

## Leader following Speed Consensus in Induction Motors using Multi-Agent and Hybrid Controller Scheme

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### Abstract

*This paper explores a novel design of utilizing Leader following multi-agent system (MAS) consensus algorithm with Field Oriented Control (FOC), induction motors using a Hybrid control scheme containing regulation, pole placement and tracking (here after RST), controller along with model reference adaptive control (MRAC). To ensure system stability, Massachusetts institute of technology (MIT), rule is incorporated to validate that the cost function gets diminished over time. To follow the speed of leader, a leader following consensus protocol of multi-agent system is merged so that we can control the speed of n induction motors. In this new approach, each motor along with its complete unit (Voltage Source Converter (VSC) and local controller) is considered as a single agent. It is proved that the consensus is reached among leader and its followers not only without delay but also with delay. The communication topology is supposed to be fixed and it is also assumed that every agent can share its information (speed data), with neighbors. The model is simulated using MATLAB and obtained results ensure the effectiveness of the design approach.*

**Keywords:** *Induction motor, Leader following multi-agent system, Consensus control, Voltage Source Converter*

### 1. Introduction

Presently, Cyber-Physical Energy Systems (CPES), attracts a lot of attention in the engineering community. Generally, CPES can be viewed as any physical system consist of computing devices, communication, and control theory to obtain better performance and efficiency, greater stability and robustness. The concept of CPES has coined about a decade ago when various engineering domain requiring integration of physical systems with the cyber world. The core of any CPES is the communication network connecting physical system with computing device. In Network Control System (NCS), each system is considered as a node connected to the controller via a communication network.

In an electrical power system, CPES is widely studied in the context of Smart grid. As advance power systems, smart grids compose of a computer system, communication network, and control methodologies. The basic purpose of the smart grid is an optimal distribution of electrical energy. When dealing with multiple numbers of systems, distributed in a network having similar dynamics designed to accomplish the desired task

than it is called MAS. In [1], for a smart grid, a distributed MAS is proposed for managing power distribution among Plug-in Hybrid Electrical Vehicles. Similarly, the application of MAS is studied with power system in [2], where it is utilized to manage distributed smart grid in the presence of a fault and to isolate that fault and keep delivering power to the de-energized locations. In [3] a decentralized autonomous agent based algorithm is proposed with energy storage to reestablish the load by determining the fault, its location and subsequently isolating it from the rest of the system. Furthermore, in [4], a virtual power plant is proposed and modeled as MAS to manage different sections of the plant. They also showed estimated demand of energy by the domestic user by integrating agents with artificial neural networks (ANN).

In the industrial sector, NCS is now getting more attention due to the flexible distributed operation i-e sensor, actuator and controller use the network to share data, and ease of installation. When dealing with motors in a network, one common application includes Networked Motion Control System (NMCS). NMCS consist of local motor driver and remote motion controller connected through a network to coordinate motion. Here the point is that mostly DC motors are used for NMCS application. Now, as far as induction motors (IM) are concern, they are frequently available in every industrial setting. In [5], driving system of motor uses NCS to control the speed of IM by tracking the rotor under network induced delay. Furthermore, in [6] NCS based speed control of multiple IM's is studied. They use Lyapunov theorem to determine the network speed controller gain and tuning is performed by fuzzy logic. In [7], speed control of network connected IM's with a delay in the network and data drop out is presented using fuzzy logic. The maximum allowable delay and data dropout are designed using Lyapunov-Krasovskii theorem. Moreover, in [8] motion control with IM is proposed. They use sliding mode control with adjusting cross coupling scheme and proved that multiple IM's (they used four IM's), connected through CAN bus can achieve synchronized motion control.

These result of IM's integrated with NCS encourage us to study the behavior of IM's connected as a MAS particularly Leader following MAS. One of the main issue related to MAS coordination is the consensus. A distinguishing feature of MAS with leader following approach is that the system is designed in such a way that the following agents are unable to communicate with one another instead they communicate with a leader so that they can reach the desired trajectory. This shows that, maneuvering the leader leads to achieve consensus. In [9], it is proved that consensus in leader following scenario having directed communication is achieved by designing a nonlinear adaptive algorithm using state information. Moreover, in [10], a problem of adaptive consensus in leader following MAS having reference input, with unknown external disturbance is addressed. A distributed disturbance free protocol is designed using MRAC methodology and subsequently upper and lower disturbance bounds are estimated by accumulating disturbance compensator.

Therefore, in this paper, we address the consensus problem with multiple leader following IM's under fixed communication topology with and without delay. The objective of this study is to design such a system in which following agents converge to the speed of leader which contain the reference speed data. The main contribution of this work is to use the dynamic model of induction motor obtained via  $d - q$  frame to extract out the parameters for RST controller which are tuned by the MRAC after that MIT rule is used to drive the error to zero.

The paper is structured as follows. The graph theory and concept of leader following MAS consensus protocol are given in Section II. Section III presents the applicability of consensus protocol with IM. The complete steady state and dynamic model of IM are presented in section IV. Controller design with IM model obtained in section IV is performed in Section V. After that, results and simulation are given in section VI and lastly, conclusion is given.

## 2. Preliminaries and Graph Theory

To achieve consensus, coordination among agents and leader is necessary and to do this, they require a communication network. Graph theory plays a key role to demonstrate the communication network and interconnection among leader and agents to illustrate information flow. A graph  $\mathcal{G}(\Lambda, \Gamma)$ , corresponds to a set of nodes  $\Lambda = \{v_1, \dots, v_N\}$  and edges  $\Gamma \subseteq \Lambda \times \Lambda$ . Edges denote information interchange among nodes while every node represents single agent. If an edge exists between nodes  $(v_i, v_j)$ , then it is said that  $v_i$  is a parent node and  $v_j$  is referred as a child. It is obvious that information flow occurs from the parent node. There also exists a set of neighbor for every node  $v_i$ , indicated by  $\mathcal{N}_i$  i-e  $\mathcal{N}_i = \{j: ij \in \Gamma\}$ . For each node, the total amount of its neighbors are described by a degree matrix  $\mathcal{D} = \text{diag}\{d_1, \dots, d_N\}$ , i-e if  $i = j$  than  $\text{deg}(v_i)$ , otherwise 0. Graph adjacency is described by another matrix called Adjacency matrix  $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$  i-e if  $(v_j, v_i) \in \Gamma$  then  $a_{ij}$  is positive otherwise  $a_{ij} = 0$ . Furthermore, if the graph doesn't have any loops then  $a_{ii} = 0$ . Moreover, when dealing with undirected graphs,  $a_{ij} = a_{ji}$  for all  $i \neq j$ . Additionally, Laplacian matrix of a graph is described as  $\Xi = [\Xi_{ij}] \in \mathbb{R}^{N \times N}$  such that  $\Xi = \mathcal{D} - \mathcal{A}$  i-e  $\Xi_{ii} = \sum_{j \neq i} a_{ij}$  and  $\Xi_{ij} = -a_{ij}$ ,  $i \neq j$ . An interesting thing about laplacian matrix is its row sum which is zero all the time and eigenvector of  $\mathbf{1}$  corresponds to simplest 0 eigenvalue. For undirected graphs, the laplacian is always positive definite as illustrated by the laplacian potential of undirected graph in (1).

$$\xi(x) = x^T \Xi x = \frac{1}{2} \sum_{i,j} a_{ij} (x_i - x_j)^2 \quad (1)$$

**Lemma 1.** In [11], the  $\Xi$  has zero as eigenvalue for connected undirected graph while the tiniest non-zero eigenvalue i-e  $\lambda_2$  holds  $\min_{x \neq 0, 1^T x = 0} \frac{x^T \Xi x}{x^T x}$ , called algebraic connectivity.

**Lemma 2.** In [12], if  $\lambda_2(\mathcal{G}) > 0$ , than  $\mathcal{G}$  is said to be connected.

In Leader following MAS, the communication between leader and followers is initiated whenever the following agent comes in its neighborhood. Let's fix node 0 for the leader, this yields another graph  $\bar{\mathcal{G}}$  describing connectivity between leader and those followers which are its neighbor. This gives rise to another diagonal matrix describing aforesaid connectivity as  $\Pi = \text{diag}\{\beta_1, \beta_2, \dots, \beta_N\}$  where  $\beta_i > 0$  if agent  $i$  is neighbor else  $\beta_i = 0$ . Our aim is to design a controller  $u_i, i = 1, 2, \dots, N$ , in such a way that the followers must converge to the trajectory of leader. We also use the graph theory concepts to develop the corresponding matrices accordingly to prove the effectiveness of our proposed system.

**Definition:** For any initial speed i-e  $\omega_i(0)$ ,  $i = 1, \dots, N$ , a leader following MAS consensus is reached for agent  $i \in \{1, \dots, N\}$ , such that the feedback  $u_i$  exists of  $\{\omega_j : j \in \mathcal{N}_i\}$  holding  $\lim_{t \rightarrow \infty} \|\omega_i(t) - \omega_0(t)\| = 0$ ,  $i = 1, \dots, N$ .

**Lemma 3.** In [13], for undirected connected  $\mathcal{G}$  and  $\bar{\mathcal{G}}$ ,  $H = \Xi + \Pi$  is symmetric positive definite if  $\bar{\mathcal{G}}$  remains connected.

**Lemma 4.** In [14],  $H = \Xi + \Pi$  is stable if  $v_0$  is reachable in  $\bar{\mathcal{G}}$

## 3. Leader Following Consensus with IM

In this paper, two types of leader following MAS consensus protocols are considered. First, we study the consensus of IM's without delay and after that we explore the system by introducing a constant delay in the network ( $\tau$ ). It is shown that the consensus between  $n$  agents (IM's), in leader following setting is achieved in such a way that the following agents converge to the same speed as that of a leader ( $\omega_0$ ), in both situations. It is also

necessary that, for any initial speed, the error must hold  $\lim_{t \rightarrow \infty} e(t) = 0$ , where  $e = [e_1^T, \dots, e_N^T]^T$  and  $e_i = \omega_i - \omega_0$ . Furthermore, it is supposed that communication among leader and follower is organized and connected as given by figure 3 in the result section. So, the algorithm to solve consensus problem without delay in leader following MAS case for  $i_{th}$  agent is given as

$$u_i(t) = \sum_{j \in \mathcal{N}_i} (\omega_i - \omega_j) + f_i(\omega_0 - \omega_i) \quad (2)$$

It is obvious that agent  $i$  can't get information from the leader and its neighbors instantly, therefore, we also address the possibility of delay in the system.

$$u_i(t) = \sum_{j \in \mathcal{N}_i} (\omega_i(t - \tau) - \omega_j(t - \tau)) + f_i(\omega_0(t - \tau) - \omega_i(t - \tau)) \quad (3)$$

In [15], it is proved that consensus with delay is reached as long as delay is within  $\pi/2\lambda_{max}$  or system Nyquist plot has zero encirclements around  $-1/\lambda_k, \forall k > 1$ .

#### 4. Induction Motor Modeling

Induction motors are commonly used in Industry particularly squirrel cage motor due to its robust nature, good performance and most importantly it is cheaper. Traditionally, PI and PID controllers are mostly used for controlling the speed of induction motors. The fixed gains of these controllers perform outstandingly in various operating situations but their performance decline in the systems involving uncertainties, non-linearities and *etc.*, To overcome these issues, there are a variety of techniques available such as Self-tuning PID control, Sliding mode control, Fuzzy logic control and *etc.*, As IM model has a lot of uncertainties, adaptive controller suits better to achieve good performance, even when we have limited information about the plant [16]. In 1958, Whitaker [17], propose Model Reference Adaptive Control (MRAC) in which parameters of the controller are adjusted by using a mechanism of adjustment incorporated in the control protocol such that output of the plant becomes analogous to reference model.

**4.1.3.-Phase Dynamic Model of Induction Motor in  $d - q$  Frame:**The Vector control of induction motor paved a way to control induction motor such that one can attain the performance as that of DC motor. The advantage of vector control is that we can control the motor torque and flux individually. When dealing with 3-phase induction machine, the most common approach is using  $d - q$  frame of reference. In  $d - q$  frame,  $d$ -axis rotates with rotor flux since it is attached to it whereas  $q$ -axis is positioned perpendicular to it. Therefore, current in  $d$ -axis control flux while  $q$ -axis current controls torque. The complete model of 3-phase induction motor using  $d - q$  frame is given below and  $d - q$  equivalent circuit of 3-phase induction motor is given in Figure 1.

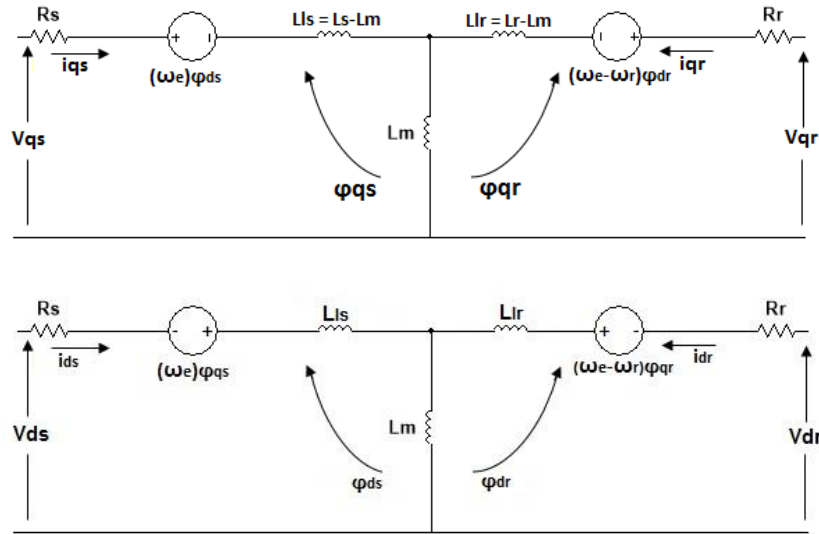
$$V_{qs} = R_s i_{qs} + \frac{d}{dt} \varphi_{qs} + \omega_e \varphi_{ds} \quad (4)$$

$$V_{ds} = R_s i_{ds} + \frac{d}{dt} \varphi_{ds} - \omega_e \varphi_{qs} \quad (5)$$

$$V_{qr} = R_r i_{qr} + \frac{d}{dt} \varphi_{qr} + \omega_e \varphi_{dr} \Rightarrow V_{qr} = R_r i_{qr} + \frac{d}{dt} \varphi_{qr} + (\omega_e - \omega_r) \varphi_{dr} \quad (6)$$

$$V_{dr} = R_r i_{dr} + \frac{d}{dt} \varphi_{dr} - \omega_e \varphi_{qr} \Rightarrow V_{dr} = R_r i_{dr} + \frac{d}{dt} \varphi_{dr} - (\omega_e - \omega_r) \varphi_{qr} \quad (7)$$

Where  $\mathcal{V}_{d_s}$ ,  $\mathcal{V}_{q_s}$ ,  $\mathcal{V}_{d_r}$ , and  $\mathcal{V}_{q_r}$  are  $d$  and  $q$  axis stator and rotor voltages,  $\varphi_{d_s}$ ,  $\varphi_{q_s}$ ,  $\varphi_{d_r}$ , and  $\varphi_{q_r}$ , are  $d$  and  $q$  axis stator and rotor fluxes,  $\mathcal{R}_s$  and  $\mathcal{R}_r$  are stator and rotor resistances,  $i_{d_s}$ ,  $i_{q_s}$ ,  $i_{d_r}$ , and  $i_{q_r}$  are  $d$  and  $q$  axis stator and rotor currents,  $\omega_r$  is actual speed of rotor and  $\omega_e$  is speed due to rotation of axis (synchronous speed).



**Figure 1. 3-Phase Model of Induction Motor. (a) q-Axis Model (b) d-Axis Model**

$$\varphi_{q_s} = \mathcal{L}_{\ell_s} i_{q_s} + \mathcal{L}_m (i_{q_s} + i_{q_r}) \quad (8)$$

$$\varphi_{q_r} = \mathcal{L}_{\ell_r} i_{q_r} + \mathcal{L}_m (i_{q_s} + i_{q_r}) \quad (9)$$

$$\varphi_{d_s} = \mathcal{L}_{\ell_s} i_{d_s} + \mathcal{L}_m (i_{d_s} + i_{d_r}) \quad (10)$$

$$\varphi_{d_r} = \mathcal{L}_{\ell_r} i_{d_r} + \mathcal{L}_m (i_{d_s} + i_{d_r}) \quad (11)$$

$$\begin{bmatrix} \mathcal{V}_{q_s} \\ \mathcal{V}_{d_s} \\ \mathcal{V}_{q_r} \\ \mathcal{V}_{d_r} \end{bmatrix} = \begin{bmatrix} \mathcal{R}_s + s\mathcal{L}_s & \omega_e \mathcal{L}_s & s\mathcal{L}_m & \omega_e \mathcal{L}_m \\ -\omega_e \mathcal{L}_s & \mathcal{R}_s + s\mathcal{L}_s & -\omega_e \mathcal{L}_m & s\mathcal{L}_m \\ s\mathcal{L}_m & (\omega_e - \omega_r)\mathcal{L}_m & \mathcal{R}_r + s\mathcal{L}_r & (\omega_e - \omega_r)\mathcal{L}_r \\ -(\omega_e - \omega_r)\mathcal{L}_m & s\mathcal{L}_m & -(\omega_e - \omega_r)\mathcal{L}_r & \mathcal{R}_r + s\mathcal{L}_r \end{bmatrix} \begin{bmatrix} i_{q_s} \\ i_{d_s} \\ i_{q_r} \\ i_{d_r} \end{bmatrix} \quad (12)$$

$$\mathcal{T}_e = \mathcal{T}_L + j \frac{d\omega_m}{dt} \Rightarrow \mathcal{T}_e = \mathcal{T}_L + \frac{2}{p} j \frac{d\omega_r}{dt} \quad (13)$$

$$\mathcal{T}_e = \frac{3}{2} \left( \frac{\mathcal{P}}{2} \right) \varphi_m i_r \quad (14)$$

$$\mathcal{T}_e = \frac{3}{2} \left( \frac{\mathcal{P}}{2} \right) (\varphi_{d_s} i_{q_s} - \varphi_{q_s} i_{d_s}) \quad (15)$$

$$\mathcal{T}_e = \frac{3}{2} \left( \frac{\mathcal{P}}{2} \right) (\varphi_{d_r} i_{q_r} - \varphi_{q_r} i_{d_r}) \quad (16)$$

The state space dynamic model of IM speed control for  $i_{th}$  agent is given as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \quad (17)$$

$$\dot{x}_i = \begin{bmatrix} \frac{-(\mathcal{R}_s + \mathcal{R}_r \left(\frac{\mathcal{L}_m}{\mathcal{L}_r}\right)^2)}{\delta \mathcal{L}_s} & 0 & \frac{\mathcal{L}_m}{\delta \mathcal{L}_s \mathcal{L}_r \xi_r} & \frac{\omega_r \mathcal{L}_m}{\delta \mathcal{L}_s \mathcal{L}_r} \\ 0 & \frac{-(\mathcal{R}_s + \mathcal{R}_r \left(\frac{\mathcal{L}_m}{\mathcal{L}_r}\right)^2)}{\delta \mathcal{L}_s} & -\frac{\omega_r \mathcal{L}_m}{\delta \mathcal{L}_s \mathcal{L}_r} & \frac{\mathcal{L}_m}{\delta \mathcal{L}_s \mathcal{L}_r \xi_r} \\ \frac{\mathcal{L}_m}{\xi_r} & 0 & -\frac{1}{\xi_r} & -\omega_r \\ 0 & \frac{\mathcal{L}_m}{\xi_r} & \omega_r & -\frac{1}{\xi_r} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ \varphi_{dr} \\ \varphi_{qr} \end{bmatrix} + \begin{bmatrix} \frac{1}{\delta \mathcal{L}_s} & 0 \\ 0 & \frac{1}{\delta \mathcal{L}_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathcal{V}_{ds} \\ \mathcal{V}_{qs} \end{bmatrix} \quad (18)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ \varphi_{dr} \\ \varphi_{qr} \end{bmatrix} \quad (19)$$

Here  $\delta = 1 - \frac{\mathcal{L}_m^2}{\mathcal{L}_s \mathcal{L}_r}$ , and  $\xi_r = \frac{\mathcal{L}_r}{\mathcal{R}_r}$

The motor specification includes 220V, 3 $\phi$ , 60Hz power supply having rated power of 2HP with  $\mathcal{P} = 4$ ,  $\mathcal{R}_s = 0.2\Omega$ ,  $\mathcal{R}_r = 0.5\Omega$ ,  $\mathcal{L}_s = 30mH$ ,  $\mathcal{L}_r = 32mH$ ,  $\mathcal{L}_m = 35mH$ ,  $\mathcal{T}_L = 50Nm$ ,  $j = 0.03Kg.m^2$ , and  $\omega_{ref} = 1600 rpm$

### 5. Controller Design

In MRAC centered systems, a reference model defines the desired output or performance of the system while control polynomials are tuned w.r.t error such that the difference between the outputs of the reference model and actual system keep minimum or ended, this approach of using MRAC control is also called the direct method. Figure 2 shows that proposed system model of IM control using MRAC.

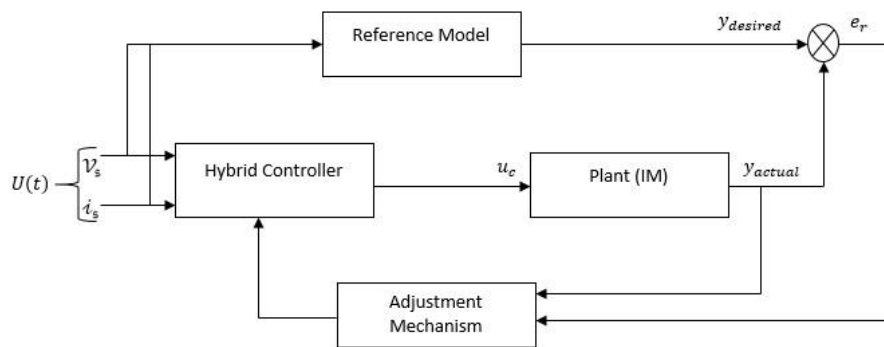


Figure 2. IM Control using MRAC

Now, by using regulation, pole placement and tracking control, the actual response of the system is defined as

$$G_M(p) = \frac{b(p)}{a(p)} = y_{actual} \quad (20)$$

The power of Numerator i-e  $b(p) = 1$  and the denominator is  $a(p) = 2$ .  $b(p)$  can be factorized into

$$b(p) = b^-(p) \times b^+(p) \quad (21)$$

Where  $b^-(p)$  is a constant and  $b^+(p)$  is variable

The desired response (output of reference model), of proposed system, is given as

$$G_M(p) = \frac{b_d(p)}{a_d(p)} = y_{desired} \quad (22)$$

$$\deg a = 2 \quad (23)$$

$$\deg b = 1 \quad (24)$$

Now, introducing MRAC gradient theory method to determine the order of RST controller.

$$\deg a_c = 2^* \deg a - 1 \Rightarrow \deg a_c = 3 \quad (25)$$

Therefore,

$$r = r_0^*p + r_1 \quad (26)$$

$$s = s_0^*p + s_1 \quad (27)$$

$$t = t_0^*p A_0 \quad (28)$$

So we obtained that degree of RST is of First Order. Now

$$\deg a_0 = \deg a - \deg b^+ - 1 \Rightarrow \deg a_0 = 0 \quad (29)$$

$$\text{Let, } a_0 = 1 \quad (30)$$

The Diophantine Equation is given as

$$a_c = ar + b^-s \Rightarrow a_0 a_m = ar + b^-s \quad (31)$$

Now, let the sensitivity error is defined as the difference between the close loop system output  $y_{actual}$  and the output of model  $y_{desired}$ . In order to eliminate the error, MIT rule is proposed in this work and is given as.

$$e_r = y_{actual} - y_{desired} \quad (32)$$

$$e_r = \left( \frac{bt}{ar+bs} \right) U_c - y_{desired} \quad (33)$$

$$\text{Where } y_{actual} = \left( \frac{bt}{ar+bs} \right) U_c \quad (34)$$

The control input is defined as

$$U_c = \left( \frac{ar+bs}{bt} \right) y_{actual} \quad (35)$$

Now, taking the partial derivative of error to observe the effect of adjustable system polynomials, called sensitivity derivative of our parameters i-e  $r_0, r_1, s_0, s_1$ , and  $t_0$ .

$$error_{(sent)} = \left( \frac{bt}{ar+bs} \right) U_c - y_{desired} \quad (36)$$

Now, begin with our first polynomial  $R$  and taking partial derivatives of parameter  $r_0$  and  $r_1$  yields

$$error_{(sent)} = \left( \frac{bt}{a(r_0 * p + r_1) + bs} \right) U_c - y_{desired} \quad (37)$$

$$\frac{\Delta error_{(sent)}}{\Delta r_0} = - \left( \frac{ap}{a_m} \right) y_{actual} \quad (38)$$

$$\frac{\Delta error_{(sent)}}{\Delta r_1} = - \left( \frac{a}{a_m} \right) y_{actual} \quad (39)$$

Let's, move on and taking our second polynomial  $S$  and taking partial derivatives of parameter  $s_0$  and  $s_1$ .

$$error_{(sent)} = \left( \frac{bt}{ar + b(s_0 * p + s_1)} \right) U_c - y_{desired} \quad (40)$$

$$\frac{\Delta error_{(sent)}}{\Delta s_0} = - \left( \frac{bp}{a_m} \right) y_{actual} \quad (41)$$

$$\frac{\Delta error_{(sent)}}{\Delta s_1} = - \left( \frac{b}{a_m} \right) y_{actual} \quad (42)$$

The sensitivity error of our last parameter i-e  $t_0$  is given as

$$error_{(sent)} = \left( \frac{b(t_0 * p) A_0}{ar + bs} \right) U_c - y_{desired} \quad (43)$$

$$\frac{\Delta error_{(sent)}}{\Delta t_0} = \left( \frac{bp}{a_m} \right) y_{actual} \quad (44)$$

For optimal control, cost function has to diminish w.r.t time. Therefore, the parameters are required to be in the negative direction of the gradient of  $\tilde{J}$ . The cost function is defined as.

$$\tilde{J}(\phi) = \frac{1}{2} error_{(sent)}^2(\phi)$$

Now, by integrating our

RST parameters in above equation yields

$$\frac{\Delta(r_0)}{\Delta t} = -\psi \frac{\Delta error_{(sent)}}{\Delta r_0} \Rightarrow \frac{\Delta(r_0)}{\Delta t} = \left( \frac{\psi a p}{a_m} \right) y_{actual} \quad (45)$$

$$\frac{\Delta(\phi)}{\Delta t} = -\psi \frac{\Delta error_{(sent)}}{\Delta t} \quad (46)$$

(47)

$$\frac{\Delta(r_1)}{\Delta t} = -\psi \frac{\Delta error_{(sent)}}{\Delta r_1} \Rightarrow \frac{\Delta(r_1)}{\Delta t} = \left( \frac{\psi a}{a_m} \right) y_{actual} \quad (48)$$

$$\frac{\Delta(s_0)}{\Delta t} = -\psi \frac{\Delta error_{(sent)}}{\Delta s_0} \Rightarrow \frac{\Delta(s_0)}{\Delta t} = \left( \frac{\psi b p}{a_m} \right) y_{actual} \quad (49)$$

$$\frac{\Delta(s_1)}{\Delta t} = -\psi \frac{\Delta error_{(sent)}}{\Delta s_1} \Rightarrow \frac{\Delta(s_1)}{\Delta t} = \left( \frac{\psi b}{a_m} \right) y_{actual} \quad (50)$$

$$\frac{\Delta(t_0)}{\Delta t} = -\psi \frac{\Delta error_{(sent)}}{\Delta t_0} \Rightarrow \frac{\Delta(t_0)}{\Delta t} = - \left( \frac{\psi b p}{a_m} \right) y_{actual} \quad (51)$$

Now, the complete controller is given as

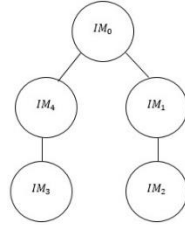
$$U(t) = \frac{t}{r} U_c - \frac{s}{r} (y_{actual} - y_{desired}) \quad (52)$$

## 5. Simulation and Results

In this portion, we give mathematical and simulation results for both the consensus conditions i-e with delay and without delay, on the basis of theory we establish in the earlier segments. For this, we consider the system of five agents having identical



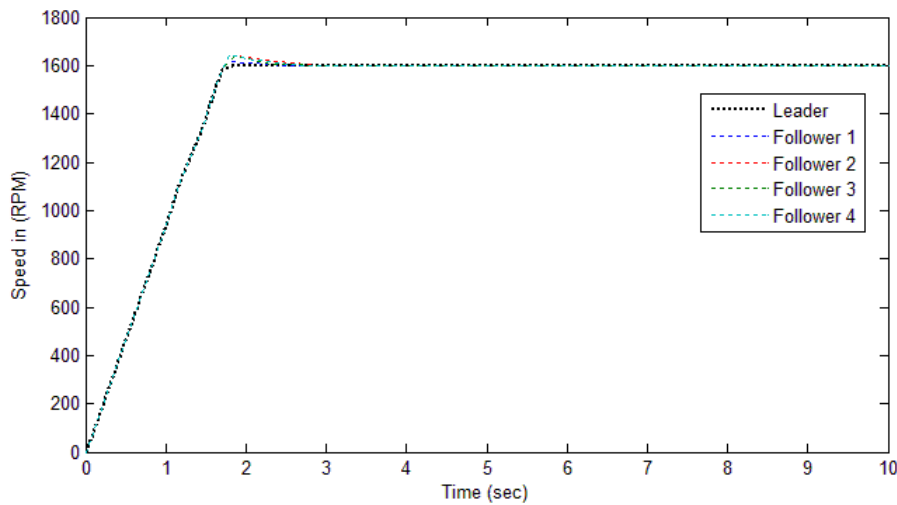
dynamics, subdivided into one leader and four followers connected in a manner as given in Figure 3. The corresponding laplacian matrix  $\Xi$  and adjacency matrix  $\Pi$  of the leader are given accordingly, thus validating Lemma 1 and 2, and laplacian matrix yield 0 and 2, the smallest and largest eigenvalues respectively. It is also shown that the matrix  $H$  is symmetric and positive definite with smallest eigenvalue of 0.382, validating lemma 3 and 4. The system polynomials are  $t = 0.467$ ,  $s = 0.4$  and  $r = 0.556$  respectively.



**Figure 3. Communication Topology between Leader and Follower**

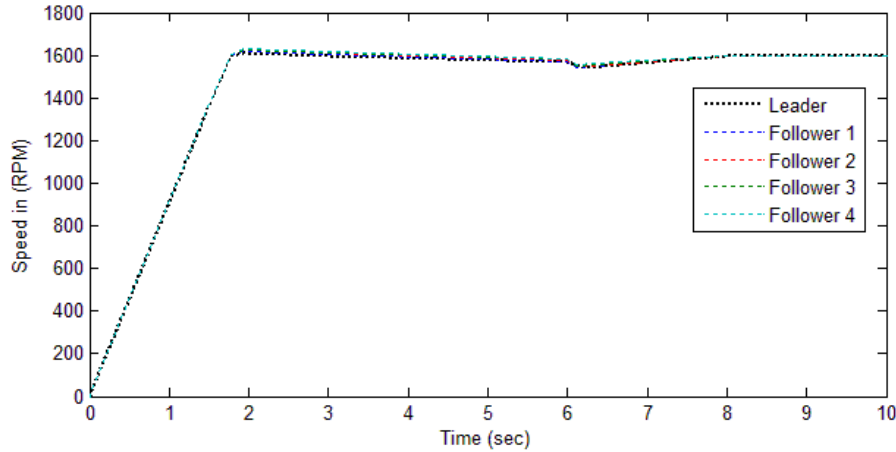
$$\Xi = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \Pi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

First, we observe the response of proposed system without considering the effect of delay. Figure 4, shows the speed trajectories for the 4 following agents under fixed topology without delay and also without any application of the load to the motors. It can be seen that consensus is reached with four followers, exhibiting robust leader following consensus performance.



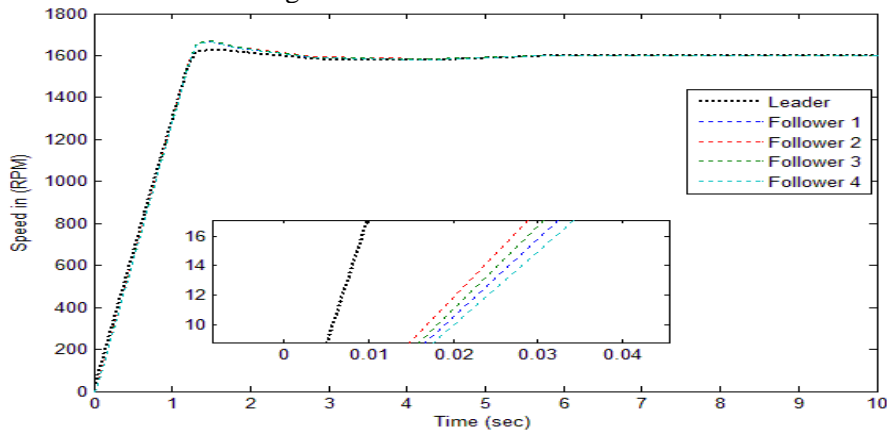
**Figure 4. Trajectory between Leader and Follower's without Load**

Furthermore, in Figure 5, we have shown the response of the same system but with the application of load ( $\mathcal{J}_L = 50Nm$ ), one can observe that each follower successfully track the leader before and after the application of load with initial overs and undershoots.



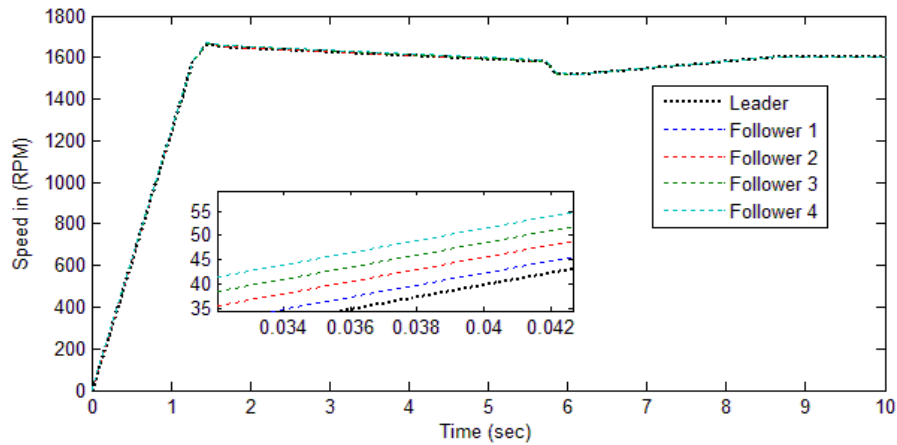
**Figure 5. Trajectory between Leader and Follower's with Load**

Now from here, we are going to observe the system response in the presence of delay. The delay is taken to be  $\tau = 0.45\text{sec}$  for all simulations. Figure 6, shows the speed trajectories of leader and all the four followers under fixed topology given in Figure 3, without applying a load to the motors. Once again, it can be observed that the proposed controller and system model performed exceptionally well under the influence of delay and consensus is reached among leader and followers.



**Figure 6. Trajectory between Leader and Followers without Load**

Moreover, Figure 7, shows the complete response of leader following agents which are influenced not only by the delay but also by the load i-e ( $\mathcal{T}_L = 50Nm$ ). It is shown that the system performed good with the application of load and can achieve consensus after momentary distorted from the leader's track.



**Figure 7. Trajectory between Leader and Followers with Load**

**Remark 3:** Figure 4-7, gives all the simulation results in different scenarios which can possibly occur in the system. One can determine that the leader following consensus can be achieved with IM's and the approach of MIT based MRAC control with regulation, pole placement and tracking provide robust performance.

## 7. Conclusion

In this paper, we address the consensus problem of multiple IM's connected as leader following MAS such that the following agents converge to the speed as that of a leader by using the leader following protocol of MAS and incorporated it into MRAC controlled IM's. The connectivity among leader and followers is modeled using graph theory. The stability of a system is proved by defining a cost function and thereafter using MIT rule to analyze its time derivative thereby endorsing system optimal performance. The paper encircles two scenarios of MAS i-e system with and without delay. It is assumed that communication topology for the system is fixed for both conditions. In both situations, it is shown via simulations that the system achieves consensus with the proposed methodology. The proposed control methodology is worthy enough for industrial application as well as for networked connected induction motors.

## Acknowledgement

This work is supported by National Natural Science Foundation of China under grant 61273114, the Innovation Program of Shanghai Municipal Education Commission under grant 14ZZ087, the Pujiang Talent Plan of Shanghai City China under grant 14PJ1403800, the International Corporation Project of Shanghai Science and Technology Commission under grants 14510722500, 15220710400.

## References

- [1] T. Logenthiran and D. Srinivasan, "Multi-agent system for managing a power distribution system with plug-in hybrid electrical vehicles in smart grid." In Innovative Smart Grid Technologies-India (ISGT India), 2011 IEEE PES, IEEE (2011).
- [2] W. Khamphanchai, S. Pisanupoj, W. Ongsakul and M. Pipattanasomporn. "A multi-agent based power system restoration approach in distributed smart grid." In Utility Exhibition on Power and Energy Systems: Issues & Prospects for Asia (ICUE), 2011 International Conference and, pp. 1-7. IEEE, (2011), pp. 346-351.
- [3] C. P. Nguyen, and A. J. Flueck. "Agent based restoration with distributed energy storage support in smart grids." IEEE Transactions on Smart Grid 3, no. 2, (2012), pp. 1029-1038.
- [4] L Hernandez, C. Baladron, J. M. Aguiar, B. Carro, A. S. Esguevillas, J. Lloret, D. Chinarro, J. J. Gomez-Sanz and D. Cook. "A multi-agent system architecture for smart grid management and forecasting of energy demand in virtual power plants." IEEE Communications Magazine 51, no. 1, (2013), pp. 106-113.

- [5] J. Ren, C. W. Li and D. Z. Zhao, "Linearizing control of induction motor based on networked control systems", *International Journal of Automation and Computing*, vol. 6, no. 2, (2009), pp. 192-197.
- [6] D. Zhao, S. Zhang, C. Li, and R. Stobart, "Scheduling and control co-design of networked induction motor control systems", In *Information and Automation (ICIA), 2013 IEEE International Conference on, IEEE*. (2013), pp. 880-885.
- [7] D. Zhao, C. Li and R. Stobart, "Hierarchical modeling and speed control of networked induction motor systems", In *2013 American Control Conference, IEEE*, (2013), pp. 2344-2349..
- [8] J. Ren, C. W. Li and D. Z. Zhao, "CAN-based synchronized motion control for induction motors", *International Journal of Automation and Computing*, vol. 6, no. 1, (2009), pp. 55-61.
- [9] C. Wang and H. Ji. "Leader-following consensus of multi-agent systems under directed communication topology via distributed adaptive nonlinear protocol", *Systems & Control Letters*, vol. 70, (2014), pp. 23-29.
- [10] Y. Liu and Y. Jia, "Adaptive leader-following consensus control of multi-agent systems using model reference adaptive control approach", *IET Control Theory & Applications*, vol. 6, no. 13, (2012), pp. 2002-2008.
- [11] Z. Li and Z. Duan, "Cooperative Control of multi-agent systems: a consensus region approach", *CRC Press*, (2014).
- [12] M. Mesbahi and M. Egerstedt, "Graph theoretic methods in multiagent networks." *Princeton University Press*, (2010).
- [13] Y. Hong, J. Hu and L. Gao. "Tracking control for multi-agent consensus with an active leader and variable topology." *Automatica*, vol. 42, no. 7, (2006), pp. 1177-1182.
- [14] K. Peng and Y. Yang, "Leader-following consensus problem with a varying-velocity leader and time-varying delays", *Physica A: Statistical Mechanics and its Applications*, vol. 388, no. 2, (2009), pp. 193-208.
- [15] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays", *IEEE Transactions on automatic control*, vol. 49, no. 9, (2004), pp. 1520-1533.
- [16] T. Yucelen, and A. J. Calise. "Derivative-free model reference adaptive control." *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 4, (2011), pp. 933-950.
- [17] H. P. Whitaker, J. Yamron and A. Kezer. "Design of model-reference adaptive control systems for aircraft." *Massachusetts Institute of Technology, Instrumentation Laboratory*, (1958).