

# An Archived Multi-Objective Simulated Annealing Algorithm for Vehicle Routing Problem with Time Windows

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## Abstract

*The vehicle routing problem with time window (VRPTW) is a well-known combinatorial optimization problem, which is to find the lowest-cost routes from a central depot to a set of geographically scattered points with various demands. This paper deals with a multi-objective variant of the VRPTW that simultaneously minimizes the number of the vehicles and the traveled distance. A metaheuristic based on simulated annealing was proposed, and the concept of archive was introduced, in order to provide a set of tradeoff solutions for the problem. The accuracy of solutions is defined as their proximity to the best known solution of Solomon's benchmarking tests. Computational results demonstrate that the proposed approach is quite effective, as it provides solutions competitive with the best known in the literature.*

**Keywords:** *Vehicle routing; Time windows; Multi-objective optimization; Simulated annealing, Archive*

## 1. Introduction

The vehicle routing problem (VRP) is a well-known combinatorial optimization problem arising in the transportation field that usually involves scheduling in the constrained environment, which was introduced by Dantzig and Ramser [1]. The problem is to find the lowest-cost routes from a central depot to a set of geographically scattered points with various demands. The VRP have received much attention in the recent years due to their wide applicability in transportation management. As a result, variants of VRP have been proposed and different formulations developed extensively in the literature. See [2] for detailed reviews.

Later, many researchers studied the vehicle routing problem with time windows (VRPTW), *i.e.*, considered the time windows constraints. Here, the time windows provide a time frame of customers and central depot. The time window is divided into two parts. The hard time window, which means a vehicle may arrive at such a destination prior to earliest specific time of window, but it cannot move until the time of the customer actually opens, and the vehicle could not arrive later than the end of service time. The soft time window allow for early or late of the specific time window service, however the extra punishment cost will be charged. Most researchers have focused on the hard time window models, so does this paper.

The objective function of the VRPTW problem seeks to minimize the sum of the total dispatching and travel costs, or the number of vehicle (NV), or traveled distance (TD), or travel time. However, minimizing one objective rarely minimizes all the other constraints, the optimization process needs to provide, not a single solution, but a range of solutions that represent trade-offs between the objectives. The VRPTW problem is a classic example of a NP-hard multi-objective optimization problem [3]. A typical VRPTW problem minimizes the number of vehicles required firstly, and then minimizes the total

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distance traveled to service the customers without violating the vehicles' capacity constraints and customers' time window requirements. Therefore, the idea of the traditional method to solve multi-objective programming problem is to transfer multi-objective problem to single-objective problem, such as weight method, constraint method, goal programming, min-max method. However, the traditional methods are very sensitive to Pareto optimal front shape and cannot handle the forefront of the recess. For the large-scale problems, it can be said the multi-objective planning techniques has never appeared in the traditional methods, and thus cannot meet the requirements of the further development of the theory and applications.

## 2. Literature Review

One of the most efficient techniques for the VRPTW problem has been the development of two-phased hybrid algorithms which is minimize the number of vehicles firstly, and then minimize the travel costs with a fixed route number in the second phase. A two-phased approach for the VRPTW is usually generated towards the design of algorithms tailored towards each sub-optimization. In the two-phased tabu search (TS), introduced by Potvin *et. al.*, [4], the first phase moved customers out of routes to reduce the total number of vehicles, and in the second phase inter- and intra-customer exchanges are done to reduce travel costs. Chiang and Russell [5] introduced a hybrid search based on simulated annealing (SA) and TS. A cluster-first, route-second method using genetic algorithms (GA) and local search optimization process was done by Thangiah [6]. Dorigo and Gambardella [7] studied a type of multi-objective implementation of the VRPTW by minimizing a hierarchical objective function, where the first objective minimized the number of vehicles and the second minimized the total travel time. This was achieved by adapting the ant colony system (ACS). Gehring and Homberger [8] introduced a two-stage hybrid search which first minimizes the number of vehicles using an evolution strategy and then the total distance is minimized using a TS algorithm. In essence, the multi-objective VRPTW problem was transformed into a single-objective optimization.

In the past decades, due to the development of the theories of the self-organizing, adaptive, self-learning and the "complexity of independence" feature, multi-objective evolutionary algorithm (MOEA) has been used to solve the multi-objective programming problem. Before 1999, Srinivas and Deb [9] developed the non-dominated sorting genetic algorithm (NSGA), and Horn and Nafpliotis [10] presented the niched pareto genetic algorithm (NPGA). These algorithms were customarily referred to as the first generation of evolutionary multi-objective optimization algorithms, which were characterized by the use of retention policies based on individual selection method Pareto rank and fitness sharing mechanism based multi-objective population. From 1999 to 2002, with elitist mechanism is characterized by the second generation of MOEA algorithms have been proposed. Zitzler *et. al.*, proposed the strong Pareto MOEA, that is SPEA-1 [11] and SPEA-2 [12]. In 2001, Erichson *et.al* developed the improved NPGA algorithm, the NPGAI [13]. Coello and Pulido developed the micro genetic algorithm (Micro-GA) [14]. After 2002, a new dominant mechanism has emerged. Laumanns *et. al.*, [16] presented  $\epsilon$  dominant concept, and Hernández-Díaz *et. al.*, [17] developed the Pareto adaptive  $\epsilon$  dominant. MOEA algorithm has the advantage of the population of the initial values and search representatives, so local search could be run at the same time and obtain the multiple optimal solutions.

The simulated annealing method is known to be a versatile and robust technique, providing excellent solutions to single objective optimization problems. Hence, this method has been adapted for the multi-objective framework by many scholars. Table 1, summarizes the main works related to the MOSA algorithm mentioned in this section.

**Table 1. The MOSA Related Work**

Author(year)	Ref.	Algorithms	Comments
Ulungu <i>et. al.</i> , (1998)	[19]	UMOSA	The criterion scalarizing fuction with the weighted sum is proposed, UMOSA builds a list PE of potentially efficient solutions containing all the generated solutions which are not dominated by any other generated solution.
Czyzak and Jaskiewicz (1998)	[20]	Pareto-SA	The inclination for dispersing the solutions from the generating sample over the whole set is obtained by controlling the weights of particular objectives.
Suman (2003)	[21]	WMOSA	The weight vector depends on the number of constraints to be satisfied by the solution vector and by the objective function vector, and the number of constraints of the problem. The weight vector is used in the acceptance criterion to handle constraints.
Suman (2004)	[22]	PDMOSA	A strategy of Pareto dominant based fitness in the acceptance criteria is used.
Tekinalp and Karsli (2007)	[23]	MC-MOSA	The algorithm has an adaptive cooling schedule and uses a population of fitness functions to accurately generate the Pareto front.
Bandyopathyay <i>et. al.</i> , (2008)	[24]	AMOSAs	The archive is incorporated into MOSA.
Sankararao and Yoo (2011)	[25]	rMOSA	Two new mechanisms are incorporated (1) to speed up the process of convergence to attain Pareto front (or a set of nondominating solutions) and (2) to get uniform nondominating solutions along the final Pareto front.
Banos <i>et. al.</i> , (2007)	[26]	MOSATS	Hybrid simulated annealing and tabu search
Maulik and Sarkar (2010)	[27]	PARAMOSA	The algorithm comprises a judicious integration of the principles of the rough sets theory and the scalable distributed paradigm with the archived multi-objective simulated annealing approach.
Ruiz <i>et. al.</i> , (2013)	[28]	MOSA-MOEA	Hybrid simulated annealing and evolutionary algorithm for multi-objective problem

Table 1, summarizes the main works related to the MOSA algorithm mentioned in this section. The main observations may be made for all solution approaches in Table 1 is that how to design acceptance rule and cooling schedule is still a hot problem in SA method.

The purpose of this paper is to present an archived-MOSA (AMOSAs) algorithm for the VRPTW problem. The proposed approach is tested on well-known Solomon's VRPTW benchmark data sets taken from the literature. The remainder of this paper is organized as follows. Implementation of the proposed AMOSAs algorithm for solving the VRPTW problem is presented in section 3. Application of the technique on Solomon's benchmark set of problems is considered in section 4. Conclusions based on the current study are finally drawn in section 5.

### 3. The AMOSAs Algorithm for VRPTW

The AMOSAs was based on simulated annealing, and the concept of archive was introduced, in order to provide a set of tradeoff solutions for the problem. The proposed AMOSAs algorithm employed here builds upon the idea of Bandyopathyay *et. al.*, [24].

#### 3.1. Parameter Setting

In this paper, at a given temperature  $T$ , a new state  $s$  is selected with a probability

$$P_{qs} = \frac{1}{1 + e^{\frac{-(E(q,T)-E(s,T))}{T}}} \quad (1)$$

Where  $q$  is the current state and  $E(s,T)$  and  $E(q,T)$  are the corresponding energy values of  $s$  and  $q$ , respectively. Note that the equation (1) automatically ensures that the probability value lies in between 0 and 1. The archive was introduced into MOSA. Two limits are kept on the size of the archive: a hard limit denoted by HL, and a soft limit denoted by SL. During the process, the non-dominated solutions are stored in the archive until the size of the archive increases to SL.

### 3.2. The Main AMOSA Process

In the initial temperature  $T_0$ , an initial solution is randomly selected the *current-pt*, and the local search is run in the neighborhood of *current-pt*. The *new-pt* is generated by perturbation. The domination status of *new-pt* is checked with respect to the *current-pt* and solutions in archive.

Based on the domination status between the *current-pt* and the *new-pt*, three different cases may arise. They are enumerated below.

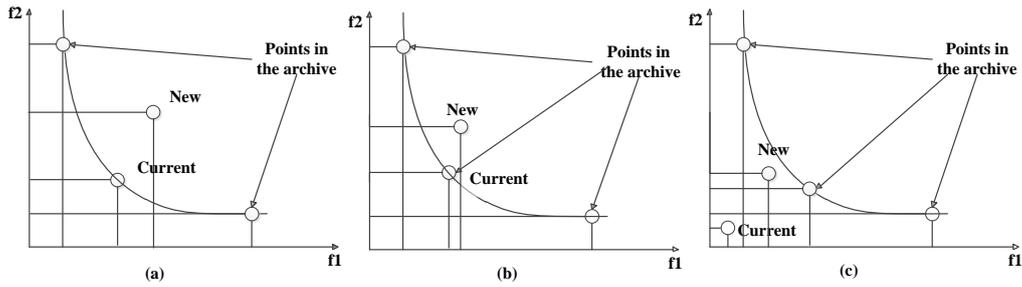
Case 1: The *current-pt* dominates the *new-pt*.

Based on the domination status of the *new-pt* and members of archive, the following three situations may rise, as shown in Figure 1.

(1) The *current-pt* dominates the *new-pt*, but  $k(k \geq 1)$  points in the archive dominate this *new-pt*. The minimum of the difference of domination amounts between the *new-pt* and  $k$  points, denoted by  $\Delta dom_{min}$ , of the archive is computed. The point from the archive which corresponds to the minimum difference is selected as the *current-pt* with probability  $= 1/(1 + \exp(-\Delta dom_{min}))$ . Otherwise, the *new-pt* is selected as the *current-pt*.

(2) The *current-pt* dominates the *new-pt*. The *current-pt* is in the archive, however the *new-pt* is nondominating with respect to the points in the archive. Therefore, the *new-pt* is accepted as the *current-pt*, and can be considered as a new nondominated solution that must be stored in archive. If the *current-pt* is in the archive, then it is removed. Otherwise, if the number of points in the archive becomes more than the SL, clustering is performed to reduce the number of points to HL. Note that the *current-pt* may or may not be on the archival front.

(3) The *current-pt* dominates the *new-pt*, and *new-pt* dominates  $k$  points ( $k \geq 1$ ) in the archive. Therefore, the *new-pt* replaces the *current-pt*, and add to archive, while all the dominated points of the archive are removed. Note that the *current-pt* may or may not be on the archival front.



**Figure 1. The Current-pt Dominates the New-pt (a) Some Solutions of Archive Dominate the New-pt, (b) the New-pt is nNondominating with Respect to the Points in the Archive, (c) the New-pt Dominates  $k$  ( $k \geq 1$ ) in the Archive**

Case 2: The *current-pt* and the *new-pt* are nondominating with respect to each other. Based on the domination status of *new-pt* and members of archive, the following three situations may rise, as shown in Figure 2.

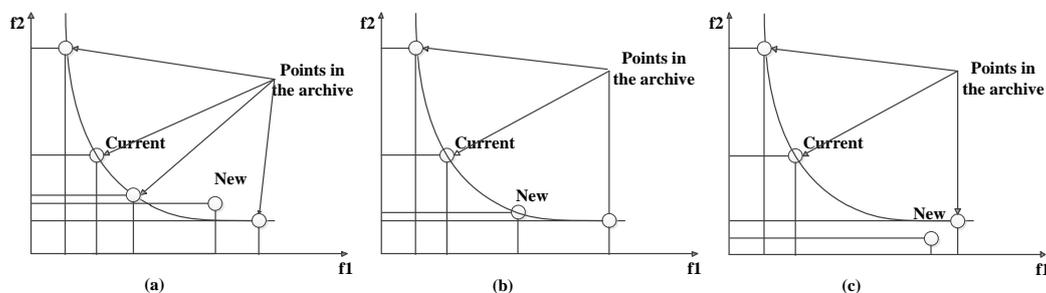
(1) The  $k$  points ( $k \geq 1$ ) in the archive dominates the *new-pt*, therefore the *new-pt* is selected as the *current-pt* with probability *new-pt*

$$P = \frac{1}{1 + \exp(\Delta dom_{avg} \times temp)} \quad (2)$$

Where,  $\Delta dom_{avg} = \sum_{i=1}^k (\Delta dom_{i,new-pt}) / k$ . Note that the *current-pt* may or may not be on the archival front.

(2) The *new-pt* is nondominating with respect to the other points in the archive. The *new-pt* is on the same front as the archive. Therefore, the *current-pt* is replaced by the *new-pt* and added to the archive. In this case, the archive becomes overfull, clustering is performed to reduce the number of points to HL.

(3) The *new-pt* dominates  $k$  points ( $k \geq 1$ ) in the archive. Therefore, the *current-pt* is replaced by the *new-pt*, and added to the archive. All the  $k$  dominated points are removed from the archive. Note that the *current-pt* may or may not be on the archival front.



**Figure 1. The Current-pt and the New-pt are Nondominating with Respect to Each Other, (a) The  $k$  points ( $k \geq 1$ ) in the Archive Dominates the Current-pt,**

**(b) the New-pt is Nondominating with Respect to the other Points in the Archive, (c) the New-pt Dominates  $k$  Points ( $k \geq 1$ ) in the Archive**

Case 3: The *new-pt* dominates the *current-pt*, and the *new-pt* dominates  $k$  points ( $k \geq 0$ ) in the archive. The situations ( $k = 0$  and  $k \geq 1$ ) are presented in Figure 3. The *new-pt* is selected as the *current-pt* with probability

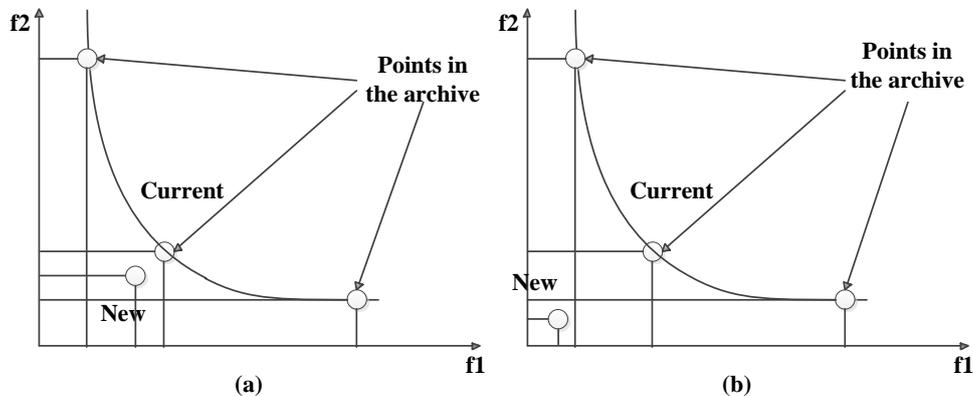
$$\frac{1}{1 + \exp(\Delta dom_{avg} \times T)} \tag{3}$$

where  $\Delta dom_{avg} = (\sum_{i=1}^k \Delta dom_{i,new-pt} + \Delta dom_{current-pt,new-pt}) / (k + 1)$ . Note that  $\Delta dom_{avg}$  denotes the average amount of domination of the *new-pt* by  $k + 1$  points, namely, the *current-pt* and  $k$  points of the archive. Also, as  $k$  increases,  $\Delta dom_{avg}$  will increase since here the dominating points that are farther away from the *new-pt* are contributing to its value.

Lemma: When  $k = 0$ , the *current-pt* is on the archive front.

Proof: If this is not the case, then some point, say A, in the archive dominates it. Since the *current-pt* dominates the *new-pt*, by transitivity, A will also dominate the *new-pt*. However, we have considered that no other point in the archive dominates the *new-pt* as  $k = 0$ . Hence proved.

However, if  $k \geq 1$ , this may or may not be true.



**Figure 2. The New-pt Dominates the Current-pt, (a) the New-pt is Nondominating with Respect to the Solutions of Archive Except Current-pt if it is in the Archive, (b) Some Solutions in the Archive Dominate the New-pt**

Based on the above situations, the pseudo-code of AMOSA for VRPTW is shown in Table 2.

**Table 2. Pseudo-Code of AMOSA for VRPTW Problem**

AMOSA Pseudo-code
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1 Initialize the archive.
2  $current-pt = \text{random}(\text{archive})$ ,  $T_{current-pt} := \gamma \times \text{cost}(current-pt)$ ,  $k := 0$ ,  $\omega := 0$ ,  $l := 0$ 
3 While ( $\omega < \omega_{max}$ )
4     While ( $l < L$ )
5          $new-pt = \text{perturb}(current-pt)$ 
6         Check the domination status of  $new-pt$  and  $current-pt$ 
7         If ( $current-pt$  dominates  $new-pt$ )
8             Check the domination status of  $new-pt$  and points in the archive.
9             If ( $new-pt$  is dominated by  $k$  ( $k \geq 1$ ) points in the archive)
10                 $p = 1/(1 + \exp(-\Delta dom_{min}))$ 
11                Set point of the archive which corresponds to  $\Delta dom_{min}$  as  $current-pt$ 
with
                probability =  $p$ 
12            Else set  $new-pt$  as  $current-pt$ .
13            If ( $new-pt$  is non-dominating with respect to the points in the archive)
14                Set  $new-pt$  as the  $current-pt$  and add it to the archive.
15            If  $current-pt$  is in the archive, remove it from the archive.
16            Else if archive size > SL
17                Cluster archive to HL number of clusters.
18            If ( $new-pt$  dominates  $k$  other points in archive.
19                Set  $new-pt$  as  $current-pt$  and add it to the archive.
20                Remove all the  $k$  dominated points from the archive.
21            If ( $current-pt$  and  $new-pt$  are non-dominating to each other)
22                Check the domination status of  $new-pt$  and points in the archive.
23                If ( $new-pt$  is dominated by  $k$  ( $k \geq 1$ ) points in the archive)
24                     $p = 1/(1 + \exp(\Delta dom_{avg} \times T))$ 
25                    Set  $new-pt$  as  $current-pt$  with probability =  $p$ 
26                If ( $new-pt$  is non-dominating w.r.t all the points in the archive)
27                    Set  $new-pt$  as  $current-pt$  and add  $new-pt$  to the archive.
28                If archive size > SL
29                    Cluster archive to HL number of clusters.
30                If ( $new-pt$  dominates  $k$  ( $k \geq 1$ ) points in the archive)
31                    Set  $new-pt$  as  $current-pt$  and add it to the archive.
32                    Remove all the  $k$  dominated points from the archive.
33                If ( $new-pt$  dominates  $current-pt$ )
34                     $p = 1/(1 + \exp(\Delta dom_{avg} \times T))$ 
35                    Set  $new-pt$  as  $current-pt$  with probability =  $p$ 
36            End while
37             $k := k + 1$ ,  $T_{k+1} := \beta \times T(k)$ 
38            If ! update current-pt
39             $\omega := \omega + 1$ 
40        End while
41        If archive size > SL
42            Cluster archive to HL number of clusters.

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## 4. Experimental Results

The proposed AMOSA metaheuristic was implemented in a JDK 7 environment and were tested on Solomon dataset [3]. The computations were performed on a laptop

equipped with Intel(R) Core (TM) i7-2620M CPU 2.7 GHZ. The Solomon datasets consists of 56 problem instances. Each of these instances comprises 100 customers. The location of the depot and the customers are given as integer values from the range 0 to 100 in a Cartesian coordinate system. It is assumed that the travel time  $t_{ij}$  are equal to the corresponding Euclidean distance  $d_{ij}$  between the customer location. The test problems are grouped into six problem types. In problem sets R1 and R2, the customer locations are generated randomly in a given area according to a uniform distribution. Problem sets C1 and C2, customers are located in a number of cluster, while RC1 and RC2 are a mix of them. Sets R1, C1 and RC1 have tighter time windows, allow fewer customers per route, and their vehicle capacities are low (200). Problem sets R2, C2 and RC2 have wider windows, allow a larger number of customers per route, and have higher vehicle capacities (700, 1000 and 1000 respectively). The parameters used during the experiments are given in Table 3.

**Table 3. Parameters Setting Summary**

Parameter	Description	Value
HL	The maximum size of the archive on termination.	50
SL	The maximum size to which the archive may be filled before clustering is used to reduce its size to HL.	2HL
$\gamma$	relates the cost and temperature	1
$\omega_{\max}$	The maximum number of iterations for which a non-improved solution is obtained	35
L	The maximum number of iterations for temperature $T$	$n^2$
$\beta$	The cooling ratio	0.03

The AMOSA is compared with the world's best solutions in the literature, which have been already published on the SINTEF website [29]. Two criteria are adopted to justify the effectiveness of the proposed approach: (i) those best results of the proposed AMOSA are better than or the same as the best known solutions, (ii) the solution gap which is given by the average results by AMOSA and the best known solution in the literature (relative to the best known Solution).

Table 4, presents a summary of our results and compare them with the best known solutions. The column labeled Best Known gives the world's best known solutions, column labeled Proposal AMOSA gives the best results and the average results in 10 runs, and column labeled Gap gives the NV gap between the best known solutions and average results of AMOSA, and the percent gap between the TD obtained by the best known solution and average TD by AMOSA (relative to the best known solution).

Bolded values in Table 4 indicate indicates that those results of the proposed AMOSA are the same as the best known solutions. For 30 instances out of 64 attempted, the best results obtained by our AMOSA are equal to the best known solutions found in the literature. For the other instances, the average TD% gap of the best results obtained by our AMOSA is 0.49%, and the maximum gap is 4.83% (calculated relative to the best known solution). In the 10 runs, the average deviation between the NV obtained by AMOSA and Best Known Solution is 0.42, and the average deviation between the TD obtained by

AMOSAs and Best Known is 0.73%. It is noteworthy that most published works in VRPs, the first phase is to minimize the NV, and the second phase is to minimize TD. In this paper, there is no theoretical or practical advantage to giving priority to the NV than TD, and we minimize the NV and TD simultaneously. Admittedly, as Desrochers *et. al.*, [30] mentioned, there was an associated cost to having more vehicles, and the associated courier to drive them. However, there was also an associated cost to the additional fuel and time used in using fewer vehicles at longer distances to service customers.

## 5. Concluding Remarks

Vehicle routing problem with time windows (VRPTW) involves the optimization of routes for multiple vehicles so as to meet all constraints and to minimize the number of vehicles needed and total distance traveled. In this paper, a metaheuristic based on simulated annealing (SA) incorporating archive concept was proposed to solve multi-objective VRPTW. The proposed approach was tested using 64 benchmark instances with 100 customers and the program was run 10 times. It was found that the best results obtained by AMOSA could obtain the world's best known solutions in 30 instances of total 64 instances. In the 10 runs, the average deviation between the NV obtained by AMOSA and Best Known is 0.42, and the average deviation between the TD obtained by AMOSA and Best Known is 0.73%. The computational results show that the proposed method is effective for solving multi-objective VRPTW problem.

However, we solved each problem ten times and the best solution of AMOSA is not achieved each time. Therefore, it is advisable to solve a problem more than once to achieve the best results. This is due to the probabilistic nature of the techniques which search a near optimal solution.

**Table 4. Comparison between the Best Known Solutions and the Proposed AMOSA on Solomon’s Benchmark Instances**

Problem	Best Known		Proposed AMOSA				Gap		Problem	Best Known		Proposed AMOSA				Gap	
	NV	TD	Best		Average		NV/ TD%	NV/ TD%		NV	TD	Best		Average		NV/ TD%	NV/ TD%
			NV	TD	NV	TD						NV	TD	NV	TD		
R101	19	1650.8	19	1650.80	19.40	1646.63	0/0.00%	0.40/-0.25%	R201	4	1252.37	<b>4</b>	<b>1252.37</b>	4.81	1247.42	0/0.00%	0.81/-0.40%
R102	17	1486.12	18	1480.31	19.10	1495.12	1/-0.39%	1.10/0.61%	R202	3	1191.7	<b>3</b>	<b>1191.7</b>	3.43	1162.12	0/0.00%	0.43/-2.48%
R103	13	1292.68	<b>13</b>	<b>1292.68</b>	14.45	1236.32	0/0.00%	1.45/-4.36%	R203	3	939.5	3	968.31	3.40	973.03	0/3.07%	0.40/3.57%
R104	9	1007.31	10	1007.45	10.70	1008.70	1/0.01%	0.70/0.14%	R204	2	825.52	<b>2</b>	<b>825.52</b>	3.00	810.31	0/0.01%	1.00/-1.84%
R105	14	1377.11	<b>14</b>	<b>1377.11</b>	15.21	1376.38	0/0.00%	1.21/-0.05%	R205	3	994.42	3	999.77	3.34	1013.64	0/0.54%	0.34/1.93%
R106	12	1252.03	<b>12</b>	<b>1252.03</b>	13.03	1247.83	0/0.00%	1.03/-0.34%	R206	3	906.14	3	943.31	3.00	951.12	0/4.10%	0.00/4.96%
R107	10	1104.66	11	1078.31	11.61	1077.27	1/-2.39%	0.61/-2.48%	R207	2	890.61	<b>2</b>	<b>890.61</b>	2.66	881.45	0/0.00%	0.66/-1.03%
R108	9	960.88	<b>9</b>	<b>960.88</b>	10.21	974.73	0/0.00%	1.21/1.44%	R208	2	726.82	2	741.85	2.32	737.37	0/2.07%	0.32/1.45%
R109	11	1194.73	11	1194.73	12.88	1183.21	0/0.00%	1.88/-0.96%	R209	3	909.16	3	951.58	3.87	930.65	0/4.67%	0.87/2.36%
R110	10	1118.84	10	1124.64	12.07	1104.21	0/0.52%	2.07/-1.31%	R210	3	939.37	3	975.23	3.00	983.34	0/3.82%	0.00/4.68%
R111	10	1096.72	10	1097.28	10.40	1097.16	0/0.05%	0.40/0.04%	R211	2	885.71	2	902.55	2.00	921.23	0/1.90%	0.00/4.01%
R112	9	982.14	9	983.43	9.90	975.66	0/0.13%	0.90/-0.66%									
C101	10	828.94	<b>10</b>	<b>828.94</b>	10.00	828.94	0/0.00%	0.00/0.00%	C201	3	591.56	<b>3</b>	<b>591.56</b>	3.00	591.56	0/0.00%	0.00/0.00%
C102	10	828.94	<b>10</b>	<b>828.94</b>	10.00	865.34	0/0.00%	0.00/4.39%	C202	3	591.56	<b>3</b>	<b>591.56</b>	3.43	614.77	0/0.00%	0.43/3.92%
C103	10	828.06	10	828.94	10.00	832.31	0/0.11%	0.00/0.51%	C203	3	591.17	<b>3</b>	<b>591.17</b>	3.00	591.17	0/0.00%	0.00/0.00%
C104	10	824.78	10	825.65	10.00	831.73	0/0.11%	0.00/0.84%	C204	3	590.6	<b>3</b>	<b>590.6</b>	3.00	590.6	0/0.00%	0.00/0.00%
C105	10	828.94	<b>10</b>	<b>828.94</b>	10.00	853.12	0/0.00%	0.00/2.92%	C205	3	588.88	<b>3</b>	<b>588.88</b>	3.00	588.88	0/0.00%	0.00/0.00%
C106	10	828.94	<b>10</b>	<b>828.94</b>	10.00	828.94	0/0.00%	0.00/0.00%	C206	3	588.49	<b>3</b>	<b>588.49</b>	3.00	588.49	0/0.00%	0.00/0.00%
C107	10	828.94	<b>10</b>	<b>828.94</b>	10.00	828.94	0/0.00%	0.00/0.00%	C207	3	588.29	<b>3</b>	<b>588.29</b>	3.00	588.29	0/0.00%	0.00/0.00%
C108	10	828.94	<b>10</b>	<b>828.94</b>	10.00	828.94	0/0.00%	0.00/0.00%	C208	3	588.32	<b>3</b>	<b>588.32</b>	3.00	588.32	0/0.00%	0.00/0.00%
C109	10	828.94	<b>10</b>	<b>828.94</b>	10.00	828.94	0/0.00%	0.00/0.00%									
RC101	14	1696.94	14	1713.54	14.00	1722.17	0/0.98%	0.00/1.49%	RC201	4	1406.94	<b>4</b>	<b>1406.94</b>	4.00	1459.86	0/0.00%	0.00/3.76%
RC102	12	1554.75	<b>12</b>	<b>1554.75</b>	13.74	1492.32	0/0.00%	1.74/-4.02%	RC202	3	1365.65	<b>3</b>	<b>1365.65</b>	3.00	1386.74	0/0.00%	0.00/1.54%
RC103	11	1261.67	11	1266.42	11.70	1284.21	0/0.38%	0.70/1.79%	RC203	3	1049.62	3	1064.26	3.00	1068.37	0/1.39%	0.00/1.79%
RC104	10	1135.48	10	1135.48	10.60	1172.42	0/0.00%	0.60/3.25%	RC204	3	798.46	<b>3</b>	<b>798.46</b>	3.00	798.46	0/0.00%	0.00/0.00%
RC105	13	1629.44	13	1629.44	14.64	1563.33	0/0.00%	1.64/-4.06%	RC205	4	1297.65	4	1313.43	4.00	1328.54	0/1.22%	0.00/2.38%
RC106	11	1424.73	<b>11</b>	<b>1424.73</b>	11.00	1434.64	0/0.00%	0.00/0.70%	RC206	3	1146.32	3	1201.63	3.00	1203.41	0/4.83%	0.00/4.98%
RC107	11	1230.48	11	1230.54	11.42	1175.48	0/0.00%	0.42/-4.47%	RC207	3	1061.14	<b>3</b>	<b>1061.14</b>	3.34	1109.31	0/0.00%	0.00/4.54%
RC108	10	1139.82	<b>10</b>	<b>1139.82</b>	10.00	1145.21	0/0.00%	0.00/0.47%	RC208	3	828.14	3	831.07	3.00	872.05	0/0.35%	0.00/5.30%

Boldface indicates that those solutions of the proposed AMOSA are better than or the same as the best known solutions.

NV in column Gap: the NV difference between the NV obtained by the Best Known and NV by AMOSA. %TD: the percent gap between the TD obtained by the Best Known and TD by AMOSA (relative to the Best Known).

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