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Delay-dependent H_∞ Control for Autonomous Underwater Vehicle

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Abstract

This paper deals with the problem of delay-dependent H_∞ control for AUV system with external disturbance. A delay-dependent H_∞ control criteria is obtained and formulated in the form of LMI. It is obtained through Lyapunov-Krasovskii functional approach and an integral inequality. Simulation example shows that the proposed method is effective.

Keywords: *autonomous underwater vehicle (AUV); time-delay system; H_∞ Control; integral inequality; linear matrix inequality (LMI)*

1. Introduction

Time-delays are frequently encountered in many fields of science and engineering, such as manufacturing systems, biology, economy and other areas[1]. Many methods have been applied to obtain less conservative delay-dependent conditions[2-6]. Over the past few years, a great amount of research has been conducted regarding the autonomous underwater vehicle (AUV). AUV is a advanced tool for ocean exploration and exploitation[7]. It is employed in risky missions such as oceanographic observations, military applications, recovery of lost man-made objects, etc [8]. But AUV's kinematic and dynamic models(Fossen) are highly nonlinear and coupled, making control problem a difficult task. In practical applications, the model equations of AUV should be adjusted according to the specific environments. In the implementation of marine observation, reconnaissance, deep-sea photography and other tasks, the AUV need operate at low speed for long time. In the condition of low speed, there is no need to impose control to the movement in surge and sway for the AUV.

In this paper, we are concerned with the design of delay-dependent H_∞ control for nonlinear AUV system. Firstly, reasonable assumptions are applied to simply the AUV system. Then we can obtain an nonlinear AUV system of four degrees of freedom. Considering time-delay and external disturbances, the AUV system can be express a uncertain time-delay system with disturbance. Secondly, by using Lyapunov-Krasovskii functional approach combined with an integral inequality, the H_∞ control law is obtained. Lastly, illustrative example shows the effectiveness of the method.

This paper is organized as follows. Section 2 presents the simplified model. Section 3 gives the problem formulation. The main results on designing the delay-independent H_∞ controllers are presented in Section 4. Simulation results are shown in Section 5. Section 6 contains the main conclusions.

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2. Simplified Model

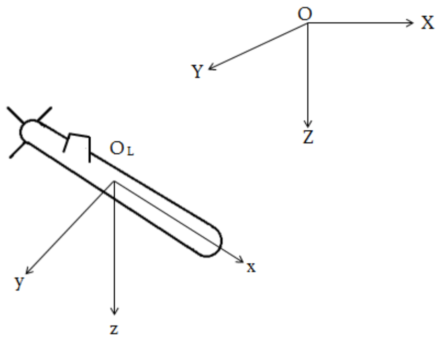

Figure 1. The AUV Model

The equations of motion of AUV are generally derived in the body coordinate frame. Figure 1 shows the inertial and body coordinate frames along with notations of various motions. The general kinematic and dynamic equations of motion of the AUV in six degrees of freedom can be written as[9]

$$\begin{cases} \dot{\eta} = J(\eta)v \\ M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \end{cases} \tag{1}$$

The meaning of symbols see [9] for further details.

In the condition of low speed, there is no need to impose control to the movement in surge and sway for the AUV. In order to simply the model, the AUV is assumed to be symmetric about all the three axes. It is assumed that the AUV is neutrally buoyant and that the centre of buoyancy coincides with the centre of gravity. Then the four degrees of freedom of the equations can be expressed as

$$\begin{cases} \dot{x} = u\cos\psi - v\sin\psi \\ \dot{y} = u\sin\psi + v\cos\psi \\ \dot{z} = w \\ \dot{\psi} = r \end{cases} \tag{2}$$

$$\begin{cases} m(\dot{u} - vr) = \frac{\rho}{2}L^4 X_{rr}r^2 + \frac{\rho}{2}L^3(X_{\dot{u}}\dot{u} + X_{vr}vr) + \frac{\rho}{2}L^2(X_{vv}v^2 + X_{ww}w^2) + T_x \\ m(\dot{u} + vr) = \frac{\rho}{2}L^4 Y_r r^2 + \frac{\rho}{2}L^3(Y_{\dot{v}}\dot{v} + Y_{ur}ur) + \frac{\rho}{2}L^2(Y_{uv}uv + Y_{vw}vw) + T_y \\ m\dot{w} = \frac{\rho}{2}L^4 Z_{rr}r^2 + \frac{\rho}{2}L^3(Z_{\dot{w}}\dot{w} + Z_{vr}vr) + \frac{\rho}{2}L^2(Z_{uw}uw + Z_{vv}v^2) + T_z \\ I_z\dot{r} = \frac{\rho}{2}L^5 N_{\dot{r}}\dot{r} + \frac{\rho}{2}L^4(N_{\dot{v}}\dot{v} + N_{ur}ur + N_{wr}wr) + \frac{\rho}{2}L^3(N_{uv}uv + N_{vw}vw) + M_z \end{cases} \tag{3}$$

where L denotes the length of AUV, ρ denotes the density of water. m and I_z denote the mass and moment of inertia of the AUV, respectively. M_z is the torque about $O_L - z$ axis. T_x, T_y and T_z denote the forces along the $O_L - x$, $O_L - y$ and $O_L - z$ axis.

$X_{(\square)}$, $Y_{(\square)}$, $Z_{(\square)}$ and $N_{(\square)}$ are classical hydrodynamic derivatives.

$m = 5454\text{kg}$	$\rho = 1000\text{kg/m}^3$	$L = 5.3\text{m}$	$I_z = 13587\text{Nms}^2$
$X_{\dot{u}} = 7.6\text{e}-3$	$Y_{\dot{v}} = -5.5\text{e}-2$	$Z_{\dot{\omega}} = -2.4\text{e}-1$	$N_{\dot{v}} = 1.2\text{e}-3$
$X_{vv} = 5.3\text{e}-2$	$Y_{\dot{r}} = 1.2\text{e}-3$	$Z_{vv} = -6.8\text{e}-2$	$N_{\dot{r}} = -3.4\text{e}-3$
$X_{\omega\omega} = 1.7\text{e}-1$	$Y_{uv} = -1.0\text{e}-1$	$Z_{rr} = -7.4\text{e}-3$	$N_{uv} = -7.4\text{e}-3$
$X_{rr} = 4.0\text{e}-3$	$Y_{ur} = 3.0\text{e}-2$	$Z_{u\omega} = -3.0\text{e}-1$	$N_{v\omega} = -2.7\text{e}-2$
$X_{vr} = 2.0\text{e}-2$	$Y_{v\omega} = 6.8\text{e}-2$	$Z_{vr} = 4.5\text{e}-2$	$N_{ur} = -1.6\text{e}-3$
	$Y_{\omega r} = -1.9\text{e}-2$		$N_{\omega r} = 7.4\text{e}-3$

Table 1. Hydrodynamic Derivatives and Inertial Parameters

Substituting hydrodynamic derivatives and inertial parameters in [10] (see Table 1)into (2) and (3) and setting

$$x = (x_i)^{\text{T}} = (x, y, z, \psi, u, v, w, r)^{\text{T}},\ i = 1, 2, ..., 8$$

$$u = (u_1, u_2, u_3, u_4)^{\text{T}}$$

$$u = \begin{pmatrix} 1.661\times10^{-4} & 0 & 0 & 0 \\ 0 & 1.049\times10^{-4} & 0 & -2.40\times10^{-6} \\ 0 & 0 & 4.29\times10^{-5} & 0 \\ 0 & -2.40\times10^{-6} & 0 & 4.84\times10^{-5} \end{pmatrix} \begin{pmatrix} T_x \\ T_y \\ T_z \\ M_z \end{pmatrix}$$

gives us

$$\dot{x}(t) = f(x) + Bu(t) \tag{4}$$

where $x \in \mathbb{R}^8$ and $u \in \mathbb{R}^4$ are the state and control inputs, respectively.

$$f(x) = \begin{pmatrix} x_5 \cos x_4 - x_6 \sin x_4 \\ x_5 \sin x_4 + x_6 \cos x_4 \\ x_7 \\ x_8 \\ 0.124x_6^2 + 0.397x_7^2 + 0.262x_8^2 + 1.153x_6x_8 \\ -0.353x_5x_8 - 0.141x_7x_8 - 0.149x_5x_6 + 0.095x_6x_7 \\ -0.041x_6^2 - 0.125x_8^2 - 0.181x_5x_7 + 0.144x_6x_8 \\ -0.313x_5x_8 + 0.138x_7x_8 - 0.031x_5x_6 - 0.095x_6x_7 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ I_4 \end{pmatrix} \in \mathbb{R}^{8\times4}$$

Based on Maclaurin Series, $f(x)$ reduces to

$$f(x)=\begin{pmatrix}0 & I_4\\ 0 & 0\end{pmatrix}_{8\times 8} x+\begin{pmatrix}-x_4x_6\\ x_4x_5\\ 0\\ 0\\ 0.124x_6^2+0.397x_7^2+0.262x_8^2+1.153x_6x_8\\ -0.353x_5x_8-0.141x_7x_8-0.149x_5x_6+0.095x_6x_7\\ -0.041x_6^2-0.125x_8^2-0.181x_5x_7+0.144x_6x_8\\ -0.313x_5x_8+0.138x_7x_8-0.031x_5x_6-0.095x_6x_7\end{pmatrix}$$

Then equation (4) can rewritten as

$$\dot{x}(t)=\left(A+\Delta A\right)x(t)+Bu(t) \tag{5}$$

In practice, there are delays in the control process for the AUV and the AUV must often operate in the presence of ocean currents. Equation (5) can be expressed as

$$\dot{x}(t)=\left(A+\Delta A\right)x(t)+\left(A_1+\Delta A_1\right)x(t-d)+Bu(t)+Cw(t) \tag{6}$$

where d is the delay for the AUV, $w(t)$ is the irregular wave disturbances.

3. Problem Formulation

Consider the following system with time-delay and disturbances:

$$\begin{cases}\dot{x}(t)=\left(A+\Delta A\right)x(t)+\left(A_1+\Delta A_1\right)x(t-d)+Bu(t)+Cw(t)\\ z(t)=Dx(t)+D_1w(t)+D_2u(t)\\ x(t)=\varphi(t),\ t\in\left[-d,0\right]\end{cases} \tag{7}$$

where $x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^m$ and $z(t)\in\mathbb{R}^p$ are the state vector, input vector and output vector of the system, respectively; $w(t)\in\mathbb{R}^q$ is external disturbances; $d>0$ is the constant delay; $\varphi(t)$ is a continuous vector valued initial function; A, A_1, B, C, D, D_1 and D_2 are constant matrices. ΔA and ΔA_1 are unknown matrices representing time-varying parameter uncertainties in the system model. We assume that the uncertainties are norm-bounded and can be described as:

$$\begin{pmatrix}\Delta A & \Delta A_1\end{pmatrix}=EF(t)\begin{pmatrix}H & H_1\end{pmatrix}$$

where E, H, H_1 are known real constant matrices, and $F(t)\in\mathbb{R}^{i\times j}$ is an unknown real and possibly time-varing matrix satisfying $F^{\mathrm{T}}(t)F(t)\le I$ for any given t.

The external disturbances for the AUV in the ocean can be described as :

$$\dot{w}(t)=Gw(t)$$

where G is known constant matrix.

For given constant $\gamma>0$, introduce performance index

$$J=\int_0^{\infty}\left(\gamma^{-1}z^{\mathrm{T}}z-\gamma w^{\mathrm{T}}w\right)\mathrm{d}t$$

where γ denotes the degree of inhibition of external disturbances.

The memoryless state feedback controller

$$u(t)=Kx(t) \tag{8}$$

is employed to stabilize (7). The closed-loop system constructed by means of (7) and (8) is given by

$$\begin{cases} \dot{x}(t) = A_K x(t) + \bar{A}_1 x(t-d) + Cw(t) \\ z(t) = D_K x(t) + D_1 w(t) \end{cases} \tag{9}$$

where

$$A_K \triangleq A + \Delta A + BK\,,\ \ \bar{A}_1 \triangleq A_1 + \Delta A_1\,,\ \ D_K \triangleq D + D_2 K\,.$$

The objectives of this paper are: Given a scalar $\gamma > 0$, establish the delay-dependent sufficient conditions such that the AUV system has a prescribed H_∞ performance γ, that is:

(i) the system (9) with $w(t) = 0$ is asymptotically stable;

(ii) the H_∞ performance $\|z(t)\|_2 \le \gamma \|w(t)\|_2$ is guaranteed.

And design a delay-dependent state feedback controller.

Before concluding this section, we introduce three lemmas which are essential for the development of our results.

Lemma 1[11] For given appropriate dimensions matrices Y, G, H, and Y is symmetric,
$Y + HF(t)G + G^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}} < 0$ for all matrices satisfying $F^{\mathrm{T}}(t)F(t) \le I$, if and only if existing a constant $\varepsilon > 0$, such that $Y + \varepsilon HH^{\mathrm{T}} + \varepsilon^{-1}G^{\mathrm{T}}G < 0$.

Lemma 2[12] Let $x(t) \in \mathbb{R}^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any matrices $M_1, M_2 \in \mathbb{R}^n$ and $X = X^{\mathrm{T}} > 0$, and a scalar function $h = h(t) \ge 0$:

$$-\int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s) X \dot{x}(s) \mathrm{d}s \le \xi^{\mathrm{T}}(t) \begin{pmatrix} M_1^{\mathrm{T}} + M_1 & -M_1^{\mathrm{T}} + M_2 \\ * & -M_2^{\mathrm{T}} - M_2 \end{pmatrix} \xi(t)$$

$$+ h\xi^{\mathrm{T}}(t) \begin{pmatrix} M_1^{\mathrm{T}} \\ M_2^{\mathrm{T}} \end{pmatrix} X^{-1} \begin{pmatrix} M_1 & M_2 \end{pmatrix} \xi(t)$$

where $\xi(t) \triangleq \begin{pmatrix} x(t) \\ x(t-h) \end{pmatrix}$.

Lemma 3[13] For given symmetric matrix $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$, and matrix $S_{11} \in \mathbb{R}^{r \times r}$, then the following conditions are equivalent:

(i) $S < 0$;

(ii) $S_{11} < 0$, $S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$;

(iii) $S_{22} < 0$, $S_{11} - S_{12}^{\mathrm{T}} S_{22}^{-1} S_{12} < 0$.

4. Main Results

In this section, we first presents the delay-dependent H_∞ conditions obtained by means of the integral-inequality method. We have the following result.

Theorem 1. For given scalar $\gamma > 0$, the system (9) has a prescribed H_∞ performance γ if there exist real matrices $P, Q, R > 0$ such that

$$\Theta = \begin{pmatrix} A_K^{\mathrm{T}}P + PA_K + Q + M_1^{\mathrm{T}} + M_1 & P\bar{A}_1 - M_1^{\mathrm{T}} + M_2 & PC & D_K^{\mathrm{T}} & dA_K^{\mathrm{T}} & dM_1^{\mathrm{T}} \\ * & -Q - M_2^{\mathrm{T}} - M_2 & 0 & 0 & d\bar{A}_1^{\mathrm{T}} & dM_2^{\mathrm{T}} \\ * & * & -\gamma I & D_1^{\mathrm{T}} & dC^{\mathrm{T}} & 0 \\ * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & -dR^{-1} & 0 \\ * & * & * & * & * & -dR \end{pmatrix} < 0 \tag{10}$$

Proof. Choose a Lyapunov-Krasovskii functional candidate as

$$V(t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-d}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$

where $P > 0, Q > 0, R > 0$. Taking the derivative of $V(t)$ with respect to t along the trajectory of (9) yields

$$\begin{aligned} \dot{V}(t) &= 2x^{\mathrm{T}}(t)P\dot{x}(t) + x^{\mathrm{T}}(t)Qx(t) - x^{\mathrm{T}}(t-d)Qx(t-d) \\ &\quad + d\dot{x}^{\mathrm{T}}(t)R\dot{x}(t) - \int_{t-d}^{t} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s \end{aligned} \tag{11}$$

Use Lemma 2 to obtain

$$\begin{aligned} -\int_{t-d}^{t} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s &\le \zeta^{\mathrm{T}}(t)\begin{pmatrix} M_1^{\mathrm{T}} + M_1 & -M_1^{\mathrm{T}} + M_2 \\ * & -M_2^{\mathrm{T}} - M_2 \end{pmatrix}\zeta(t) \\ &\quad + d\zeta^{\mathrm{T}}(t)\begin{pmatrix} M_1^{\mathrm{T}} \\ M_2^{\mathrm{T}} \end{pmatrix} R^{-1}\begin{pmatrix} M_1 & M_2 \end{pmatrix}\zeta(t) \end{aligned} \tag{12}$$

where $\zeta(t) \triangleq \begin{pmatrix} x(t) \\ x(t-d) \end{pmatrix}$.

Substituting (12) to (11) gives

$$\dot{V}(t) \le \eta^{\mathrm{T}}(t)\left[\Gamma + d\begin{pmatrix} A_K^{\mathrm{T}} \\ \bar{A}_1^{\mathrm{T}} \\ C^{\mathrm{T}} \end{pmatrix} R\begin{pmatrix} A_K & \bar{A}_1 & C \end{pmatrix} + d\begin{pmatrix} M_1^{\mathrm{T}} \\ M_2^{\mathrm{T}} \\ 0 \end{pmatrix} R^{-1}\begin{pmatrix} M_1 & M_2 & 0 \end{pmatrix}\right]\eta(t)$$

where

$$\Gamma = \begin{pmatrix} A_K^{\mathrm{T}}P + PA_K + Q + M_1^{\mathrm{T}} + M_1 & P\bar{A}_1 - M_1^{\mathrm{T}} + M_2 & PC \\ * & -Q - M_2^{\mathrm{T}} - M_2 & 0 \\ * & * & 0 \end{pmatrix}$$

$$\eta(t) \triangleq \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix}$$

Noting $\gamma^{-1}z^{\mathrm{T}}(t)z(t) - \gamma w^{\mathrm{T}}(t)w(t) = \eta^{\mathrm{T}}(t)\begin{pmatrix} \gamma^{-1}D_K^{\mathrm{T}}D_K & 0 & \gamma^{-1}D_K^{\mathrm{T}}D_1 \\ * & 0 & 0 \\ * & * & \gamma^{-1}D_1^{\mathrm{T}}D_1 - \gamma I \end{pmatrix}\eta(t)$

and using Lemma 3, (10) is equivalent to

$$\Pi=\begin{pmatrix}\Pi_{11} & \Pi_{12} & \Pi_{13}\\ * & \Pi_{22} & \Pi_{23}\\ * & * & \Pi_{33}\end{pmatrix}<0 \tag{13}$$

where

$$\Pi_{11}=A_K^{\mathrm{T}}P+PA_K+Q+M_1^{\mathrm{T}}+M_1+dA_K^{\mathrm{T}}RA_K+dM_1^{\mathrm{T}}R^{-1}M_1+\gamma^{-1}D_K^{\mathrm{T}}D_K$$
$$\Pi_{12}=P\bar{A}_1-M_1^{\mathrm{T}}+M_2+dA_K^{\mathrm{T}}R\bar{A}_1+dM_1^{\mathrm{T}}R^{-1}M_2$$
$$\Pi_{13}=PC+dA_K^{\mathrm{T}}RC$$
$$\Pi_{22}=-Q-M_2^{\mathrm{T}}-M_2+d\bar{A}_1^{\mathrm{T}}R\bar{A}_1+dM_2^{\mathrm{T}}R^{-1}M_2$$
$$\Pi_{23}=d\bar{A}_1^{\mathrm{T}}RC$$
$$\Pi_{33}=dC^{\mathrm{T}}RC+\gamma^{-1}D_1^{\mathrm{T}}D_1-\gamma I$$

In zero initial conditions, consider

$$J_\tau=\int_0^\tau\left(\gamma^{-1}z^{\mathrm{T}}z-\gamma w^{\mathrm{T}}w\right)\mathrm{d}t$$

For any non-zero external disturbance $w(t)$, we can derive

$$J_\tau\le\int_0^\tau\left(\gamma^{-1}z^{\mathrm{T}}z-\gamma w^{\mathrm{T}}w+\dot{V}(t)\right)\mathrm{d}t$$
$$=\int_0^\tau\eta^{\mathrm{T}}(t)\begin{pmatrix}\Pi_{11} & \Pi_{12} & \Pi_{13}\\ * & \Pi_{22} & \Pi_{23}\\ * & * & \Pi_{33}\end{pmatrix}\eta(t)\mathrm{d}t<0$$

i.e.

$$\int_0^\tau z^{\mathrm{T}}(t)z(t)\mathrm{d}t<\gamma^2\int_0^\tau w^{\mathrm{T}}(t)w(t)\mathrm{d}t\le\gamma^2\int_0^\infty w^{\mathrm{T}}(t)w(t)\mathrm{d}t$$

Theorem 2. For given scalar $\gamma>0$, $k_1\in\mathbb{R}$, $k_2\neq 0$, if there exist symmetric matrices $X>0, \bar{R}>0, \bar{Q}>0$, a matrix Y and a scalar $\varepsilon>0$ such that the following LMI is satisfied:

$$\Omega\triangleq\begin{pmatrix}\Omega_{11} & \Omega_{12} & C & \Omega_{14} & \Omega_{15} & dk_1\bar{R} & XH^{\mathrm{T}}\\ * & \Omega_{22} & 0 & 0 & dk_2^{-1}XA_1 & d\bar{R} & k_2^{-1}XH_1^{\mathrm{T}}\\ * & * & -\gamma I & D_1^{\mathrm{T}} & dC^{\mathrm{T}} & 0 & 0\\ * & * & * & -\gamma I & 0 & 0 & 0\\ * & * & * & * & \Omega_{55} & 0 & 0\\ * & * & * & * & * & -d\bar{R} & 0\\ * & * & * & * & * & * & -\varepsilon I\end{pmatrix}<0 \tag{14}$$

where

$$\Omega_{11}=XA^{\mathrm{T}}+AX+\bar{Q}+2k_1X+Y^{\mathrm{T}}B^{\mathrm{T}}+BY+\varepsilon EE^{\mathrm{T}}$$
$$\Omega_{12}=k_2A_1X-k_1k_2^{-1}X+X$$
$$\Omega_{14}=XD^{\mathrm{T}}+Y^{\mathrm{T}}D_2^{\mathrm{T}}$$
$$\Omega_{15}=d\left(XA^{\mathrm{T}}+Y^{\mathrm{T}}B^{\mathrm{T}}\right)+\varepsilon dEE^{\mathrm{T}}$$
$$\Omega_{22}=-\left(k_2^{-1}\right)^2\bar{Q}-2k_2^{-1}X$$
$$\Omega_{55}=-d\bar{R}+\varepsilon d^2EE^{\mathrm{T}}$$

Then the state feedback control law

$$u(t) = YX^{-1}x(t) \tag{15}$$

with an H_∞ norm γ.

Proof. In order to obtain a controller gain K, we first let

$$M_1 = k_1 P,\ \ M_2 = k_2 P$$

where $k_1 \in \square$, $k_2 \neq 0$.

(10) can be rewritten as

$$\Psi + \begin{pmatrix} PE \\ 0 \\ 0 \\ 0 \\ dE \\ 0 \end{pmatrix} F(t)\begin{pmatrix} H & H_1 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} H^{\mathrm{T}} \\ H_1^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} F^{\mathrm{T}}(t)\begin{pmatrix} E^{\mathrm{T}}P & 0 & 0 & 0 & dE^{\mathrm{T}} & 0 \end{pmatrix} < 0 \tag{16}$$

where

$$\Psi = \begin{pmatrix} \Psi_{11} & \Psi_{12} & PC & D^{\mathrm{T}} + \left(D_2 K\right)^{\mathrm{T}} & d\left(A + BK\right)^{\mathrm{T}} & dk_1 P \\ * & -Q - 2k_2 P & 0 & 0 & dA_1^{\mathrm{T}} & dk_2 P \\ * & * & -\gamma I & D_1^{\mathrm{T}} & dC^{\mathrm{T}} & 0 \\ * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & -dR^{-1} & 0 \\ * & * & * & * & * & -dR \end{pmatrix}$$

where

$$\Psi_{11} = A^{\mathrm{T}}P + PA + Q + 2k_1 P + \left(BK\right)^{\mathrm{T}} P + P\left(BK\right)$$

$$\Psi_{12} = PA_1 - k_1 P + k_2 P$$

Applying Lemma 1 in (16), the matrix inequality holds for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \le I$ if and only if there exists a constant $\varepsilon > 0$ such that

$$\Psi + \varepsilon \begin{pmatrix} PE \\ 0 \\ 0 \\ 0 \\ dE \\ 0 \end{pmatrix} \begin{pmatrix} E^{\mathrm{T}}P & 0 & 0 & 0 & dE^{\mathrm{T}} & 0 \end{pmatrix} + \varepsilon^{-1} \begin{pmatrix} H^{\mathrm{T}} \\ H_1^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} H & H_1 & 0 & 0 & 0 & 0 \end{pmatrix} < 0 \tag{17}$$

It follows from the Lemma 3 that (17) is equivalent to

$$\Lambda=\begin{pmatrix}\Lambda_{11} & \Lambda_{12} & PC & D^{\mathrm{T}}+\left(D_2K\right)^{\mathrm{T}} & \Lambda_{15} & dk_1P & H^{\mathrm{T}}\\ * & -Q-2k_2P & 0 & 0 & dA_1^{\mathrm{T}} & dk_2P & H_1^{\mathrm{T}}\\ * & * & -\gamma I & D_1^{\mathrm{T}} & dC^{\mathrm{T}} & 0 & 0\\ * & * & * & -\gamma I & 0 & 0 & 0\\ * & * & * & * & -dR^{-1}+\varepsilon d^2EE^{\mathrm{T}} & 0 & 0\\ * & * & * & * & * & -dR & 0\\ * & * & * & * & * & * & -\varepsilon I\end{pmatrix}<0 \quad (18)$$

where

$$\Lambda_{11}=A^{\mathrm{T}}P+PA+Q+2k_1P+\left(BK\right)^{\mathrm{T}}P+P\left(BK\right)+\varepsilon PEE^{\mathrm{T}}P$$
$$\Lambda_{12}=PA_1-k_1P+k_2P$$
$$\Lambda_{15}=d\left(A+BK\right)^{\mathrm{T}}+\varepsilon dPEE^{\mathrm{T}}$$

Multiplying (18) by $\mathrm{diag}\left\{P^{-1},k_2^{-1}P^{-1},I,I,I,R^{-1},I\right\}$, and setting $X=P^{-1},\bar{R}=R^{-1},\bar{Q}=P^{-1}QP^{-1},Y=KP^{-1}$, we find (18) is equivalent to (14). The desired controller is defined with $u(t)=YX^{-1}x(t)$.

Remark 1. Theorem 2 provides a sufficient condition for delay-independent H_∞ control of uncertain time-delay AUV system. Inequality (14) is a LMI in scalar ε, matrix variables $X,\bar{R},\bar{Q},Y$. Hence, the delay-independent H_∞ control problem can be transformed to the feasible problem of a LMI, and the latter can be solved through feasp function in LMI Toolbox.

Remark 2. We can obtain minimum the degree of inhibition for given time-delay by solving the following optimization problem

$$\begin{aligned}&\min_{X,\bar{R},\bar{Q},\varepsilon}\ \gamma\\ &s.t.\ (14)\\ &X>0\end{aligned}\qquad(19)$$

Problem (19) is a convex optimization problem with LMI constraints. We can solve it by means of mincx solver in LMI Toolbox.

Remark 3. For given some degree of inhibition γ, we can obtain maximum allowable time-delay d^* by solving the following optimization problem

$$\begin{aligned}&\max_{X,\bar{R},\bar{Q},\varepsilon}\ d\\ &s.t.\ (14)\\ &X>0\end{aligned}\qquad(20)$$

Problem (20) can be solved by gevp solver in LMI Toolbox.

5. Simulation Example

In this section, we present a simulation example to illustrate the effectiveness of the proposed method. Consider the system described by (7) with

$$A=\begin{pmatrix}-5 & 0\\ 6 & -10\end{pmatrix},\ A_1=\begin{pmatrix}-1 & 0\\ 2 & 1\end{pmatrix},\ B=\begin{pmatrix}-1\\ 3\end{pmatrix},\ C=\begin{pmatrix}4\\ 3\end{pmatrix},$$

$$D=-1,\quad D_1=\begin{pmatrix}4\\3\end{pmatrix},\quad D_2=\begin{pmatrix}3\\-1\end{pmatrix},\quad E=-0.1,$$
$$H=\begin{pmatrix}0.2 & 0\\1 & 2\end{pmatrix},\quad H_1=\begin{pmatrix}-1 & 2\\0 & -0.3\end{pmatrix}$$

$d=0.3$, $F(t)=0.5\sin t$, external disturbance $w(t)$ satisfying $\dot{w}(t)=\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}w(t)$.

Initial conditions are $x_1(0)=10$, $x_2(0)=10$. We choose $\gamma=5$, $k_1=0.6$, $k_2=0.2$. By Theorem 2, we can derive the controller gain

$$K=\begin{pmatrix}0.6432 & -0.5679\end{pmatrix}$$

The curves of state and control of system are shown in Figure 2 and Figure 3.

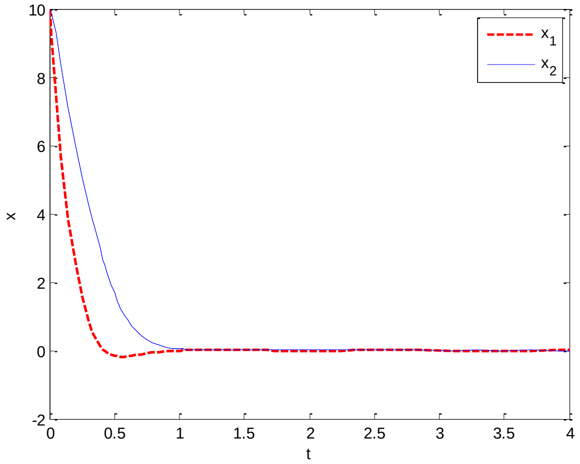

Figure 2. The Curve of State Variables

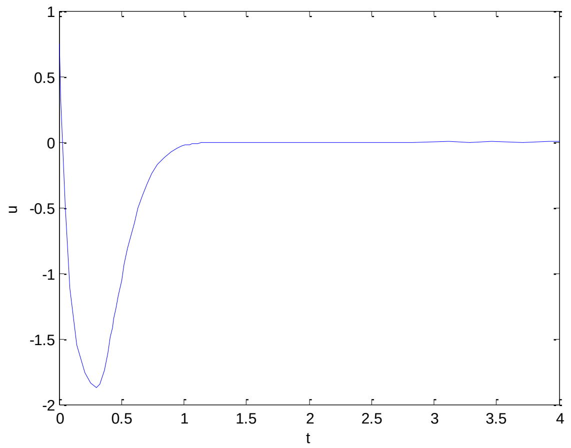

Figure 3. The Curve of Control $u(t)$

Table 2 shows the relations of given scalar γ and maximum allowable time-delay d^*.

γ	4	5	6	8	10	13	20
d^*	0.98	1.05	1.07	1.09	1.11	1.12	1.15

Table 2. Maximum Allowable Time-delay d^* in Different γ

From Table 2, we can conclude that with the increase of γ, maximum allowable time-delay d^* is also increased.

6. Conclusions

The problem of delay-dependent H_∞ control for AUV system with external disturbance has been addressed. Reasonable assumptions of the AUV model with six degrees of freedom have been applied to simply the AUV system. An nonlinear AUV system of four degrees of freedom has been obtained. The H_∞ control law is obtained by using Lyapunov-Krasovskii functional approach combined with an integral inequality. Simulation example illustrates the effectiveness of the proposed method.

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