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A Rapid Online Calculation Method of Three-dimensional Plastic Deformation in Strip Rolling

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Abstract

Three-dimensional strip plastic deformation model is a key part of shape setup model in hot rolling. The numerical efficiency, accuracy and stability of the method are the main factors that may restrict its online application. Considering the theoretical limitations of existing models, basic equations include equilibrium equations with the effect of shear stress, constitutive equations based on incremental theory, and lubricated friction with the respective coefficients of friction at longitudinal and transverse direction. After normalization of parameters and the asymptotic analysis, the strip plastic deformation model is established, and solved by finite volume method and BICGSTAB algorithm to get the transverse distribution of rolling force of each stand. Compared to the finite element model established by ANSYS software, the new model has reliable results and a short computation time of 31 milliseconds, which show its feasibility of online application.

Keywords*: plastic deformation; shape control; three-dimensional difference; asymptotic analysis; finite volume method*

1. Introduction

Strip shape setup model mainly consists of the strip plastic deformation model, the roll elastic deformation model and the buckling model [1, 2]. The three-dimensional strip plastic deformation model has been a key part of the setup model, because it involves several complex rolling theories, such as friction conditions, nonlinear constitutive relations of material and nonlinear metal flow, *etc* [3, 4].

Three-dimensional finite difference method is widely used for solving the plastic deformation, due to its complete theory, concise form and high calculation speed. Ref. [5] proposed a theoretical three-dimensional difference method for narrow strip (width of 30mm). Ref. [6] applied the method to a greater width (width-thickness ratio about 150) after a modification to reduce the computing time and solve the divergence of iteration, achieving stress distribution and width spread of the strip. Ref. [7] improved the method for wide strip (width-thickness ratio about 300), using pre-displacement method to calculate the friction in the stagnate area and adopting certain stress boundary conditions. Both modified methods introduced transverse shear stress whose accuracy depends on the strip edge shape in the deformation zone. When the edge shape is not properly set, the transverse shear stress could be abnormal and thus lead to convergence of iteration. Besides, the sensitivity to the edge shape increases with the width. Ref. [8] and [9] made certain assumptions and simplifications, and established an online plastic deformation model based on finite difference method to predict the transverse distribution of rolling force and tensile stress, with a better adaptability to the strip width. However, both these online model neglected the effect of transverse shear stress on the deformation, which caused an incomplete theoretical foundation and reduced the result credibility. Therefore, an online strip deformation model of high efficiency, precision and adaptability is very necessary for improving the shape theory, shape setup model and product quality.

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Considering the limitations of the above difference models, in this paper, boundary conditions was determined according to the characteristics of strip hot rolling process and the plastic deformation theory [10]; lubricated friction theories were applied to the plastic deformation model [11, 12]; eventually, an three-dimensional plastic deformation model which can adopt any strip width was established based on asymptotic analysis theory [13], and was solved by finite volume method and advanced fast algorithm to improve calculation accuracy and speed.

2. Limitations of Existing Models

The three-dimensional difference models in [6] and [7] represented the classic strip plastic deformation model which had a relatively rigorous theory and credible results. But they could only apply to the narrow strip situation and their accuracy and stability depended on an appropriate assumption of strip edge shape. The theoretical limitations of these models are as follows:

1) Shear strain. Though both models used different formula for shear strain increment, they considered shear strain increment on only one plane. Besides, Ref. [6] retained only the transverse item, ignoring the longitudinal item, as (1); Ref. [7] had both items, but introduced a correction coefficient, as (2).

$$\mathrm{d}\gamma_{xz} = \frac{\partial}{\partial x}(\mathrm{d}W) \tag{1}$$

$$\mathrm{d}\gamma_{xz} = \alpha_{\gamma}\left[\frac{\partial}{\partial x}(\mathrm{d}W) + \frac{\partial}{\partial z}(\mathrm{d}U)\right] \tag{2}$$

where x is the length coordinate (longitudinal); z is the width coordinate (transverse); γ_{xz} is the shear strain on xoz plane; dU and dW are the components of increment of strip sliding displacement relative to the roll surface at x-axis and z-axis respectively; α_{γ} is the correction coefficient of shear strain.

2) The ratio of transverse strain and longitudinal strain. In both models, the ratios of transverse strain ε_z and longitudinal strain ε_x were a constant α, as (3). However, in the hot rolling practice, transverse strain and longitudinal strain change with rolling conditions such as strip steel grade, width and thickness.

$$\mathrm{d}\varepsilon_z / \mathrm{d}\varepsilon_x = \varepsilon_z / \varepsilon_x = \alpha \tag{3}$$

3) Coefficient of friction. In both models, the coefficients of friction at x-axis and z-axis were the same constant. But in practice, the coefficients vary due to the different stress condition at each direction. In [6], the Coulomb friction stress at the contact surface between roll and strip was as (4) and (5).

$$\begin{cases} q_x = q\dfrac{\mathrm{d}U}{\sqrt{\mathrm{d}U^2 + \mathrm{d}W^2}} \\ q_z = q\dfrac{\mathrm{d}W}{\sqrt{\mathrm{d}U^2 + \mathrm{d}W^2}} \end{cases} \tag{4}$$

$$q = \begin{cases} \mu p & (\mu p \le \sigma_F) \\ \sigma_F & (\mu p > \sigma_F) \end{cases} \tag{5}$$

where q is the friction stress, and q_x and q_z are the friction stress component at x-axis and z-axis respectively; μ is the coefficient of friction; p is the rolling pressure; σ_F is the strip yield stress.

The contact surface between roll and strip consisted of two parts in [7], the sliding friction area and stagnant area (adhering area). Equation (5) was still applied in the former area, but in the latter area, the friction stress was obtained by (6), introducing the pre-displacement limit M. A smoother distribution of friction stress near the neutral point could be obtained by this method, which was closer to the experimental results. However, the coefficients of friction at x-axis and z-axis were not treated separately, and the accuracy can be improved.

$$q = \mu p \frac{\sqrt{\mathrm{d}U^2 + \mathrm{d}W^2}}{M} \tag{6}$$

3. Rapid Strip Plastic Deformation Model

In this section, improvements were made in the shear strain calculation, the relationship between stress and strain of plastic process, the calculation of friction between strip and roll, and the assumptions of rolling condition; an online three-dimensional plastic deformation model was established to get the transverse distribution of rolling force, which is the key to shape calculation.

3.1. Assumptions

Based on the plastic deformation theory and the characteristics of strip hot rolling process, certain assumptions of the deformation model were made as follows:

1) The rolling process is stable.

2) Neglecting the elastic deformation, only consider the plastic deformation of the hot strip.

3) Stress, strain and deformation rate have symmetrical distribution about the geometric center of strip.

3.2. Equilibrium Equations

Shear stresses in all directions were taken into consideration for the unit cell in the deformation section. Fig. 1 shows the stress state of the strip internal cells and external cells at the contact surface between roll and strip.

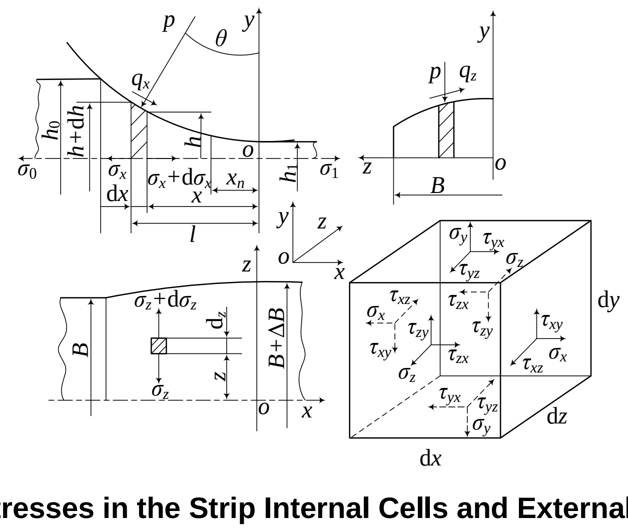

Figure 1. Stresses in the Strip Internal Cells and External Cells at the Contact Surface

For the strip internal cells, since there is no external force, equilibrium equations are as follows:

$$\begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} + \dfrac{\partial \tau_{xz}}{\partial z} = 0 \\ \dfrac{\partial \tau_{xy}}{\partial x} + \dfrac{\partial \sigma_y}{\partial y} + \dfrac{\partial \tau_{yz}}{\partial z} = 0 \\ \dfrac{\partial \tau_{xz}}{\partial x} + \dfrac{\partial \tau_{yz}}{\partial y} + \dfrac{\partial \sigma_z}{\partial z} = 0 \end{cases} \tag{7}$$

where σ_i is the normal stresses and τ_{ij} is the shear stresses; i, j=x, y, z.

For the external cells, the upper contact surface equation is expressed as:

$$y = \frac{1}{2} h(x, z) \tag{8}$$

$$\begin{cases} \vec{\mathbf{n}} = \left[-\dfrac{1}{2}\dfrac{\partial h}{\partial x},\ 1,\ -\dfrac{1}{2}\dfrac{\partial h}{\partial z} \right]^{\mathrm{T}} \\ \vec{\mathbf{I}} = \left[1,\ \dfrac{1}{2}\dfrac{\partial h}{\partial x},\ 0 \right]^{\mathrm{T}} \\ \vec{\mathbf{m}} = \left[0,\ \dfrac{1}{2}\dfrac{\partial h}{\partial x},\ 1 \right]^{\mathrm{T}} \end{cases} \tag{9}$$

where $h(x, z)$ is the strip thickness at the point (x, z); $\vec{\mathbf{n}}$ is the outward normal vector of the surface; $\vec{\mathbf{I}}$ and $\vec{\mathbf{m}}$ are the direction vector of friction stress components q_x and q_z respectively. And the equilibrium equations are as follows:

$$\begin{cases} \dfrac{\sigma_x}{2}\dfrac{\partial h}{\partial x} - \tau_{xy} + \dfrac{\tau_{xz}}{2}\dfrac{\partial h}{\partial z} + \dfrac{p}{2}\dfrac{\partial h}{\partial x} + q_x \dfrac{|\vec{\mathbf{n}}|}{|\vec{\mathbf{I}}|} = 0 \\ \sigma_y - \dfrac{\tau_{xy}}{2}\dfrac{\partial h}{\partial x} - \dfrac{\tau_{yz}}{2}\dfrac{\partial h}{\partial z} + p - \dfrac{q_x}{2}\dfrac{\partial h}{\partial x}\dfrac{|\vec{\mathbf{n}}|}{|\vec{\mathbf{I}}|} - \dfrac{q_z}{2}\dfrac{\partial h}{\partial z}\dfrac{|\vec{\mathbf{n}}|}{|\vec{\mathbf{m}}|} = 0 \\ \dfrac{\sigma_z}{2}\dfrac{\partial h}{\partial z} - \tau_{yz} + \dfrac{\tau_{xz}}{2}\dfrac{\partial h}{\partial x} + \dfrac{p}{2}\dfrac{\partial h}{\partial z} + q_z \dfrac{|\vec{\mathbf{n}}|}{|\vec{\mathbf{I}}|} = 0 \end{cases} \tag{10}$$

3.3. Constitutive Equations

The relationship between stress and strain of plastic process, and the relationship between their increments were expressed by the constitutive equations based on incremental theory (also known as flow theory). Incremental theory assumes that plastic strain increment in any small time increment is proportional to the instantaneous stress deviator tensor. Neglecting the elastic deformation, the constitutive equations are expressed as:

$$\frac{\mathrm{d}\varepsilon_x}{s_{xx}} = \frac{\mathrm{d}\varepsilon_y}{s_{yy}} = \frac{\mathrm{d}\varepsilon_z}{s_{zz}} = \frac{\mathrm{d}\gamma_{xy}}{s_{xy}} = \frac{\mathrm{d}\gamma_{xz}}{s_{xz}} = \frac{\mathrm{d}\gamma_{yz}}{s_{yz}} = \lambda \tag{11}$$

where $d\varepsilon_i$ is the normal strain increment, and $d\gamma_{ij}$ is the shear strain increment, $i, j= x, y, z$; λ is proportional coefficient, varying with the loading history of the deformation process; s_{ij} is the stress deviator tensor which can be obtained by

$$\begin{cases} s_{xx} = \sigma_x - \sigma_m & s_{xy} = \tau_{xy} \\ s_{yy} = \sigma_y - \sigma_m & s_{xz} = \tau_{xz} \\ s_{zz} = \sigma_z - \sigma_m & s_{yz} = \tau_{yz} \end{cases} \tag{12}$$

$$\sigma_m = (\sigma_x + \sigma_y + \sigma_z)/3 \tag{13}$$

where σ_m is the mean stress.

Replace strain increment with deformation rate, the constitutive equations can also be expressed as:

$$\begin{cases} \lambda s_{xx} = \dfrac{\partial u_x}{\partial x} & \lambda s_{xy} = \dfrac{1}{2}(\dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x}) \\ \lambda s_{yy} = \dfrac{\partial u_y}{\partial y} & \lambda s_{yz} = \dfrac{1}{2}(\dfrac{\partial u_y}{\partial z} + \dfrac{\partial u_z}{\partial y}) \\ \lambda s_{zz} = \dfrac{\partial u_z}{\partial z} & \lambda s_{xz} = \dfrac{1}{2}(\dfrac{\partial u_z}{\partial x} + \dfrac{\partial u_x}{\partial z}) \end{cases} \tag{14}$$

where u_i is the deformation rate, $i= x, y, z$.

3.4. Constitutive Equations

Von Mises yield criterion was applied in the plastic deformation process:

$$s_{xx}^2 + s_{yy}^2 + s_{zz}^2 + 2s_{xy}^2 + 2s_{xy}^2 + 2s_{yz}^2 = 2k_s^2 = \frac{2}{3}\sigma_F^2 \tag{15}$$

where k_s is the shear deformation resistance obtained by tensile test.

3.5. Constant-volume Constraint

For the internal cells, the constant-volume constraint is expressed as:

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0 \tag{16}$$

while for the external cells, based on the surface equation (8), the constraint is expressed as:

$$u_y = \frac{u_x}{2}\frac{\partial h}{\partial x} + \frac{u_z}{2}\frac{\partial h}{\partial z} \tag{17}$$

3.6. Friction

The friction between strip and roll was obtained based on Newton's law of viscosity. In hot rolling with lubrication, an oil film forms between strip and roll, and lubricated friction occurs on the contact surface. There is frictional viscous resistance to longitudinal and transverse extension of the strip. And Newton's law of viscosity applies in the situation: when layers within a fluid are moving relative to each other, the internal resistance is proportional to deformation rate, and dependent on the viscosity of the fluid. With coefficient of friction which represents viscosity, the friction can be expressed as:

$$\begin{cases} q_x = f_x u_x(x,z) \\ q_z = f_z u_z(x,z) \end{cases} \tag{18}$$

where f_x and f_z are the longitudinal and transverse coefficients of friction respectively. f_x is set as a constant 0.2; while f_z decreases from the strip center to the edge [15], which can be expressed as:

$$f_z(z) = 10^{0.6175z-0.4325} \tag{19}$$

3.7. Boundary Conditions

On the surface of strip edge within the contact arc l_c, the calculus of the transverse stress is zero:

$$\int_0^{l_c} h\sigma_z \mathrm{d}x = 0 \tag{20}$$

On the cross section of strip at entry and exit side, the calculus of the longitudinal stress equals to the backward tension T_0 and forward tension T_1 respectively:

$$\begin{cases} 2\int_0^{B/2} \sigma_0(z)h_0(z)\mathrm{d}z = T_0 \\ 2\int_0^{B/2} \sigma_1(z)h_1(z)\mathrm{d}z = T_1 \end{cases} \tag{21}$$

where h_0 is the strip entry thickness and h_1 is the exit thickness; σ_0 and σ_1 are the tensile stress at the entry and exit side respectively; B is the strip width.

3.8. Asymptotic Analysis

Normalized the strip thickness to h_r (thickness at the neutral point), length and width to l_c, stresses to k_s, and rate parameters to v_0 (strip entry velocity); and remained the variable names after normalization (see Table 1).

Table 1. Normalization of Parameters

Dimensions	Stresses	Rates	Others
$\begin{cases} x = xl_c \\ y = yh_r \\ z = zl_c \end{cases}$	$\begin{cases} p = pk_s \\ s_{ij} = s_{ij}\delta k_s \\ \sigma_i = (s_{ii}\delta - p)k_s \\ \tau_{ij} = \tau_{ij}\delta k_s \\ q_i = q_i\delta k_s \end{cases}$	$\begin{cases} u_x = u_x v_0 \\ u_y = u_y v_0 \\ u_z = u_z v_0 \end{cases}$	$\lambda = \lambda \dfrac{v_0}{\delta k_s l_c}$

where Ratio δ is defined as:

$$\delta = \frac{h_r}{l_c} \cong \frac{h_r}{\sqrt{R(h_0 - h_1)}} \tag{22}$$

where R is the roll radius. And the constitutive equations can be expressed as:

$$\begin{cases} -\frac{\partial p}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\delta\frac{\partial s_{xx}}{\partial x}+\delta\frac{\partial \tau_{xz}}{\partial z}=0 \\ -\frac{\partial p}{\partial y}+\delta\frac{\partial s_{yy}}{\partial y}+\delta^2\frac{\partial \tau_{xy}}{\partial x}+\delta^2\frac{\partial \tau_{yz}}{\partial z}=0 \\ -\frac{\partial p}{\partial z}+\frac{\partial \tau_{yz}}{\partial y}+\delta\frac{\partial s_{zz}}{\partial z}+\delta\frac{\partial \tau_{xz}}{\partial x}=0 \end{cases} \tag{23}$$

Since the strip thickness is much smaller than roll radius, δ should be far less than 1. Replaced the items in the constitutive equations with asymptotic expansion:

$$f = f^{(0)} + \delta f^{(1)} + \delta^2 f^{(2)} + \cdots \tag{24}$$

Neglecting the small items with δ, simplified the equations as:

$$\begin{cases} \frac{\partial \tau_{xy}^{(0)}}{\partial y}=\frac{\partial p^{(0)}}{\partial x} \\ \frac{\partial p^{(0)}}{\partial y}=0 \\ \frac{\partial \tau_{yz}^{(0)}}{\partial y}=\frac{\partial p^{(0)}}{\partial z} \end{cases} \tag{25}$$

Which shows that rolling pressure generally varies with the shear stress on the *xoz* plane and keeps constant in the thickness direction. Thus the rolling pressure can be expressed as:

$$p^{(0)} = p^{(0)}\left(x, z\right) \tag{26}$$

3.9. Governing Equation

After the asymptotic analysis, combined all the basic equations and constraints, and the governing equation was expressed as:

$$f_x h^2 (\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial z^2}) + 2hf_x(\frac{\partial ph}{\partial x^2}+\frac{\partial ph}{\partial z^2}) = \frac{(f_x+f_z)v_0}{h^2}(\frac{\partial h}{\partial x}+f_z\frac{\partial h}{\partial z}) \tag{27}$$

3.10. Discretization and Solving

In order to solve the partial differential equations of rolling pressure $p(x, z)$, discretization was performed based on FVM (finite volume method) [15], with a constant pitch at x-axis (length direction) and variable pitches at z-axis (width direction), as shown in Fig. 2, and large-scale sparse linear equations were built. Then used BICGSTAB (biconjugate gradient stabilized method) preconditioned by incomplete LU decomposition to achieve a fast calculation of the equations, and the transverse distribution of rolling force was obtained.

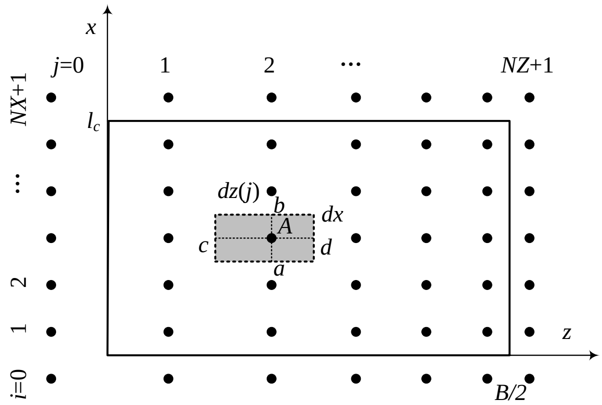

Figure 2. Discretization of the Deformation Zone

According to the inputs, outputs and structure of the model, the calculation flow is shown in Fig. 3.

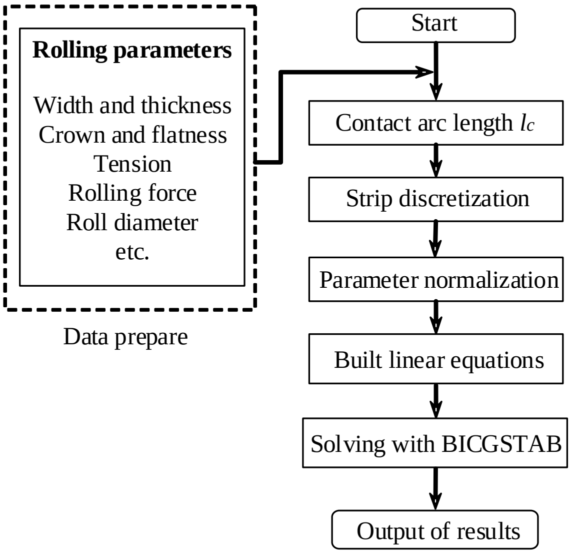

Figure 3. Calculation Flow of the Plastic Deformation Model

4. Results and Verification

4.1. Distribution of Rolling Force

Based on the theoretical model and calculation flow above, the rapid three-dimensional computational model of plastic deformation (RM) was built using the C ++ programming language. And the transverse distributions of rolling force at the exit of each stand were calculated with the rolling parameters in Table 2, assuming that the strip entry crown of first stand is 1% of the slab thickness and the entry flatness of each stand is 0.

Table 2. Parameters of Calculation Model of Three-dimensional Plastic Deformation

Stand	F1	F2	F3	F4	F5	F6	F7
Entry thickness (mm)	72.06	35.50	14.53	5.67	3.99	2.90	2.28
Exit thickness (mm)	35.50	14.53	5.67	3.99	2.90	2.28	2.02
Exit crown (mm)	0.371	0.299	0.135	0.103	0.075	0.060	0.034

Rolling force (ton)	2927	2911	2013	1767	1543	1276	878
Strip exit velocity (m/s)	0.629	1.536	3.938	5.590	7.690	9.796	11.001
Forward tension (MPa)	4	5	8.5	15	16	16.5	17.3
Work roll diameter (mm)	944	862	678	716	574	560	603
Strip width (mm)				1270			
Roll Young's modulus (MPa)				2.1e5			
Steel grade				Q235B			

As shown in Fig. 4, the rolling force is relatively steady at the central part of the strip, and has a sharp decline at the strip edge. And with the increase of width-thickness ratio, there are distribution patterns such as parabola, "cat ear" and "pot bottom", which are consistent with the classical results [6].

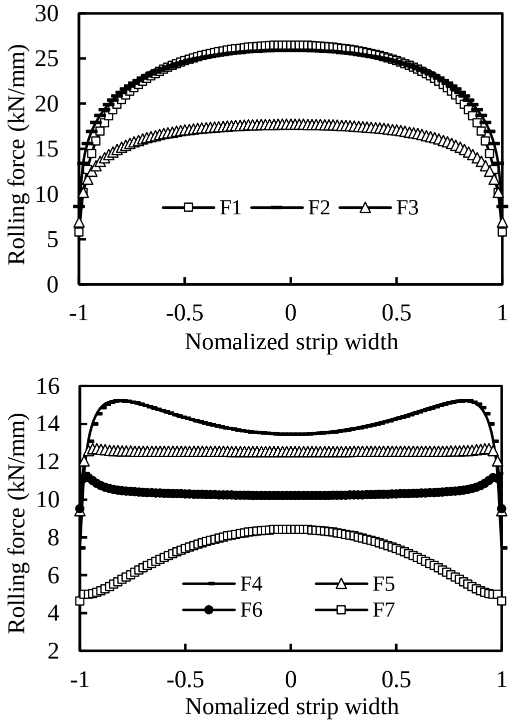

Figure 4. Transverse Distribution of Rolling Force at the Exit of Each Stand

4.2. Comparison to FEM Model

It is difficult to measure the rolling force distribution in the rolling process, and verify the results directly. As general simulation tools, the accuracy of commercial finite element analysis software has been verified by many practical production processes. Based on the implicit statics method, a roll-strip integrated model was established in the finite element software ANSYS (details of the simulation conditions can be found in [17]) with the rolling parameters of the fourth stand (F4) in Table 2. Fig. 5 compares the results of RM model and FEM model. The distribution and magnitude of rolling force obtained by both models matches well generally, indicating a satisfactory accuracy of RM model.

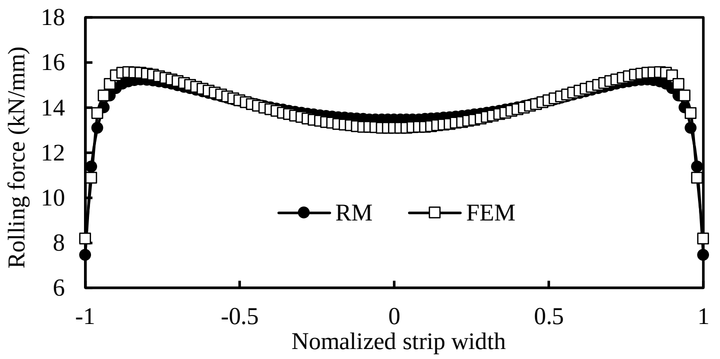

Figure 5. Comparison between the Results of RM Model and FEM Model

Though the calculation results of the two models were very close to each other, there was a big difference in the calculation time. As shown in Table 3, it took 3.5 hours for FEM to perform one case, while 31 milliseconds for RM. Thus, RM model is able to meet the requirement of online application, which is completing numerous calculation cases of various strip sizes, steel grades and mill set points in a short time.

Table 3. Comparison of Time Consumption

Method	RM	FEM
CPU	Intel Duo @ 1.80Hz	Intel Quad @2.4GHz
RAM	1.0 GB	4.0 GB
Time consumption	31 ms	3.5 h

5. Conclusions

Boundary conditions was properly set according to the characteristics of strip hot rolling process; lubricated friction theory and asymptotic analysis theory were applied to the three-dimensional plastic deformation model; the new model had a complete theoretical foundation and less sensitivity to the setting of strip edge shape, and was adaptable to any strip width. Stable, accurate and fast calculation was achieved using finite volume method and advanced algorithm.

The distribution and magnitude of rolling force obtained by the RM model were consistent with the classical results, and verified by the results of FEM model. And for RM model, the calculation time of one case is 31 milliseconds, which is very important to apply in the online shape model.

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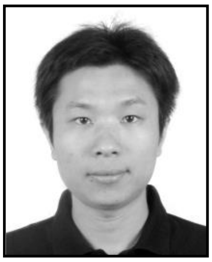

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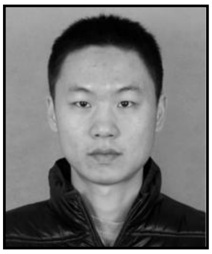

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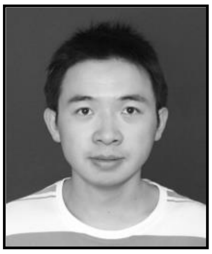

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