defined the degree of similarity between two generalized trapezoidal fuzzy numbers based on center-of-gravity (COG) points. Suppose there are two trapezoidal fuzzy numbers Ã = (a1, a2, a3, a4; $h_{\tilde{A}}$) and $\tilde{B}$ = (b1, b2, b3, b4; $h_{\tilde{B}}$), where 0 ≤ a1 ≤ a2 ≤ a3 ≤ a4 ≤ 1 and 0 ≤ b1 ≤ b2 ≤ b3 ≤ b4 ≤ 1. Then, the COG points of Ã and $\tilde{B}$ are COG(Ã) = (x^*_A, y^*_A) and COG($\tilde{B}$) = (x^*_B, y^*_B), respectively, where

$$y^*_A = \begin{cases} \dfrac{h_A \times \left(\frac{a_3 - a_2}{a_4 - a_1} + 2\right)}{6} & \text{if } a1 \neq a4 \text{ and } 0 < h_A \leq 1 \\ \dfrac{h_A}{2} & \text{if } a1 = a4 \text{ and } 0 < h_A \leq 1 \end{cases}$$

$$x^*_A = \frac{y^*_A(a3 + a2) + (a4 + a1)(h_A - y^*_A)}{2h_A}$$

Similarly, (x^*_B, y^*_B) is for $\tilde{B}$. Then, the degree of similarity $S_{CC}(A, B)$ between Ã and $\tilde{B}$ can be calculated as follows:

$$S_{CC}(A, B) = \left[1 - \frac{\sum_{i=1}^{4} \left| a_i - b_i \right|}{4} \right] \times \left(1 - \left| x^*_A - x^*_B \right| \right)^{B(S_A, S_B)} \times \frac{\min(y^*_A, y^*_B)}{\max(y^*_A, y^*_B)}$$

In above formula, $B(S_A, S_B)$ was defined as follows:

$$B(S_A, S_B) = \begin{cases} 1 & S_A + S_B > 0 \\ 0 & S_A + S_B = 0 \end{cases}$$

where S_A and S_B are the lengths of the based of $\tilde{A}$ and B, respectively, defined as follows:

$$S_A = a_4 - a_1$$

$$S_B = b_4 - b_1$$

Recently, Deng *et. al.*. define the degree of similarity between two generalized trapezoidal fuzzy numbers based on radius of gyration (ROG) points [11]. Let the ROG points of $\tilde{A}$ and B be $ROG(\tilde{A}) = (r_x^A, r_y^A)$ and $ROG(B) = (r_x^B, r_y^B)$, respectively. Then, the degree of similarity $S_{DSDL}(A, B)$ between $\tilde{A}$ and B can be calculated as follows:

$$S_{DSDL}(A,B) = \left[1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}\right] \times \left(1 - \left|r_y^A - r_y^B\right|\right)^{B(S_A, S_B)} \times \frac{\min(r_x^A, r_x^B)}{\max(r_x^A, r_x^B)}$$

where $B(S_A, S_B)$ has the same definition as in $S_{CC}(A, B)$.

3. Proposed Similarity Measure and its Properties

In this section, we first explain how to measure the degree of similarity between the i-th components in two generalized trapezoidal fuzzy numbers. Then, we show how to aggregate such information to derive a novel similarity measure for generalized trapezoidal fuzzy numbers.

3.1. Similarity Measure between a Pair Of Components in Two Fuzzy Numbers

As mentioned in Section 2, a generalized trapezoidal fuzzy number is uniquely determined by its five components. Assume that there are two generalized trapezoidal fuzzy numbers Ã and B, where Ã = (a1, a2, a3, a4; h_A), B = (b1, b2, b3, b4; h_B), 0 ≤ a1 ≤ a2 ≤ a3 ≤ a4 ≤ 1, and 0 ≤ b1 ≤ b2 ≤ b3 ≤ b4 ≤ 1. To measure the similarity between Ã and B, a natural idea is to measure the similarity between each pair of components in Ã and B, and then aggregate all such information to evaluate the overall similarity between these two fuzzy numbers. Clearly, the more similar their components (respectively), the more similar two generalized trapezoidal fuzzy numbers.

Therefore, the critical problem is how to develop a "reasonable" similarity measure between two components (i.e. two real numbers between 0 and 1). In the following, we will use "components" and "real numbers" interchangely. Given two components x and y, we now explain how to define their degree of similarity Sim(x, y). Before we present the mathematical equations, let us first enumerate some properties that the similarity measure has to satisfy:

- The numerical value of Sim(x, y) must be normalized between 0 and 1 so that the similarity between two components can be compared in a meaningful way.
- The value 0 is assigned if and only if the paired numbers are (0, 1) or (1, 0), and the value 1 is assigned if and only if the paired numbers are (x, x) for any x;
- It is symmetrical. In other words, the similarity between x and y is the same as the similarity between y and x.
- It should be consistent with intuition. We are most interested in two cases. The first case is shown in Figure 2(a), where x < y < z. Because x is closer to y than z, intuitively, x is more similar to y than z. Hence, the numerical value of Sim(x, y) should be larger than that of Sim(x, z). Similarly, the numerical value of Sim(y, z) should be larger than that of Sim(x, z). The second case is shown in Figure 2(b), where y1 > y2 and Δ = y1 – x1 = y2 – x2 > 0. In this case, we can conclude that x1 > x2. As it can be seen, although y1 – x1 = y2 – x2, the "common part" of x1 and y1 is larger than that of x2 and y2 (i.e. x1 > x2). In other words, the effect of Δ on the

similarity between x1 and y1 is smaller than that on the similarity between x2 and y2. From this viewpoint, x1 and y1 is more similar than x2 and y2. Therefore, the numerical value of Sim(x1, y1) should be larger than that of Sim(x2, y2).

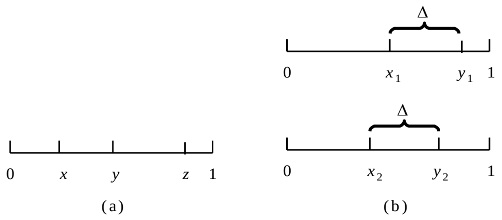

Figure 2. Special Relationships between Components

Therefore, we define the degree of similarity between two components as follows:

Definition 3. Let $0 \le x \le 1$ and $0 \le y \le 1$. The similarity *Sim* between *x* and *y* is defined as:

$$Sim(x, y) = 1 - |\sqrt{x} - \sqrt{y}|$$

In the above definition of function Sim, we take the absolute of the difference between the square roots of two components to measure the similarity of the two components. A small absolute of difference corresponds to a high degree of similarity, whereas two components with a large absolute of difference are considered to be rather dissimilar.

We have defined the similarity measure of components. We now prove that it has the expected properties mentioned above.

Theorem 1. The similarity measure $Sim(x, y) = 1 - |\sqrt{x} - \sqrt{y}|$ satisfies that:

(1) $0 \le$ Sim(x, y) ≤ 1, $\forall$x, y $\in [0, 1]$;

(2) Sim(x, y) = Sim(y, x), $\forall$x, y $\in [0, 1]$;

(3) Sim(x, y) = 1 iff x = y, $\forall$x, y $\in [0, 1]$;

(4) Sim(x, y) = 0 iff x = 0 and y =1 or x = 1 and y = 0;

(5) $\forall$x, y, z $\in [0, 1]$, if x < y < z, then Sim(x, y) > Sim(x, z) and Sim(y, z) > Sim(x, z);

(6) $\forall$x1, y1, x2, y2 $\in [0, 1]$, if y1 – x1 = y2 – x2 > 0 and y1 > y2, then Sim(x1, y1) > Sim(x2, y2).

Proof.

(1) Because $0 \le x \le 1$ and $0 \le y \le 1$, $0 \le |\sqrt{x} - \sqrt{y}| \le 1$, we have $0 \le 1 - |\sqrt{x} - \sqrt{y}| \le 1$. So, $0 \le Sim(x, y) \le 1$.

(2), (3), and (4) can be obtained directly from its definition.

(5) Because $x < y < z$, we have $|\sqrt{x} - \sqrt{y}| < |\sqrt{x} - \sqrt{z}|$ and $|\sqrt{y} - \sqrt{z}| < |\sqrt{x} - \sqrt{z}|$, i.e. $1 - |\sqrt{x} - \sqrt{y}| > 1 - |\sqrt{x} - \sqrt{z}|$ and $1 - |\sqrt{y} - \sqrt{z}| > 1 - |\sqrt{x} - \sqrt{z}|$. So, $Sim(x, y) > Sim(x, z)$ and $Sim(y, z) > Sim(x, z)$.

(6) Because $y_1 > y_2$ and $y_1 - x_1 = y_2 - x_2$, we have $x_1 > x_2$. Let $\Delta = y_1 - x_1 = y_2 - x_2$. Since $\Delta > 0$, $x_1(x_2 + \Delta) > x_2(x_1 + \Delta)$. Then, we have $x_1 + x_2 + \Delta + 2\sqrt{x_1(x_2 + \Delta)} > x_1 + x_2 + \Delta + 2\sqrt{x_2(x_1 + \Delta)}$. From $\Delta = y_1 - x_1 = y_2 - x_2$, we have $y_1 = x_1 + \Delta$ and $y_2 = x_2 + \Delta$. So, $x_1 + y_2 + 2\sqrt{x_1 y_2} > x_2 + y_1 + 2\sqrt{x_2 y_1}$. Therefore, $\sqrt{x_1} + \sqrt{y_2} > \sqrt{y_1} + \sqrt{x_2}$, i.e. $\sqrt{y_2} - \sqrt{x_2} > \sqrt{y_1} - \sqrt{x_1}$. On the other hand, $y_1 > x_1$ and $y_2 > x_2$. So, $1 - |\sqrt{y_2} - \sqrt{x_2}| < 1 - \sqrt{y_1} - \sqrt{x_1}$. Hence, we have $Sim(x_1, y_1) > Sim(x_2, y_2)$. □

Note that in our context the function f(x, y) = $\frac{Min\{x, y\}}{Max\{x, y\}}$ cannot be used a similarity measure. There are two reasons: first, it is meaningless when x = y = 0; second, f(x, y) = 0 for any x and y with Min{x, y} = 0. On the other hand, the function g(x, y) = 1 – |x – y| also cannot be used as a similarity measure in our context. The reason is the following: ∀x1, y1, x2, y2 ∈ [0, 1], if y1 – x1 = y2 – x2 > 0 and y1 > y2, g(x1, y1) = g(x2, y2).

3.2. Similarity Measure between Generalized Trapezoidal Fuzzy Numbers

By Definition 3, we may get the degree of similarity of the i-th components in two generalized trapezoidal fuzzy numbers Ã = (a1, a2, a3, a4; h_A) and B̃ = (b1, b2, b3, b4; h_B). Then, the degrees of similarity of five pairs of components can be aggregated to obtain the overall similarity between Ã and B̃.

Definition 4.The similarity between two generalized trapezoidal fuzzy numbers Ã = (a1, a2, a3, a4; h_A) and B̃ = (b1, b2, b3, b4; h_B), where 0 ≤ a1 ≤ a2 ≤ a3 ≤ a4 ≤ 1 and 0 ≤ b1 ≤ b2 ≤ b3 ≤ b4 ≤ 1, is defined as follows:

$$S(A, B) = \left(Sim(h_A, h_B) \times \prod_{i=1}^{4} Sim(a_i, b_i) \right)^{1/5}$$

In the above definition of function S(Ã, B̃), we take the geometric mean of the degrees of similarity of five pairs of components in Ã and B̃ as the similarity between two generalized trapezoidal fuzzy numbers. The larger the value of S(Ã, B̃), the more the similarity between the generalized trapezoidal fuzzy numbers Ã and B̃.

The proposed similarity between two generalized trapezoidal fuzzy numbers has the following properties.

Property 1. For any two generalized trapezoidal fuzzy numbers Ã = (a1, a2, a3, a4; h_A) and B̃ = (b1, b2, b3, b4; h_B), 0 ≤ S(Ã, B̃) ≤ 1.

Proof. It can be obtained directly from its definition.

Property 2. Two generalized trapezoidal fuzzy numbers Ã = (a1, a2, a3, a4; h_A) and B̃ = (b1, b2, b3, b4; h_B) are identical if and only if S(Ã, B̃) = 1.

Proof. (1) If Ã and B̃ are identical, then a1 = b1, a2 = b2, a3 = b3, a4= b4, and $h_A = h_B$. The degree of similarity between Ã and B̃ can be calculated as follows:

$$S(\tilde{A}, \tilde{B}) = \left(Sim(h_A, h_B) \times \prod_{i=1}^{4} Sim(a_i, b_i) \right)^{1/5} = \left[\left(1 - \left|\sqrt{h_A} - \sqrt{h_A}\right|\right) \times \prod_{i=1}^{4} \left(1 - \left|\sqrt{a_i} - \sqrt{a_i}\right|\right) \right]^{1/5} = 1$$

(2) If $S(\tilde{A}, \tilde{B}) = 1$, then $S(\tilde{A}, \tilde{B}) = \left(Sim(h_A, h_B) \times \prod_{i=1}^{4} Sim(a_i, b_i) \right)^{1/5} = 1$

It implies that a1 = b1, a2 = b2, a3 = b3, a4 = b4, and $h_A = h_B$. Therefore, two generalized trapezoidal fuzzy numbers Ã and B̃ are identical.

Property 3. $S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A})$.

Proof. Because

$$S(\tilde{A}, \tilde{B}) = \left(Sim(h_A, h_B) \times \prod_{i=1}^{4} Sim(a_i, b_i) \right)^{1/5} \text{ and } S(\tilde{B}, \tilde{A}) = \left(Sim(h_B, h_A) \times \prod_{i=1}^{4} Sim(b_i, a_i) \right)^{1/5}$$

where $Sim(h_A, h_B) = Sim(h_B, h_A)$ and $Sim(a_i, b_i) = Sim(b_i, a_i)$, we have $S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A})$.

Property 4. If Ã = (a, a, a, a; h) and B̃ = (b, b, b, b; h), then S(Ã, B̃) = $\left(1 - |\sqrt{a} - \sqrt{b}|\right)^{4/5}$.

Proof. It can be obtained directly from its definition.

We now illustrate the use of the above similarity measure by an example. Consider three generalized trapezoidal fuzzy numbers shown in Figure 3, where Ã = (0.225, 0.3, 0.375, 0.45; 1.0), B = (0.2, 0.25, 0.35, 0.45; 0.95), and C = (0.475, 0.55, 0.65, 0.7; 0.9).

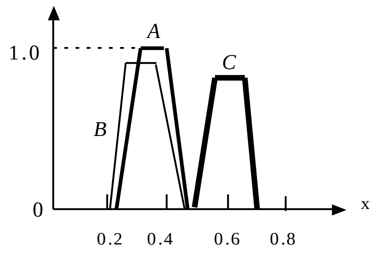

Figure 3. Three Generalized Trapezoidal Fuzzy Numbers

From Figure 3, intuitively, we can see that Ã is more similar to B than C . By Definition 4, we have

$$S(\tilde{A}, B) = [(1-|\sqrt{1}-\sqrt{0.95}|)\times(1-|\sqrt{0.225}-\sqrt{0.2}|)\times(1-|\sqrt{0.3}-\sqrt{0.25}|)$$

$$\times(1-|\sqrt{0.375}-\sqrt{0.35}|)\times(1-|\sqrt{0.45}-\sqrt{0.45}|)]1/5$$

$$= 0.9757$$

$$S(\tilde{A}, C) = 0.8341$$

Because S(Ã, B) > S(Ã, C), the proposed similarity measure draws a conclusion that is consistent with intuition.

4. Numerical Comparisons with Previous Works

In this section, twenty-four sets of generalized fuzzy numbers, shown in Figure 4 (from [10]), Figure 5 (from [11]) and Figure 6, are used to compare the proposed similarity measure with existing six similarity measures. A comparison of the calculation results of the proposed similarity measure with existing measures is given in Table 1. Note that from set 10 of Figure 4, we can see that Ã and B two generalized triangular fuzzy numbers, so we should use the formula for generalized triangular fuzzy numbers to compute their similarity, *i.e.*, $S_C(A,B)$ = 0.8667. However, both [10] and [11] incorrectly used the formula for generalized trapezoidal fuzzy numbers to compute their similarity ($S_C(A,B)$ = 0.9). The same problem is also in Set 12 of Figure 4.

From Figure 4, Figure 5, Figure 6, and Table 1, we can see some drawback of existing similarity measures:

(1) From Set 1 of Figure 4, we can see that $\tilde{A}$ and B are different generalized fuzzy numbers. However, from Table 1, we can see that $S_{HC}(A,B)$ = 1.

(2) From Figure 2, we can see that Set 5 and Set 6 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_{ZCB}(A,B)$, $S_C(A,B)$, $S_{HC}(A,B)$, and $S_L(A,B)$, Set 3 and Set 4 get the same degree of similarity, respectively.

(3) From Set 5 of Figure 4, we can see that $\tilde{A}$ and B are different generalized fuzzy numbers. However, from Table 1, we can see that $S_C(A,B)$ = $S_{HC}(A,B)$ = $S_L(A,B)$ = 1.

(4) From Set 6 of Figure 4 and Table 1, we can see that if we apply $S_{ZCB}(A,B)$ and $S_L(A,B)$, we cannot calculate the degree of similarity between two identical real

values due to the fact that the denominator will become zero, such that $S_{ZCB}(A,B)$ = $S_L(A,B)$ = ∞ and it is a wrong result.

(5) From Figure 4, we can see that Set 7 and Set 8 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_C(A,B)$ and $S_{HC}(A,B)$, Set 7 and Set 8 get the same degree of similarity, respectively. Furthermore, if we apply $S_{ZCB}(A,B)$ or $S_L(A,B)$ to Set 8, we cannot correctly calculate the degree of similarity due to $S_{ZCB}(A,B)$ = 0, and it is an incorrect result; if we apply $S_{ZCB}(A,B)$ to Set 7, we cannot calculate the degree of similarity between two real values due to $S_{ZCB}(A,B)$ = ∞.

(6) From Figure 4, we can see that Set 8 and Set 9 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_C(A,B)$ and $S_{HC}(A,B)$, Set 8 and Set 9 get the same degree of similarity, respectively.

(7) From Figure 4, we can see that Set 10 and Set 11 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_{ZCB}(A,B)$ and $S_{HC}(A,B)$, Set 10 and Set 11 get the same degree of similarity, respectively.

(8) From Figure 5, we can see that Set 13 and Set 14 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_C(A,B)$, $S_{HC}(A,B)$, and $S_{CC}(A,B)$, Set 13 and Set 14 get the same degree of similarity, respectively. Furthermore, if we apply $S_{ZCB}(A,B)$ to Set 13 and Set 14, we cannot correctly calculate the degree of similarity due to fact that $S_{ZCB}(A,B)$ = 0, and it is an incorrect result.

(9) From Figure 5, we can see that Set 15 and Set 16 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_C(A,B)$, $S_{HC}(A,B)$, and $S_{CC}(A,B)$, Set 13 and Set 14 get the same degree of similarity, respectively. Furthermore, if we apply $S_{ZCB}(A,B)$ to Set 15, we cannot correctly calculate the degree of similarity due to fact that $S_{ZCB}(A,B)$ = 0, and it is an incorrect result.

(10) From Figure 5, we can see that Set 17 and Set 18 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_C(A,B)$, $S_{HC}(A,B)$, and $S_{CC}(A,B)$, Set 17 and Set 18 get the same degree of similarity, respectively. Furthermore, if we apply $S_{ZCB}(A,B)$ to Set 17 and Set 18, we cannot correctly calculate the degree of similarity due to fact that $S_{ZCB}(A,B)$ = 0, and it is an incorrect result.

(11) From Figure 6, we can see that Set 19 and Set 20 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_C(A,B)$ and $S_{DSDL}(A,B)$, Set 19 and Set 20 get the same degree of similarity, respectively.

(12) From Figure 6, we can see that Set 21 and Set 22 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply any existing similarity measure, Set 21 and Set 22 get the same degree of similarity.

(13) From Figure 6, we can see that Set 23 and Set 24 are different sets of fuzzy numbers. However, from Table 1, we can see that if we apply $S_C(A,B)$, $S_{CC}(A,B)$, and $S_{DSDL}(A,B)$, Set 23 and Set 24 get the same degree of similarity, respectively. Moreover, we can see that $\tilde{A}$ and B in Set 24 are different generalized fuzzy numbers. However, from Table 1, we can see that $S_{HC}(A,B)$ = 1.

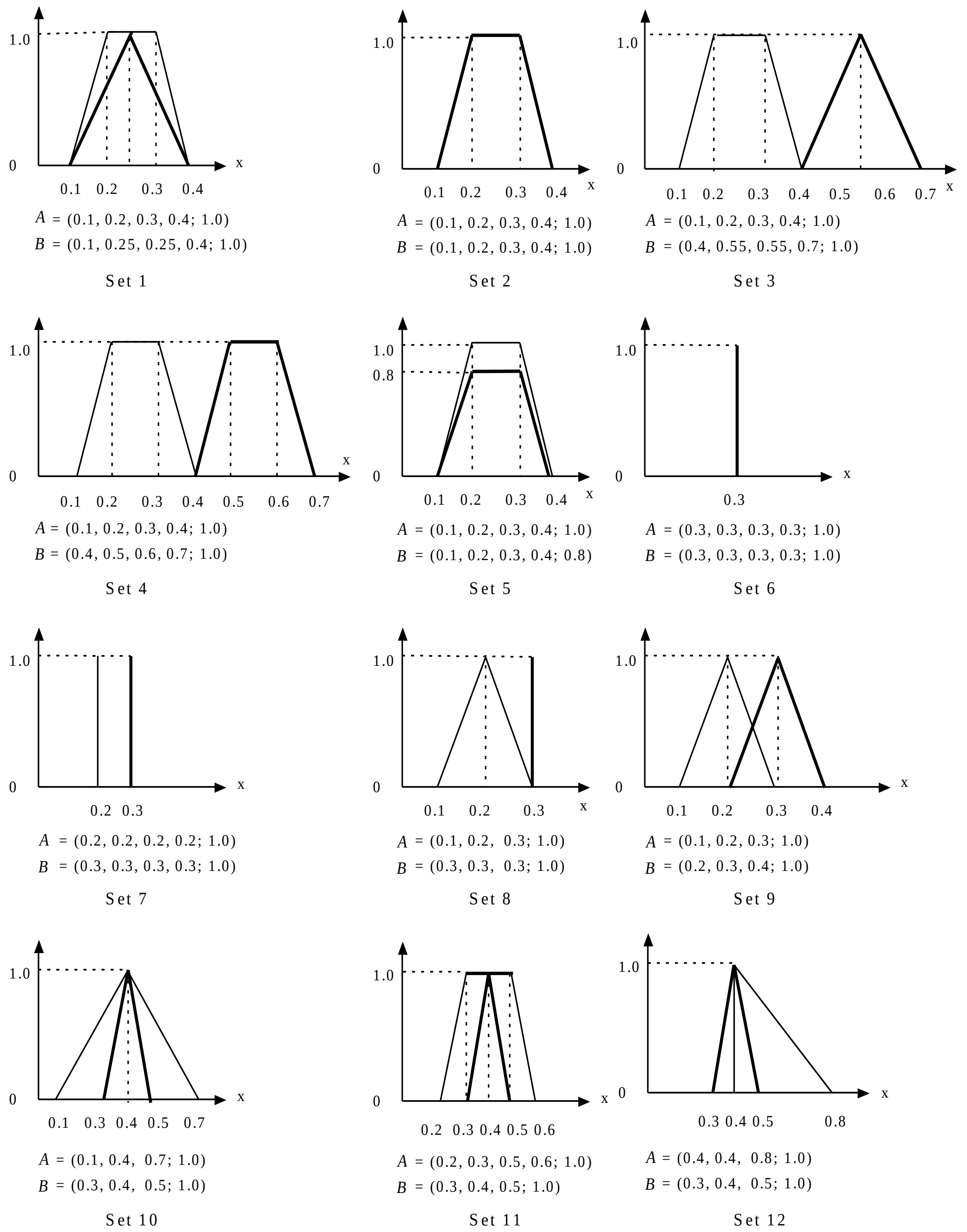

Figure 4. Twelve Sets of Fuzzy Numbers

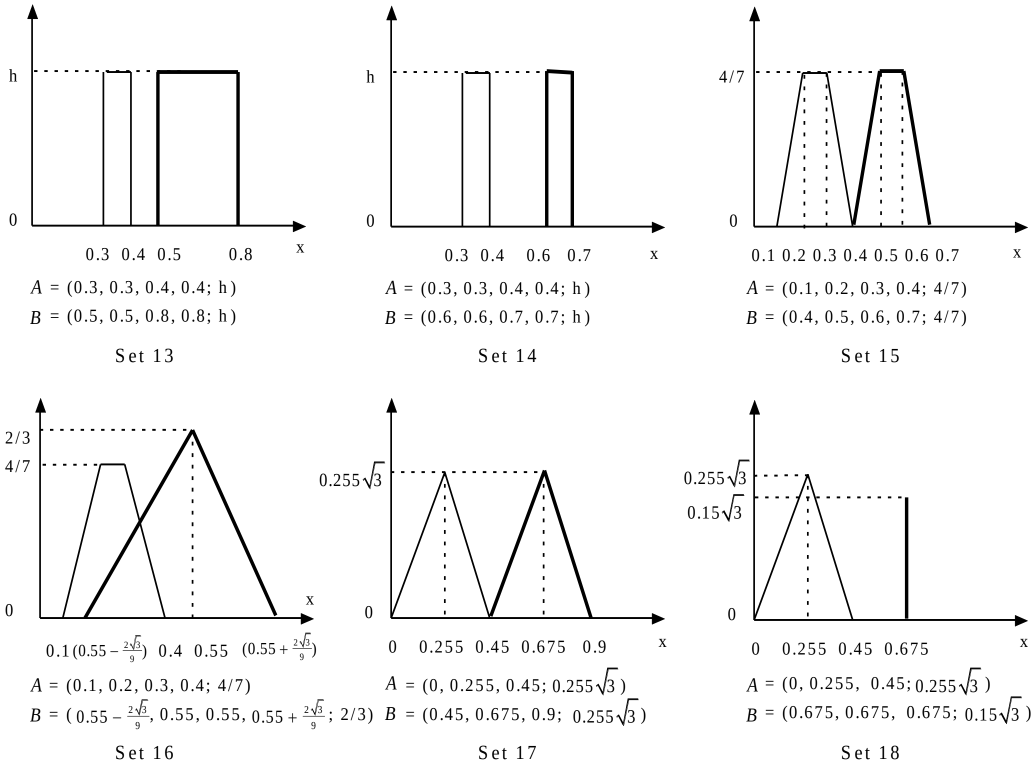

Figure 5. Six Sets of Fuzzy Numbers

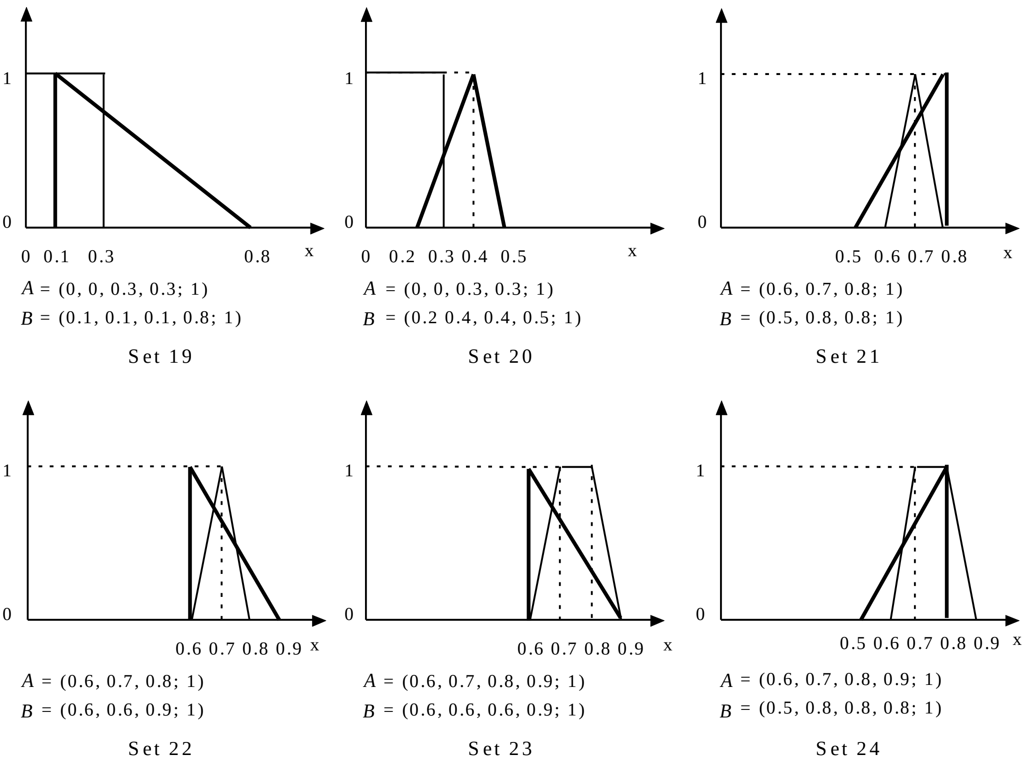

Figure 6. Another Six Sets of Fuzzy Numbers

Table 1. A Comparison of the Calculation Results of the Proposed Similarity Measure with the Existing Methods

	$S_{ZCB}(A,B)$	$S_C(A,B)$	$S_{HC}(A,B)$	$S_L(A,B)$	$S_{CC}(A,B)$	$S_{DSDL}(A,B)$	$S(A,B)$
Set1	0.75	0.975	1	0.9167	0.8357	0.7954	0.9796
Set2	1	1	1	1	1	1	1
Set3	0	0.7	0.7692	0.5	0.42	0.4028	0.7909
Set4	0	0.7	0.7692	0.5	0.49	0.4931	0.7919
Set5	0.8	1	1	1	0.8	0.8	0.9779
Set6	*	1	1	*	1	1	1
Set7	*	0.9	0.9091	0	0.9	0.9	0.9188
Set8	0	0.9	0.9091	0.5	0.54	0.5754	0.9093
Set9	0.1429	0.9	0.9091	0.6667	0.81	0.8112	0.9156
Set10	0.3333	0.8667	1	0.8333	0.9	0.8854	0.9227
Set11	0.3333	0.9	1	0.75	0.72	0.6914	0.9339
Set12	0.2	0.8667	0.9375	0.8	0.78	0.7744	0.9425
Set13	0	0.7	0.7692	0.4	0.49	0.4868	0.8262
Set14	0	0.7	0.7692	0.25	0.49	0.4904	0.8234
Set15	0	0.7	0.7692	0.5	0.49	0.4931	0.7919
Set16	0.1476	0.7	0.7692	0.6407	0.49	0.4576	0.7980
Set17	0	0.55	0.6897	0.5	0.3025	0.3090	0.6326
Set18	0	0.55	0.6897	0.3333	0.3025	0.2945	0.5642
Set19	0.3582	0.775	0.9375	0.7188	0.4219	0.4391	0.7484
Set20	0.0588	0.775	0.8108	0.55	0.4047	0.4391	0.6899
Set21	0.4545	0.9333	0.9524	0.75	0.925	0.9228	0.9629
Set22	0.4545	0.9333	0.9524	0.75	0.925	0.9228	0.9639
Set23	0.4737	0.925	0.9091	0.75	0.7532	0.7181	0.9624
Set24	0.5556	0.925	1	0.8125	0.7532	0.7181	0.9636

Note:"*" means that the similarity measure cannot calculate the degree of similarity between two generalized fuzzy numbers.
"▇" means incorrect results

From above analysis, we can see that existing similarity measures have some limitations. $S_C(A,B)$, $S_{HC}(A,B)$, and $S_L(A,B)$ can only be applied to normal fuzzy numbers because they do not take into account the 5th component (h) of generalized fuzzy numbers. Although $S_{ZCB}(A,B)$ can be applied to generalized fuzzy numbers, it cannot correctly the degree of similarity if two generalized fuzzy numbers have no common interaction. In general, $S_{CC}(A,B)$ and $S_{DSDL}(A,B)$ outperform previous similarity measures. However, they also have some drawbacks. In some cases, where two different generalized fuzzy numbers have the same COG (or ROG) points, $S_{CC}(A,B)$ (or $S_{DSDL}(A,B)$) cannot correctly determine the degree of similarity of fuzzy numbers. On the other hand, both $S_{CC}(A,B)$ and $S_{DSDL}(A,B)$ have complex definitions and are difficult to interpret. Compared with existing similarity measures, the proposed measure has a simple definition and is easier to interpret. More importantly, as it can be seen, the proposed similarity measure can overcome the drawbacks of existing similarity measures.

5. Application to New Product Go/Nogo Decision-Making

New product development is both a complex process and a substantial business risk. In [17], a method for new product screening using fuzzy logic was proposed: first, the criteria ratings and their corresponding importance were assessed in linguistic terms described by fuzzy numbers; then, fuzzy weighted average was employed to aggregate these fuzzy numbers into a fuzzy-possible-success rating (FPSR) of the product; finally, the Euclidean distance was employed to translate the FPSR back into linguistic terms to derive at a new product screening decision. However, this method has some drawbacks:

- Linguistic terms were described by normal fuzzy numbers rather than generalized fuzzy numbers. As a result, the degrees of confidence of the opinions of the evaluators were not taken into account. In practice, evaluators often evaluate new products in uncertain environments and based on incomplete information. If the degrees of confidence of the opinions of the evaluators are considered, a new product screening decision will be more objective, with much less chance of personal bias.
- It used traditional fuzzy weighted average (FWA) to aggregate evaluators' opinions. However, the FWA has some methodological problems: first, it increases the imprecision unnecessarily, *i.e.*, the FWA may be more imprecise than necessary and therefore this imprecision is not meaningful; second, the position and balance point of FWA is inappropriate in some cases.

Thus, in the following, we combine the proposed similarity measure of generalized trapezoidal fuzzy numbers and a new fuzzy weighted average (NFWA) method to deal with a new product go/nogo decision problem to overcome these drawbacks.

Suppose a committee of m evaluators (*i.e.*, E_t, t = 1, 2, …, m) conducts a new product screening decision. Let F_i, i = 1, 2, …, n, be factors for screening decision, R_{ti}, i = 1, 2, …, n, represent the fuzzy numbers approximating the linguistic factor rating given to F_i by evaluator E_t, and W_{ti}, i = 1, 2, …, n, represent the fuzzy numbers approximating the linguistic importance weighting given to F_i by evaluators E_t. In this paper, a seven-member linguistic set is used to represent the linguistic terms. Table 2 illustrates the linguistic terms and their corresponding generalized trapezoidal fuzzy numbers. Then, the new product go/nogo decision process can be described as follows.

- Step 1: Calculate the average factor rating R_i and the average importance weighting W_i. Assume that the values $R_{ti} = (a_{ti}, b_{ti}, c_{ti}, d_{ti}; h_{ti})$ and $W_{ti} = (a'_{ti}, b'_{ti}, c'_{ti}, d'_{ti}; h'_{ti})$ are generalized trapezoidal fuzzy numbers, then R_i and W_i are calculated as follows:

$$R_i = \left(\frac{1}{m}\sum_{t=1}^{m} a_{ti}, \frac{1}{m}\sum_{t=1}^{m} b_{ti}, \frac{1}{m}\sum_{t=1}^{m} c_{ti}, \frac{1}{m}\sum_{t=1}^{m} d_{ti}; \frac{1}{m}\sum_{t=1}^{m} h_{ti} \right) \tag{1}$$

$$W_i = \left(\frac{1}{m}\sum_{t=1}^{m} a'_{ti}, \frac{1}{m}\sum_{t=1}^{m} b'_{ti}, \frac{1}{m}\sum_{t=1}^{m} c'_{ti}, \frac{1}{m}\sum_{t=1}^{m} d'_{ti}; \frac{1}{m}\sum_{t=1}^{m} h'_{ti} \right) \tag{2}$$

- Step 2: Use the NFWA method to integrate the evaluating items R_i and W_i of each factor F_i, where i = 1, 2, …, n, to obtain the FPSR of the product, R, shown as follows:

$$R = \frac{\sum_{i=1}^{n} W_i \otimes R_i}{\sum_{i=1}^{n} W_i} \tag{3}$$

Here, R is calculated through the NFWA method. Denote the α-cuts of the fuzzy ratings R_i and the fuzzy weights W_i as

$$R_{i\alpha} = [R_{i\alpha a}, R_{i\alpha b}]$$
$$W_{i\alpha} = [W_{i\alpha a}, W_{i\alpha b}]$$

Then, the α-cut of R is given by

$$R_\alpha = [R_{\alpha a}, R_{\alpha b}]$$

where

$$R_{\alpha a} = \min\left(\frac{\sum_{i=1}^{n} R_{i\alpha a} \cdot w_i}{\sum_{i=1}^{n} w_i} \right) \text{ and } R_{\alpha b} = \max\left(\frac{\sum_{i=1}^{n} R_{i\alpha b} \cdot w_i}{\sum_{i=1}^{n} w_i} \right)$$

Here, $w_i \in \{W_{i\alpha a}, W_{i\alpha b}\}$ for all $i \in \{1, 2, \ldots, n\}$ and all $\alpha \in [0, 1]$. The set of w_i that is used in the numerator has to be the same as the one in the denominator. By enumerating different α values, the membership function R can be constructed. Finally, the degree of confidence of R is the minimum of the degrees of confidence of R_i and w_i for all $i \in \{1, 2, \ldots, n\}$.

- Step 3: Use the proposed similarity measure to evaluate the degree of similarity between the fuzzy number R and each linguistic term shown in Table 2. Translate the fuzzy number R into a linguistic term, which has the largest degree of similarity to R.

Table 2. A Seven-Member Linguistic Term Set

Linguistic terms	Generalized fuzzy numbers
Very low (VL)	(0.0, 0.0, 0.05, 0.2; 1.0)
Low (L)	(0.0, 0.15, 0.25, 0.4; 1.0)
Fairly low (FL)	(0.2, 0.3, 0.4, 0.5; 1.0)
Medium (M)	(0.3, 0.45, 0.55, 0.7; 1.0)
Fairly high (FH)	(0.5, 0.6, 0.7, 0.8; 1.0)
High (H)	(0.6, 0.75, 0.85, 1.0; 1.0)
Very high (VH)	(0.8, 0.95, 1.0, 1.0; 1.0)

Now, we illustrate the new product go/nogo decision-making process of the proposed method by an example. In [17], the new product evaluation and selection criteria were developed as shown in Table 3. The factors were classified into four categories: product-marketing competitive advantages; product superiority; technological appropriateness; and product risk. Note that these factors in the former three categories have a positive impact on the FPSR of the product but these factors in the last category have a negative impact on the FPSR of the product.

Suppose that four experts use the thirteen criteria shown in Table 3 to make a screening of a new product TM-21 [17], where both the criteria ratings and their corresponding importance are assessed in linguistic terms described by fuzzy numbers in Table 2. Table 4 shows the ratings of criteria assigned by four experts E_1, E_2, E_3, and E_4, where the values w_{ij} denotes the degree of confidence that evaluator E_j evaluates the rating of criterion C_i, where $1 \le i \le 13$ and $1 \le j \le 4$. At the same time, the committee members assess the relative importance of all the criteria, on the basis of their experience and knowledge. The results are show in Table 5.

Table 3. Product Evaluation and Selection Criteria

Criteria		Description
Competitive marketing advantages	Marketing timing (C_1)	Matches desired entry timing needed by target segments
	Price superiority (C_2)	Offers value for money to target segments
	Marketing competencies (C_3)	Conforms to our salesforce, channels of distribution and logistical strengths
	Marketing attractiveness (C_4)	Permits the company to enter into a growing, high-potential market
Superiority	Functional competency (C_5)	Has unique or special functions to meet and attract target segments
	Featured differentia (C_6)	Has unique or special features to attract target segments
Technology suitability	Design quality (C_7)	Is designed for the quality needed by target segements
	Material specialization (C_8)	Uses materials of high quality and low rejection
	Manufacturing compatibility (C_9)	Can be produced by our best manufacturing technology and flexibility
	Supply benefit (C_{10})	Allows the company to use very best suppliers
Risk	Market competitiveness (C_{11})	Allows many competitive products in the market
	Technological uncertainty (C_{12})	Uses new technological skills that cannot be addressed by research
	Monetary risk (C_{13})	Products total dollar risk profile of product

Table 4. Ratings of Criteria and the Degree of Confidence Assigned by Experts using Linguistic Terms

Creteria	Experts				
	E_1	E_2	E_3	E_4	Average (R_i)
C_1	FH (w_{11} = 1.0)	VH (w_{12} = 0.6)	VH (w_{13} = 0.9)	H (w_{14} = 0.8)	(0.675, 0.8125, 0.8875, 0.95; 0.825)
C_2	M (w_{21} = 0.9)	FH (w_{22} = 0.7)	FL (w_{23} = 0.8)	M (w_{24} = 0.9)	(0.325, 0.45, 0.55, 0.675; 0.825)
C_3	M (w_{31} = 1.0)	FL (w_{32} = 0.8)	FL (w_{33} = 0.9)	M (w_{34} = 1.0)	(0.25, 0.375, 0.475, 0.6, 0.925)
C_4	H (w_{41} = 0.9)	H (w_{42} = 0.9)	VH (w_{43} = 0.6)	VH (w_{44} = 0.9)	(0.7, 0.85, 0.925, 1; 0.825)
C_5	VH (w_{51} = 0.8)	VH (w_{52} = 0.9)	H (w_{53} = 1.0)	VH (w_{54} = 0.8)	(0.75, 0.9, 0.9625, 1; 0.875)
C_6	H (w_{61} = 0.9)	H (w_{62} = 0.5)	H (w_{63} = 1.0)	VH (w_{64} = 0.7)	(0.65, 0.8, 0.8875, 1; 0.775)
C_7	H (w_{71} = 1.0)	H (w_{72} = 0.9)	H (w_{73} = 1.0)	VH (w_{74} = 0.9)	(0.65, 0.8, 0.8875, 1; 0.95)
C_8	FH (w_{81} = 0.8)	H (w_{82} = 0.8)	FH (w_{83} = 0.8)	H (w_{84} = 0.9)	(0.55, 0.675, 0.775, 0.9; 0.825)
C_9	VH (w_{91} = 0.8)	H (w_{92} = 0.6)	H (w_{93} = 0.9)	FH (w_{94} = 0.9)	(0.625, 0.7625, 0.85, 0.95; 0.8)
C_{10}	M (w_{101} = 0.9)	FH (w_{22} = 0.9)	M (w_{103} = 0.9)	FH (w_{104} = 1.0)	(0.4, 0.525, 0.625, 0.75; 0.925)
C_{11}	FH (w_{111} = 0.9)	H (w_{112} = 0.8)	FH (w_{113} = 0.9)	H (w_{114} = 0.9)	(0.55, 0.675, 0.775, 0.9; 0.875)
C_{12}	FH (w_{121} = 0.9)	H (w_{122} = 0.9)	FH (w_{123} = 1.0)	FH (w_{124} = 0.8)	(0.525, 0.6375, 0.7375, 0.85; 0.9)
C_{13}	FL (w_{131} = 1.0)	FH (w_{132} = 0.8)	M (w_{133} = 0.9)	FL (w_{134} = 0.9)	(0.3, 0.4125, 0.5125, 0.625; 0.9)

In the following, we use the proposed new product go/nogo decision-making method to deal with the screening of the product.

- Step 1: in this step, for each criterion, we calculate the average rating and the average importance weighting given by four experts based on Table 4, Table 5, Equation (1), and Equation (2). The averaged fuzzy numbers are shown in Table 4 and Table 5, respectively.
- Step 2: in this step, we aggregate fuzzy rating and fuzzy weighting of all the criteria to obtain the FPSR. Since the criteria C_{11}, C_{12}, and C_{13} have a negative impact on the FPSR of the product, the corresponding average ratings should first be transformed by the following formula:

$$R_i = (1, 1, 1, 1; 1) \ominus R_i$$

where $\ominus$ is the fuzzy numbers subtraction operation defined in [1]. Given two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4; h_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; h_B)$, we have

$$\tilde{A} \ominus \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min(h_A, h_B))$$

After such processing, we have

$$\tilde{R}_{11} = (0.1, 0.225, 0.325, 0.45; 0.875)$$

$$\tilde{R}_{12} = (0.15, 0.2625, 0.3625, 0.475; 0.9)$$

$$\tilde{R}_{13} = (0.375, 0.4875, 0.5875, 0.7; 0.9)$$

Finally, based on Table 4, Table 5, and Equation (3), we obtain the FPSR of the product shown as follows:

$$\tilde{R} = (0.4496, 0.6021, 0.6989, 0.8047; 0.675)$$

- Step 3: We use the proposed similarity measure to evaluate the degree of similarity between the fuzzy number $\tilde{R}$ and each linguistic term shown in Table 2:

$$S(\tilde{R}, \text{VL}) = 0.4191$$
$$S(\tilde{R}, \text{L}) = 0.6046$$
$$S(\tilde{R}, \text{FL}) = 0.7951$$
$$S(\tilde{R}, \text{M}) = 0.8869$$
$$S(\tilde{R}, \text{FH}) = 0.9534$$
$$S(\tilde{R}, \text{H}) = 0.8870$$
$$S(\tilde{R}, \text{VH}) = 0.8254$$

Because $S(\tilde{R}, \text{FH})$ has the largest value, the generalized trapezoidal fuzzy number $\tilde{R}$ is translated into the linguistic term "Fairly High", where the degree of similarity is 0.9534. That is, the possible success of the TM-21 development is Fairly High. This result coincides with the one presented in [17].

Table 5. Importance Weightings of Criteria and the Degree of Confidence Assessed by Experts Using Linguistic Terms

Creteria	Experts				
	E_1	E_2	E_3	E_4	Average (W_i)
C_1	H (w_{11} = 0.8)	FH (w_{12} = 1.0)	H (w_{13} = 1.0)	H (w_{14} = 0.7)	(0.575, 0.7125, 0.8125, 0.95; 0.875)
C_2	L (w_{21} = 0.7)	FH (w_{22} = 0.7)	FH (w_{23} = 0.6)	M (w_{24} = 0.7)	(0.325, 0.45, 0.55, 0.675; 0.675)
C_3	H (w_{31} = 0.9)	H (w_{32} = 1.0)	FH (w_{33} = 0.9)	FH (w_{34} = 0.8)	(0.55, 0.675, 0.775, 0.9; 0.9)
C_4	FH (w_{41} = 0.8)	H (w_{42} = 0.8)	H (w_{43} = 0.9)	H (w_{44} = 0.6)	(0.575, 0.7125, 0.8125, 0.95; 0.775)
C_5	H (w_{51} = 1.0)	FH (w_{52} = 0.9)	H (w_{53} = 1.0)	FH (w_{54} = 0.8)	(0.55, 0.675, 0.775, 0.9; 0.925)
C_6	M (w_{61} = 1.0)	L (w_{62} = 0.8)	M (w_{63} = 0.9)	M (w_{64} = 0.7)	(0.225, 0.375, 0.475, 0.625; 0.85)
C_7	FH (w_{71} = 1.0)	FH (w_{72} = 0.9)	H (w_{73} = 0.8)	H (w_{74} = 0.7)	(0.55, 0.675, 0.775, 0.9; 0.85)
C_8	FH (w_{81} = 0.8)	M (w_{82} = 0.7)	M (w_{83} = 0.8)	L (w_{84} = 0.7)	(0.275, 0.4125, 0.5125, 0.65; 0.75)
C_9	FH (w_{91} = 0.9)	M (w_{92} = 1.0)	L (w_{93} = 1.0)	M (w_{94} = 0.9)	(0.275, 0.4125, 0.5125, 0.65; 0.95)
C_{10}	M (w_{101} = 0.8)	FH (w_{22} = 0.8)	M (w_{103} = 0.9)	M (w_{104} = 1.0)	(0.35, 0.4875, 0.5875, 0.725; 0.875)
C_{11}	H (w_{111} = 0.8)	FH (w_{112} = 1.0)	H (w_{113} = 0.8)	H (w_{114} = 1.0)	(0.575, 0.7125, 0.8125, 0.95; 0.9)
C_{12}	FH (w_{121} = 0.9)	FH (w_{122} = 0.9)	H (w_{123} = 1.0)	FH (w_{124} = 0.8)	(0.525, 0.6375, 0.7375, 0.85; 0.9)
C_{13}	M (w_{131} = 0.8)	FH (w_{132} = 1.0)	M (w_{133} = 1.0)	L (w_{134} = 1.0)	(0.275, 0.4125, 0.5125, 0.65; 0.95)

Compared to the new product screening method presented in [17], our method based on the proposed similarity measure is more realistic and more reasonable because it takes into account the degrees of confidence of evaluators. On the other hand, for this particular example, if S_{CC} or S_{DSDL} instead of the proposed similarity measure S is used in Step 3, we can draw the same conclusion, i.e. the possible success of the TM-21 development is Fairly High. However, in general cases, the proposed similarity measure S is more suitable to new product screening problems than existing similarity measures due to the fact that the proposed similarity can overcome their drawbacks.

6. Conclusions

In this paper, we propose a novel measure for computing the degree of similarity between two generalized trapezoidal fuzzy numbers. The proposed similarity measure has a simple definition and is more intuitively understandable. We compare the proposed similarity measure with existing similarity measures by numerical examples. The results show that the proposed measure is effective and promising due to the fact that it can overcome the drawbacks of existing similarity measures. Finally, we use the proposed similarity measure of generalized trapezoidal fuzzy numbers for handling new product screening problems.

It should be noted that although in this paper the proposed measure is used to measure the degree of similarity of generalized trapezoidal fuzzy numbers, it can be easily extended to generalized fuzzy numbers. To obtain the degree of similarity between two generalized fuzzy numbers, a direct idea is to use the degree of similarity of left and right membership functions rather than the degree of similarity of components in these fuzzy numbers. In the future work, we will explore this extension and investigate its applications in web mining.

Acknowledgment

This work was supported by Philosophy and Sociology of Science Foundation of Zhejiang Province (13NDJC036YB). The author also gratefully acknowledges helpful comments and suggestions of the reviewers, which have improved the presentation.

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