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# A Reliability Location Model for New Blood Banks in Post-Disaster Reconstruction Areas in China

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***Abstract***

*Locating blood banks is a strategic decision problem. It is necessary to consider the risk of facility disruptions in the design stage because blood banks may fail as a result of public emergencies. Taking the timeliness and reliability of blood relief as goals, a reliability P-median location model for new blood banks was developed by considering different probabilities of facility failure in different regions. According to the characteristics of the model, a linearization technique was applied to convert the model and a Lagrangian Relaxation (LR) algorithm was proposed to solve the problem. A case study was given based on one of affected areas in the Wenchuan Earthquake, i.e., Aba prefecture in Sichuan province in China. A group of numerical examples were given to test and verify the model and algorithm. The results show that the proposed algorithm is efficient.*

***Keywords****: facility location, facility disruptions, reliability, P-median, Lagrangian relaxation, blood bank*

## 1. Introduction

Unconventional emergencies attacks, such as large scale devastating earthquake, may severely damage the blood relief system of stricken areas. It is urgently required to reconstruct the blood relief system in post-earthquake reconstruction areas. Blood bank location planning is the main step of blood relief system reconstruction. Rational planning of blood bank layout is a basic prerequisite to guarantee the regional daily blood demands effectively, as well as an important condition for disaster prevention and disaster alleviation. Rational blood bank location planning is of great significance to carry out disaster rescue and raise quick response to emergencies.

Blood relief plays an important role in medical treatment. Different from other ordinary materials, the timeless and reliability are highlighted in blood supply. Consequently, locating the blood banks has a major characteristic of locating relief supplies reserve bases. In reality, some unconventional emergencies, such as large scale devastating earthquake, tsunami, terrorist attacks, *etc.*, may cause the blood banks disruptions, and result in a booming demand for emergency blood. Thus, the facility disruptions and supply timeliness must be fully considered when planning the location of new blood banks in post-disaster reconstruction areas.

In order to improve the blood relief system in post-disaster reconstruction areas, and ensure timeless and reliability of the interregional blood relief, the decision makers plan to build a new blood banking system based on existing blood banks. Then, how to locate

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the new blood banks will directly determine the efficiency of blood relief, and emergency response ability in post-disaster reconstruction areas.

The traditional Facility-Location Problem (FLP) or the Location-Allocation Problem (LAP), usually take the minimal system cost as target. These researches have been intensified [1-2]. And relief supplies reserve bases location problem often aims to maximize the time benefits or minimize the disaster losses. Many scholars have studied this problem using different methods from different angles. Berman and Gavious [3] discussed the location of terror response facilities under the worst scenario. Rawls and Turnquist [4] proposed an optimization model whose solution provides a pre-positioning strategy for facility locations and resource stocking under uncertainty about eventual demands. They proposed the Lagrangian L-shaped method to decompose the model into a series of smaller and easier solved sub-problems. Görmez, et al.[5], considered the problem of locating disaster response and relief facilities in the city of Istanbul, where an earthquake is expected to occur in the near future. Two objectives were considered in their model: minimize the average distance traveled to serve a refugee and minimize the number of new facilities to establish. Lin, *et. al.*,[6], proposed the location of temporary depots around the disaster-affected area, under the background of an earthquake with significant damage and an extensive time period over which supplies need to be delivered. A two-phase heuristic approach is proposed to solve the model in this paper. Pavankumar *et. al.*,[7], formulated the maximal covering location model with a loss function to address a large-scale emergency of a hypothetical anthrax attack. The demand uncertainty and capacity constraints are considered in the model. And a locate-allocate heuristic algorithm was designed in [7].

But the literature about blood bank location-allocation problem is very few. Typical works are listed as follows. Or and Pierskalla [8] proposed the Blood Transportation-Allocation Problem (BTAP) to solve the following problems: how many central blood banks to set up,where to locate them, how to allocate the hospitals to the banks, and how to route the periodic supply operation. The BTAP took the minimum system cost as objective function. Sahin *et. al.*,[9], designed a hierarchical organization structure for the blood services of Turkish Red Crescent (TRC). Sapountzis [10] proposed a method for allocating units of blood from a Regional Blood Transfusion Service to the hospitals of its area, taking into account the characteristics of the hospitals. The method is shown to reduce expiries significantly. Sivakumar *et. al.*,[11], introduced multi-phase composite analytical model for integrated allocation-routing problem of blood banks from the perspective of supply chain. Wang *et. al.*,[12], proposed a P-median location problem of new blood banks in post-disaster reconstruction areas considering timeliness of blood relief.

All of the above literature implicitly assumes that, once constructed, the facilities always operate as planned without failure. That is to say, the facilities are absolutely reliable. But realistically, facility disruptions (also known as facility failures) occur frequently. Such failures may lead to excessive transportation costs as customers must be served from facilities much farther than their regularly assigned facilities. Thus, the reliable location problem (RLP) arises.

The reliable location model was first introduced by Snyder and Daskin (2005) to handle facility disruption. The goal was to choose facility locations that were both inexpensive under traditional objective functions and also reliable. Currently, research on RLP mainly extends two kinds of classical facility location problem: uncapacitated fixed-charge location problem (UFLP), and P-median problem (PMP). Respectively named as reliability uncapacitated fixed-charge location problem (RUFLP), and reliability p-median problem (RPMP). Snyder and Daskin[13] formulated RPMP and RUFLP based on traditional PMP and UFLP. In their models, each customer is assigned to several backup facilities, and some facilities will fail with a given probability. They developed an optimal Lagrangian relaxation algorithm to solve proposed models. Cui, Ouyang, and Shen [14] proposed a compact linear mixed-integer program (MIP) formulation and a continuum approximation (CA) formulation, relaxing the uniform failure probability assumption in Snyder and Daskin (2005). In Lee and Chang [15], the failure probability of facilities can be different, but each customer was permitted to choose only one backup facility. Li and Ouyang [16] studied a RFP. They allow the failure probabilities to be spatially correlative. Two relevant articles extended RPMP in [13] are Berman (2007)[17], Berman(2009)[18].

However, the above literatures of RFP are designed for conventional facilities, without considering the reliable location of emergency facilities. Though the probability of failure is small, but once occurs, the damage is tremendous. In addition, blood bank location is a strategic decision that cannot be changed in a short time. It is critical to account for the failures of facility in designing the blood banking network.

In this paper, a reliability P-median location model for new blood banks was developed by considering different probabilities of facility failure in different regions. The goal is to ensure timeliness and reliability of blood relief. The main difference from the existing relief supplies reserve bases location model is that the incomplete reliability of facilities is considered in this paper. Compared with existing RPMP models, the differences in this paper can be depicted as follows: (1) Previous RPMP models are centered on conventional facility location problem. The goal is to establish a low cost and reliable infrastructure network. But we study the blood bank location problem, aims to establish an infrastructure network with high timeless and reliability of blood relief. (2)The dependent relations between blood banks are considered. The timeliness constraints between blood banks, between blood banks and demand points, are considered too. In addition, the facilities in this RPMP model have different failure probabilities. The nonlinear “transitional probability” equations are used. Then the complexities of the objective function and relevant constrains are greatly increased, causing the increment of difficulties in solving the model. In this paper, a linearization technique was applied to convert the model. The converted model can be solved using commercial software packages like ILOG CPLEX for small scale problems. This fact motivates the development of a Lagrangian relaxation algorithm (LR) to solve large-scale problems.

The rest of this paper is organized as follows. The formulation for the problem is presented in Section 2. The model hypothesis and formulation are presented in section 3. Our solution strategy is discussed in Section 4. Computational experiments and case study are conducted and summarized in Section 5. Finally, Section 6 contains conclusions and suggestions for future work.

## 2. Problem Description

In China, blood collection, storage and supply are all managed by specialized institutions. These specialized institutions that can provide qualification of blood using to clinic are defined as “blood bank”. These blood banks can be categorized into 3 types: blood center, central blood bank, and ordinary blood bank. Blood centers are set up in provincial capitals, capital cities of ethnic minority autonomous regions, or municipality directly under the central government. Central blood banks are located in cities of setting up districts which people government located in. If there exist cities and counties that cannot be served by blood center or central blood bank within 3 hours, it is need to set a blood bank in a county-level medical institution, responsible for regional blood relief task. These blood banks are different from blood centers and central blood banks, are called “ordinary blood banks”.

All the above blood banks have the right to collect whole blood and blood components, responsible for making, storing and suppling clinical demands of blood products to medical institutions and recollecting outdated blood products within the administrative scope of the local municipality or county (city). The blood center and central blood bank can set up subsidiaris of not independence, fixed points of blood collection or store, according to actual needs of areas they serve. These fixed points of blood collection and store have no authority to test blood products. The hospitals apply clinical blood products to local blood bank. If blood shortage occur in local blood bank, then local blood bank can apply emergency blood transferring information to the upper level. Therefore, if the region has a broader area, the time to deliver the collected blood to blood bank, and the time to ship the blood to the hospital are very long. In this case, the timeliness of blood relief will be greatly affected.

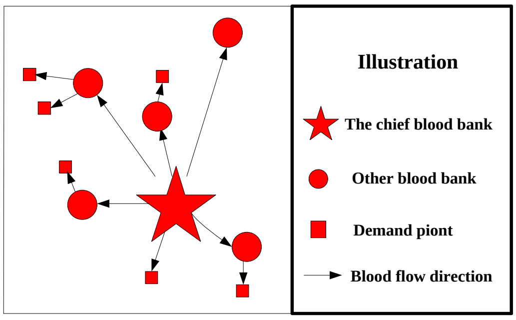


**Figure 1. Attachment Relationships Among Blood Banks**

In China, if a region has a blood center located in, it is no longer an option to set up a central blood bank. The blood center and central blood bank are higher authorities of ordinary blood bank in aspects of regional rank, scale and functions. Similar to literature [12], the blood center and central blood bank are collectively called as “the chief blood bank”. Compared with other blood banks, the chief blood bank has a larger scale and a more complete function. Considering the importance of the chief blood bank, the facility is well protected from attack. Then the chief blood bank is absolutly reliable. That is to say, the chief blood bank **is** nonfailable. In an administrative region, there is a blood banking network contain a chief blood bank and some other ordinary blood banks. The other blood banks are dependent on the chief blood bank. That is, other blood banks can get a certain proportion of blood supply from the chief blood bank. To be specific: When emergencies occur, the blood demands increase sharply, and the ability of blood collection and supply are restricted. Because blood processing such as collection, test and preparation, need some time, so in order to ensure the clinical blood demands on time, blood of other blood banks demanded can be transferred from the chief blood bank, as showed in Fig.1.

Similar to traditional RPMP, each demand point is assigned to up to R≥1 facilities and can be served by these and only these facilities. Multi-backup facilities assignment is helpful to cope with the failure, and improve system reliability. Because each conventional blood bank that serve a demand point may fail, so a “virtual” blood bank is introduced in this paper. This “virtual” blood bank indexed by j=E, is nonfailable. If a demand point is assigned to blood bank E in “level-r”, a penalty value is generated.

A “level-r” assignment is one for which there are r closer failable facilities that are open. If r=0, this is a primary assignment; otherwise, it is a backup assignment. Each demand point i has a level-r assignment for each $r=1,\ldots,l-1$, unless i is assigned to a level-s facility that is nonfailable(“virtual” blood bank), where $s<r$. In other words, demand point i is assigned to one facility at level 0, another facility at level 1, and so on until i has been assigned to all open facilities at some level, or i has been assigned to facility E.[13] The penalty value can be understood as the time that blood emergency collection, packaging, transportation and receiving spent from other areas outside the region.

 

This paper considers a region containing some subordinate administrative regions. Demand points in this region send blood demand information to blood banks. The chief blood bank in the regional administrative center has been set up. Other subordinate administrative regions have no blood bank built can be took as the candiate points. Both the candidate points and the chief blood bank may fail. The problem to be solved in this article is：How to locate the blood banks? And how to allocate the service areas of the blood banks?

## 3. Model Development

### 3.1. Model Hypothesis

The following assumptions are formed. (1) The failure probabilities of all blood banks are priori probabilities and mutually independent. (2) The blood demands in a region depend on the population of that region, so the blood transportation time consuming can be indicated by the weighted demand distance (*i.e.*, multiply the total population of demand point by the distance from the demand point to blood bank)[20],[21].(3) Each demand point is served by one blood bank. If there is a blood bank locating in the seat of demand point, then this demand point is served by its own facility, and the demand points that no blood bank located in are known as the pure demand points.

### 3.2. Notations

Parameters are shown as follows.

$A$ ：the set of old blood banks, $A=\{1,\cdots,n\}$ ,

$C$ ：the set of candidate locations of new blood banks, $C=\{n+1,\cdots,n+m\}$ ,

$D$ ：the set of demand points， $D=A\cup C$,

$o$ ：denote the chief blood bank, $o\in A$,

$l$ : the total number of old blood banks and new blood banks, determined by decision maker according to the relevant policies or actual requirements, and $l\le n+m$,

$g_i$ : the population in demand piont $i$ , $i\in D$ ,

$d_{ij}$ : the distance between demand point $i$ and blood bank $j$ , $i,\ j\in D$,

$Dis$ : the maximun distance allowed a new blood bank open away from the chief blood bank;

$H$ : the service radius of a blood bank;

$\xi_j$ : the dependent coefficient of blood bank $j$ to the chief blood bank. The proportion of the amount of transferred blood from the chief blood bank to total amount of blood demands in blood bank $j$ ;

$E$ : denote the “virtual” blood bank. When all asigned regular backup facilitis fail, $i$ is assigned to blood bank E, meaning that the demand in $i$ cannot be satisfied;

$q_j$ ：the probability of failure in blood bank $j$ , and $0\le q_j\le 1$， $q_E=0$ ,

$\phi_i$ ：the penalty value of not serving the customer per unit of missed demand, and $\phi_i=d_{iE}$ ,

$R$: each demand point should have at most $R$ regular assignments, $1\le R\le l-1$; A “level-$r$” assignment for a demand point $i\in D$ will serve it if and only if all of its assigned facilities at levels $0,...,r-1$ have failed. At optimality, each demand point $i\in D$ should have exactly $R$ assignments, unless $i$ is assigned to the facility E at certain level $s<R$. If $i$ is indeed assigned to exactly $R$ regular facilities at levels

$0,...,R-1$, it must also be assigned to facility $E$ at level $R$ to capture the possibility that all of the $R$ regular facilities may fail.

The variables used in this model are the location variables( $X_j$ ), the assignment variables ( $Y_{ijr}$ ), and the probability variables ( $P_{ijr}$ ):

$Y_{ijr}$ : binary variable that equals 1 if demand node $i$ is assigned to blood bank $j$ as a level-$r$ assignment, and 0 otherwise, $i,\ j \in D$,

$X_j$ : binary variable that equals 1 if a blood bank is opened at location $j$, and 0 otherwise, $j \in D$;

$P_{ijr}$ : probability that blood bank $j$ serves demand point $i$ at level $r$.

### 3.3. Model Formulation

The RPMP model of new blood banks is formulated as follows:

$$\min z_1 = \sum_{i \in D} \sum_{j \in D+E} \sum_{r=0}^{R} g_i d_{ij} Y_{ijr} P_{ijr} + \sum_{i \in D} \sum_{j \in D} \sum_{r=0}^{R} g_i \xi_j d_{jo} Y_{ijr} P_{ijr} \tag{1}$$

$$\text{s.t.} \sum_{j \in D} Y_{ijr} + \sum_{s=0}^{r} Y_{iEs} = 1, \forall i \in D, r = 0, \ldots, R \tag{2}$$

$$\sum_{j \in D} X_j = l \tag{3}$$

$$\sum_{r=0}^{R-1} Y_{ijr} \le X_j, \forall i \in D, j \in D \tag{4}$$

$$d_{ij} Y_{ijr} \le H, \forall i, j \in D, r = 0, \ldots, R \tag{5}$$

$$d_{jo} X_j X_o \le Dis, \forall j \in D \tag{6}$$

$$\sum_{r=0}^{R} Y_{iEr} = 1, \forall i \in D \tag{7}$$

$$P_{ij0} = 1 - q_j, \forall i \in D, j \in D + E \tag{8}$$

$$P_{ijr} = (1 - q_j) \sum_{k \in D} \frac{q_k}{1 - q_k} P_{i,k,r-1} Y_{i,k,r-1} \quad \forall i \in D, j \in D + E, r = 1, \ldots, R \tag{9}$$

$$X_j = 1, \forall j \in A \tag{10}$$

$$X_E = 1 \tag{11}$$

$$X_j \in \{0,1\}, \forall j \in D \tag{12}$$

$$Y_{ijr} \in \{0,1\}, \forall i, j \in D, r = 0, \ldots, R \tag{13}$$

 

The objective function (1) is the sum of the expected demand weighted distance, simultaneously considering the timeliness and reliability of blood relief. In the formulation (1), the first term denotes the expected weighted demand distance between blood banks and demand points, and the second term denotes the expected weighted demand distance between the chief blood bank and other ordinary blood banks. Constraints (2) enforce that for each demand point $i$ and each level $r$, either $i$ is assigned to a regular blood bank at level $r$ or it is assigned to the virtual blood bank $E$, at certain level $s \le r$ (if $r = 0$, $\sum_{s=0}^{r-1} Y_{iEs} = 1$). Constraint (3) requires $l$ blood banks(containing old blood banks and new blood banks) to be opened. Constraints(4) prohibit an assignment to a blood bank that has not been opened. Constraints(5) is the Service distance constraints of each blood bank. Constraints (6) limit the distance between the new blood banks and the chief blood bank less than $Dis$, in order to ensure the timeliness of blood transfer. Constraints(7) require each demand point to be assigned to blood bank E at a certain level. Constraints (8) and (9) are the "transitional probability" equations. $P_{ijr}$ is just the probability that $j$ remains open if $r = 0$. For $1 \le r \le R$, $P_{ikr} = \frac{q_k(1-q_j)}{1-q_k} P_{i,k,r-1}$, given that facility $k$ serves customer $i$ at level $r-1$. Note that constraints (2) imply that $Y_{i,k,r-1} = 1$, for at most one $k \in D$, which guarantees correctness of the transitional probabilities [14]. Constraints (10)-(11) denote old blood banks and the virtual blood bank have been opened. Constraints (12)-(13) are binary constraints.

The above model is a nonlinear mixed integer programming model. The nonlinear terms are $P_{ijr}Y_{ijr}$, $i \in D, j \in D+E, r = 1,\ldots,R$, in formulation (1) and (9), make this model difficult to solve. Similar to [14], the linearization technique introduced by Sherali and Alameddine (1992)[22] is applied by replacing each $P_{ijr}Y_{ijr}$ with a new variable $W_{ijr}$. For $i \in D$, $j \in D+E$ and $r = 0,\ldots,R$, a set of new constraints is added to the formulation to enforce $W_{ijr} = P_{ijr}Y_{ijr}$.

$$W_{ijr} \le P_{ijr} \tag{14}$$

$$W_{ijr} \le Y_{ijr} \tag{15}$$

$$W_{ijr} \ge 0 \tag{16}$$

$$W_{ijr} \ge P_{ijr} + Y_{ijr} - 1 \tag{17}$$

The linearized formulation (LRPMP) is stated below:

$$\min z_2 = \sum_{i \in D} \sum_{j \in D+E} \sum_{r=0}^{R} g_i d_{ij} W_{ijr} + \sum_{i \in D} \sum_{j \in D} \sum_{r=0}^{R} \xi_j g_i d_{jo} W_{ijr} \tag{18}$$

$$\text{s.t. } P_{ijr} = (1-q_j) \sum_{k \in D} \frac{q_k}{1-q_k} W_{i,k,r-1}, \forall i \in D, r = 1,\ldots,R \tag{19}$$

(2)-(4),(5)-(8),(10-17)

## 4. Solution Methodology

The linear mixed-integer program (LRPMP) can be solved using commercial software packages like ILOG CPLEX, but generally such an approach takes an excessively long time even for moderately sized problems[14]. This fact motivates the development of a Lagrangian relaxation algorithm.

Relaxing constraints (4) with lagrangian multipliers $\lambda_{ij}$ yields the following model:

$$\min z_3 = \sum_{i\in D}\sum_{j\in D+E}\sum_{r=0}^{R} g_i d_{ij} W_{ijr} + \sum_{i\in D}\sum_{j\in D}\sum_{r=0}^{R} \xi_j g_i d_{jo} W_{ijr} - \sum_{i\in D}\sum_{j\in D}\lambda_{ij} X_j + \sum_{i\in D}\sum_{j\in D}\sum_{r=0}^{R-1}\lambda_{ij} Y_{ijr} \quad (20)$$

s.t. (2)-(3),(5)-(8),(10)-(17),(19)

According to the characteristics of the model variables, the model can be transformed into two independent subproblems: location subproblem(SP1), and assignment subproblem(SP2).

For SP1, the objective function is $\min(-\sum_{i\in D}\sum_{j\in D}\lambda_{ij} X_j)$ , can be re written as $\max\sum_{i\in D}\sum_{j\in D}\lambda_{ij} X_j$, Thus the SP1 can be written in the following form:

$$\max z_4 = \sum_{i\in D}\sum_{j\in D}\lambda_{ij} X_j \quad (21)$$

s.t. (3),(6),(10)-(12)

While SP2 can be written as follows:

$$\min z_5 = \sum_{i\in D}\sum_{j\in D+E}\sum_{r=0}^{R} g_i d_{ij} W_{ijr} + \sum_{i\in D}\sum_{j\in D}\sum_{r=0}^{R} \xi_j g_i d_{jo} W_{ijr} + \sum_{i\in D}\sum_{j\in D}\sum_{r=0}^{R-1}\lambda_{ij} Y_{ijr} \quad (22)$$

s.t. (2),(5),(7)-(8),(13)-(17),(19)

The algorithms for the two subproblems are discussed below.

(1)SP1 algorithm: ①to solve location variable $X_j$, for each open blood bank $j$, first calculate the values of $\sum_i \lambda_{ij}$. ②Rank the open blood banks in descending order according to the values of $\sum_i \lambda_{ij}$ . For the first $(l - \sum_k X_k, \forall k \in A)$ facilites, if constraint $d_{jo} X_j X_o \le Dis, \forall j \in D$ satisfied, let $X_j = 1$ , and old blood banks $X_k = 1, \forall k \in A$. ③for other candidate locations, make $X_j = 0$ . Then the optimal value of $X_j$ can be found easily.

(2)SP2 algorithm. To find the optimal $Y_{ijr}$ , the demand points assignment decision, note that the problem is separable in demand points $i$ [2]. For given lagrangian multipliers $\lambda_{ij}$, the assignment problem *of* demand point $i$ is referred to as the relaxed subproblem (RSP). For ease of notation, we omit the subscript $i$ in $Y_{ijr}$ , $P_{ijr}$ and $W_{ijr}$ .

$$\min z_6 = \sum_{j\in D+E}\sum_{r=0}^{R} g_i d_{ij} W_{jr} + \sum_{j\in D}\sum_{r=0}^{R} \xi_j g_i d_{jo} W_{jr} + \sum_{j\in D}\sum_{r=0}^{R-1}\lambda_{ij} Y_{jr} \quad (23)$$

$$\sum_{j\in D} Y_{jr} + \sum_{s=0}^{r} Y_{Es} = 1, \forall r = 0,\ldots,R \quad (24)$$

 

$$d_j Y_{jr} \le H, \forall j \in D, r = 0,\ldots,R \tag{25}$$

$$\sum_{r=0}^{R} Y_{Er} = 1 \tag{26}$$

$$P_{ij0} = 1 - q_j, \forall i \in D, j \in D + E \tag{27}$$

$$P_{ijr} = (1-q_j)\sum_{k\in D}\frac{q_k}{1-q_k}W_{i,k,r-1}, \forall i \in D,\ r = 1,\ldots,R \tag{28}$$

$$Y_{jr} \in \{0,1\}, \forall j \in D, r = 0,\ldots,R \tag{29}$$

The RSP can be solved by the approximate algorithm proposed by Cui(2009) [14]. This Algorithm can be described as follows.

We replace the variable probability $P_{jr}$ with fixed numbers. Let $j_0$, $j_1$ …… $j_{|D|-1}$ be an ordering of the facilities such that $q_{j0} \le q_{j1} \le \cdots \le q_{|D|-1}$. For $0 \le r \le R$, define

$$\partial_r = (1-q_{jr})\prod_{l=0}^{r-1} q_{jl} \tag{30}$$

$$\beta_r = \prod_{l=0}^{r-1} q_{jl} \tag{31}$$

We define a reformulation of the relaxed subproblem (RRSP) by replacing $P_{jr}$ with $\partial_r$ if $j \in D$, and replacing $P_{Er}$ with $\beta_r$ :

$$\min z_7 = \sum_{j\in D}\sum_{r=0}^{R}[g_i d_{ij}\partial_r + \lambda_{ij} + \xi_j g_i d_{jo}\partial_r]Y_{jr} + \sum_{r=0}^{R-1} g_i d_{iE}\beta_r Y_{Er} \tag{32}$$

$$\sum_{j\in D} Y_{jr} + \sum_{s=0}^{r} Y_{Es} = 1, \forall r = 0,\ldots,R \tag{33}$$

$$d_{ij} Y_{jr} \le H, \forall i, j \in D, r = 0,\ldots,R \tag{34}$$

$$\sum_{r=0}^{R} Y_{Er} = 1 \tag{35}$$

$$Y_{jr} \in \{0,1\}, \forall j \in D, r = 0,\ldots,R \tag{36}$$

Cui (2009)[14] proved that we can solve (RRSP) for a lower bound of (RSP). The RRSP can be solved in strongly polynomial time using the Hungarian algorithm (Kuhn 1955) [23].

Optimization between SP1 and SP2 can be coordinated by Lagrange multipliers. In this paper the Lagrange multipliers are updated based on subgradient method.

Then, the lagrange relaxation problem can be solved by the standard subgradient method, the calculation process is stated as follows:

Initial multiplier $\lambda_{ij}$, the maximum number of iterations *N*, step-size $\theta_t$.

Solve the location subproblem SP1.

Solve the assignment subproblem SP2 based on RRSP solution method.

compute the lower bound: LRPMP($\mu$)= SP1($\mu$)+ SP2($\mu$), SP1($\mu$) and SP2($\mu$) respectively denote the target value of SP1 and SP2. Then Judging whether the current solution is a feasible solution. If the current solution is feasible, then stop, and this solution is the optimal solution of LRPMP; Otherwise, run into step⑤ to update the lower bound.

Compute the subgradient and step-size according to the current solution. And update the Lagrange multiplier $\lambda_{ij}$. If meet the condition of convergence or stop, then stop; otherwise, return to ②.

The Lagrange multipliers $\lambda_{ij}$ are updated by setting[24]:

$$\lambda_{ij}^{t+1} = \max\left\{\lambda_{ij}^{t} + \theta_t s^t, 0\right\}, t = t+1 \tag{37}$$

In expression (37), if $s^t$ =0, $\lambda_{ij}^{t}$ reached the iteration termination condition, then stop computation; otherwise repeat it.

In expression (37), $s^t$ is the subgradient, can be formulated as expression (38). And, $\theta_t$ is the step-size, can be computed as expression (39).

$$s^t = \sum_{r\in R} Y_{ijr} - X_j, \forall i, j \tag{38}$$

$$\theta_t = \frac{z_{UP}(t) - z_{LB}(t)}{\left\|s^t\right\|^2}\beta_t \tag{39}$$

In expression (39), $z_{UP}(t)$ and $z_{LB}(t)$ respectively denote the upper bound and lower bound of the solution. $\beta_t$ is an adjustable parameter, and $0 \le \beta_t \le 2$. In the algorithm implementation process, firstly let $\beta_1 = 1.8$. In the iterative process, if the solution quality did not improve in 20 successive iterations, let $\beta_{k+1} = 0.8\beta_k$. If the solution quality did not improve in 100 successive iterations, then stop the algorithm[25].

Determination of feasible solution and upper bound. When the lower bound determined, add the relaxed constraint(22) to relaxed subproblem(RRSP), then compute new $X_j$ and $Y_{ijr}$. Because in above algorithms, the SP2 was approximately described. So the upper bound in iterative process can be expressed by the objective function value caculated based on the approximate model. Substitute the equation $z_4$ and $z_7$ with feasible solution of $X_j$ and $Y_{ijr}$. To be specific, the upper bound equal to $z_4$+$z_7$. This feasible solution of $X_j$ and $Y_{ijr}$, can satisfy all the constraints of the original problem (RPMP), thus, is the feasible solution of RPMP. Compute the value of $z_1$ by the feasible solution, this value of $z_1$ is an upper bound of the original problem. We can get the objective value of RPMP with the the best feasible solution when the algorithm terminates.

Stopping criteria. The Lagrangian process terminates when any of thefollowing criteria are met:

Define tolerance $\gamma = [z_{UP}(t) - z_{LB}(t)] / z_{UP}(t)$. If $\gamma$ < the optimal tolerance;

If the Lagrange multiplier $\lambda_{ij}$ terminate iteration.

## 5. Computational Results

A group of numerical examples were given to test and verify the feasibility and effectiveness of the model and algorithm. The example is derived from the location decision problem after the “5.12” Wenchuan earthquake in Aba Prefecture in Sichuan Province in China. The algorithm was coded in matlab2010ra tested on an Intel Pentium 2.90GHz processor with 4.0 GBs RAM under Windows 10.

 

There are 13 cities or counties under command of Aba refecture in Sichuan Province. Wenchuan, Mao and other 7 cities or counties are severely afflicted areas in "5.12" earthquake. At present, all the collected blood in Aba Prefecture must be sent to Maerkang to test and prepare. Aba is a huge geographical region, the service radius of old blood banks are too large. Post-disaster reconstruction have greatly improved Aba's social-economic and health, but put forward a higher requirement to blood relief system. The numerical examples are given based on above background. According to different values of $l$ and $R$, we designed 16 numerical examples. The parameter values, such as the population of each city or county in Aba $g_i$, the distance between the counties and cities $d_{ij}$, the dependent coefficient $\xi_j$, the maximun distance constraint to open a new blood bank $Dis$; the service radius of a blood bank $H$ are available from literature [12]. The probability of failure $q_j$ is randomly generated from a uniform distribution between 0 and 0.1. The penalty value $\phi_i$ is set as 10000.

**Table 1. Results of the Numerical Examples**

| *L* | *R* | LB | UB | Gap | Open locations | Time (S) |
|---|---|---|---|---|---|---|
| 4 | 2 | 9129.31 | 9129.31 | 0 | 1,2,5,13,14 | 9.63 |
| | 3 | 9083.54 | 9083.54 | 0 | 1,2,5,13,14 | 9.87 |
| 5 | 2 | 8330.14 | 8330.14 | 0 | 1,4,8,11,13,14 | 11.00 |
| | 3 | 8104.44 | 8104.44 | 0 | 1,2,5,8,13,14 | 13.55 |
| 6 | 2 | 6881.12 | 6882.69 | 0 | 1,5,6,8,11,13,14 | 4.53 |
| | 3 | 6866.16 | 6873.15 | 0 | 1,5,6,8,11,13,14 | 4.05 |
| | 4 | 6819.94 | 6819.94 | 0 | 1,5,6,8,11,13,14 | 4.50 |
| 7 | 2 | 6507.09 | 6507.09 | 0 | 1,3,5,6,8,11,13,14 | 4.91 |
| | 3 | 6305.46 | 6413.89 | 1.69% | 1,3,5,6,8,11,13,14 | 5.67 |
| | 4 | 6036.03 | 6037.06 | 0.01% | 1,3,5,6,8,11,13,14 | 6.13 |
| 8 | 2 | 5257.71 | 5257.71 | 0 | 1,3,5,6,7,8,11,13,14 | 8.05 |
| | 3 | 5252.84 | 5252.84 | 0 | 1,3,5,6,7,8,11,13,14 | 11.71 |
| | 4 | 5236.87 | 5236.87 | 0 | 1,3,5,6,7,8,11,13,14 | 9.89 |
| 9 | 2 | 4310.94 | 4310.94 | 0 | 1,3,5,6,7,8,11,12,13, 14 | 12.25 |
| | 3 | 4235.33 | 4235.33 | 0 | 1,3,5,6,7,8,11,12,13, 14 | 14.10 |
| | 4 | 4164.56 | 4164.56 | 0 | 1,3,5,6,7,8,11,12,13, 14 | 10.63 |

Note: facility 14 is virtual blood bank

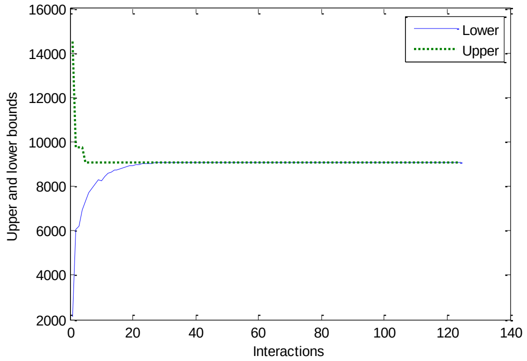


**Figure 2. Convergence of Upper and Lower Bounds Derived from LR Algorithm**

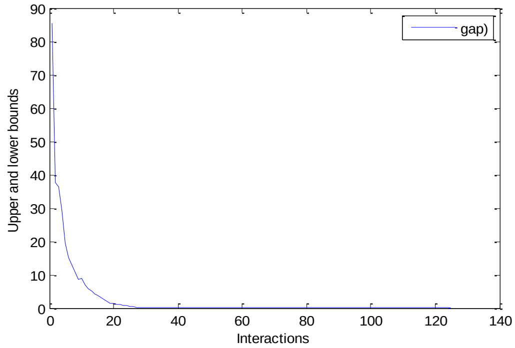


**Figure 3. Convergence of Relative Gap between Upper and Lower Bounds Derived from LR algorithm**

Parameter values for the Lagrangian relaxation are set as follows: the maximum number of iterations $N$=500, optimal tolerance $\gamma = 0.01$, initial Lagrange multiplier $\lambda_{ij}^0 = h_i \overline{d} / 10^4$, and $\overline{d}$ is the mean distance between all blood banks and demand points[13]. The algorithm results and performance are summarized in Table 1. Convergence of LR algorithm are showed in Fig.2 and Fig.3(take test of $R = 3$, $l = 4$ for an example).The experimental results show that, the relative percentage differences between upper bounds and lower bounds are mainly less than 2%. The calculation time is less than 15 seconds. Therefore, the LR algorithm presented in this paper has a good performance. Moreover, we also notice that the maximum reassignment level R does not obviously affect the optimal facility locations in our test instances, which is the same as the research conclusion of RLP in literature [13] and [14].

## 6. Conclusions

Facility networks have the risk of disruptions. Despite the disruption phenomenon doesn't happen very often, but once occurs, the damage is tremendous. Because blood bank location is a strategic decision that cannot be changed in a short time, it is vitally important to consider the non-complete reliability of facility in designing the network.

In this paper, the RPMP of new blood banks in post-disaster reconstruction areas in China was developed, and was formulated as a mixed integer nonlinear programming model. Then, a linearization technique was applied to convert the model, and we developed a LR algorithm. Test results show that LR algorithm performs satisfactorily.

This paper does not consider the protective measures effect on failure probability. Functional relations between protective measures and failure probabilities an be established for further research to study the RLP of new blood banks under various protective strategies. We can also make further researches to optimal decision rules in dynamic environment, considering the frequency and duration of facilitiy disruption. In addition, we can incorporate reliability into integrated decision-making problems, such as Location-Routing Problem (LRP), Location-Routing-Inventory Problem(LRIP), etc. Finally, the RPMP is NP-hard problem. The problems will become more complicated when these problems extended, so the more efficient algorithms will become a study emphasis in further research.

 

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