

Application and Simulation of Distribution Function in Grape Drying Process

Cong Wang¹

¹*Fisher College of Business, The Ohio State University, Columbus city, Ohio State, 43210, the United States of America
cwwangcong@hotmail.com*

Abstract

In order to investigate the influence factors of Weibull distribution function and its application in drying, the drying process of grape slices in different microwave vacuum drying power (330, 460, 590W) and vacuum degree (-50, -70, -90kPa) was taken as the research object, and the Weibull distribution function was used to simulate and analyze its drying kinetic curves, the results showed that the Weibull distribution function was able to simulate microwave vacuum drying process of grape slice well: scaling parameter was related with microwave power and vacuum degree, and decreased with the rise of them; the influence of drying variable on the form parameter β was relatively low. The estimated effective moisture diffusivity coefficient in the dry process was obtained by calculating, its value increased with the rise of microwave power and vacuum degree in the scope from 3.45078×10^{-7} to $6.74613 \times 10^{-7} \text{m}^2/\text{s}$; through Arrhenius equation, the activation energy of drying was calculated respectively 1.34701, 1.49099 and 1.57108 W/g under the vacuum degree of -50, -70 and -90kPa. This study has provided the technical basis for the application of Weibull distribution function in microwave vacuum drying technology of grape slices.

Keywords: *Grape; Drying; Weibull distribution function; Moisture effective diffusion coefficient*

1. Introduction

Grape is one of people's most favorite fruit; it is rich in nutrient element and also sugar and water, has a very strong respiratory intensity and is susceptible to be invaded by the microorganism to spoil. So in a few weeks after the grape harvest, to timely consume and process it is the main measure to reduce economic losses. In the main planting area of the whole world's grape, drying the fresh grapes into raisin is the main processing method^[1]. The grape's drying process is related to the efficiency of heat and mass transfer, energy consumption and product quality and other important indicator, so the effective prediction, control and optimization on the drying process has important significance. In order to study the drying process, the paper used Weibull distribution function. Weibull distribution function has a good applicability and compatibility, and is widely used in materials science, pharmacology and thermodynamics and other fields [2-6]. In recent years, some researchers have introduced the Weibull function into the drying dynamics, and also have made some research progress. This paper focused on the influences of drying method, drying temperature and pretreatment of blanching on drying kinetics of grapes, and Weibull function was used to simulate the drying process, to explore the influencing factors of each parameter in Weibull distribution function, to expound its physical meanings in the drying process, to provide the basis for prediction and control of drying and control of grape, also provide theoretical support for the extensive use of Weibull distribution function in drying and process.

2. Materials and Methods

2.1. Instrument and Equipment

ESWIR dryer (model SAK- W04805), temperature controlled drum wind dryer (model 6WSHGJ- 40), electronic balance (model BS- 30KA), Adam moisture test apparatus (model PMB53) and vernier caliper (model XL. 0 - 200).

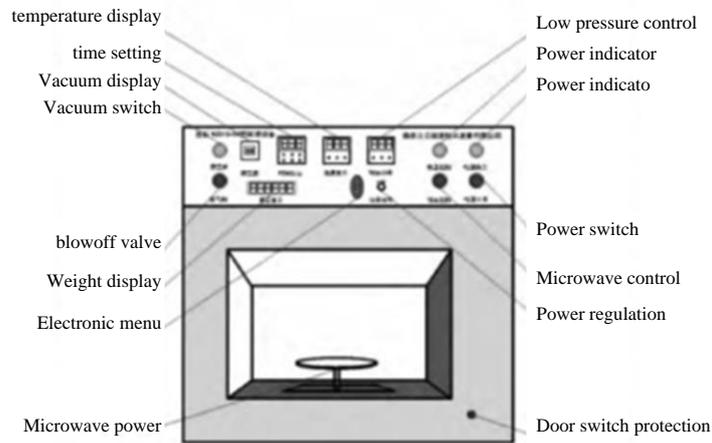


Figure 1. ESWIR Drying Equipment Figure

2.2. Materials

The raw material used for the experiment was Thompson seedless grapes and purchased from Beijing local open fair, the average diameter was (1.2 ± 0.15) cm, the average quality was (3.20 ± 0.17) g, the water content of moist basis was $79.78\% \pm 0.5\%$ (used the method of drying to measure, dry for 24h in 105°C). Before the experiment, The grapes were kept in the refrigerator of $(4 \pm 1)^\circ\text{C}$.

2.3. Drying Method of Grape

Chose the grapes of uniform size as experimental raw materials, used scissors to cut the grapes into a single grain and remained fruit stem. There were two kinds of grape drying methods: the first one, the grapes weren't processed, and were sent to air-jet impingement drying machine (see the literature [12] for the testing equipment) to dry, the temperature of drying medium and relative humidity were respectively 50°C , 11.6%; 55°C , 10.1%; 60°C , 7.4% and 65°C , 5.6%, and the constant wind speed was 15 m/s; the second one, in order to speed the drying rate, restrain browning, according to the pre-stage test, implemented the blanching pretreatment of superheated vapor, the handling time were respectively 30, 60, 90, 120 s, after processing, dried it in air-jet impingement drying machine, temperature of drying medium and relative humidity were 65°C , 5.6 %; timed from drying to measure the quality of grapes until the moisture content of grape moist basis was reduced to 20% and stopped the experiment, repeated 3 times for the experiment of in each group.

Table 1. Experimental Conditions and Parameter Setting for Grapes Drying

No.	Processing method	Temperature/t
1	Temperature controlled air-blast	50
2	Temperature controlled air-blast	60
3	Temperature controlled air-blast	70
4	Temperature controlled infrared	50
5	Temperature controlled infrared	60
6	Temperature controlled infrared	70

2.4. Calculation Method of Drying Parameters

Drying curve in the process of grape drying used the curve of moisture ratio (MR) with the change of drying time. MR is used to indicate the moisture content of the material under certain conditions and different drying time, which can be used to reflect the speed of drying rate of the material. The moisture ratio MR of grape in different drying time was calculated by formula (1) [8].

$$MR = \frac{M_t - M_e}{M_0 - M_e} \quad (1)$$

In the formula, M_0 is moisture content of the initial dry basis ($\text{g} \cdot \text{g}^{-1}$); M_e is moisture content of the dry basis at equilibrium ($\text{g} \cdot \text{g}^{-1}$); M_t is moisture content of dry basis at any drying time t ($\text{g} \cdot \text{g}^{-1}$); M_e is very small relative to M_t and M_0 , which can be ignored in engineering application. So the calculation of moisture ratio MR of material can adopt the simplified formula (2).

$$MR = \frac{M_t}{M_0} \quad (2)$$

The moisture content of dry basis was calculated by formula (3)

$$MR = \frac{W_t - G}{G} \quad (3)$$

In the formula, W_t is the gross mass at any drying time t ; G is the quality of dry matter (g). The drying rate (DR) was calculated by formula (4) [11].

$$DR = \frac{M_{t1} - M_{t2}}{t_1 - t_2} \quad (4)$$

In the formula M_{t1} is moisture content of drying base ($\text{g} \cdot \text{g}^{-1}$) at t_1 ; M_{t2} is moisture content of drying base ($\text{g} \cdot \text{g}^{-1}$) at t_2 .

2.5. Data Processing and Model Analysis

Weibull distribution function was represented by formula (5)

$$MR = \exp\left[-\left(\frac{t}{\alpha}\right)^\beta\right] \quad (5)$$

In the formula, α and β are empirical constants relevant to drying medium, the moisture ratio of grapes at the moment of t is determined by scaling parameter α and form parameter β .

Adopted SPSS 16.0 data analysis software for data processing, and used nonlinear regression analysis to match the mathematical model equation and the experimental data, the fitting degree of mathematical model was evaluated by determination coefficient R^2 , root-mean-square error RMSE and sum of deviation square χ^2 , the larger R^2 was, the smaller the value of RMSE and χ^2 was, and the better was the fitting degree. See the calculation formula in (6) and (7), (8) [13].

$$R^2 = 1 - \frac{\sum_{i=1}^N (MR_{pre,i} - MR_{exp,i})^2}{\sum_{i=1}^N (MR_{pre,i} - MR_{exp,i})^2} \quad (6)$$

$$RMSE = \left[\frac{1}{N} \sum_{i=1}^N (MR_{pre,i} - MR_{exp,i})^2 \right]^{\frac{1}{2}} \quad (7)$$

$$\chi^2 = \frac{\sum_{i=1}^N (MR_{pre,i} - MR_{exp,i})^2}{N - n} \quad (8)$$

In the formula, $MR_{exp,i}$ is the actual measured moisture ratio in drying experiment, $MR_{pre,i}$ is the predicted moisture ratio by using model; N is the group number of measured data in experiment; n is the number of parameter in function.

The whole drying process belongs to the falling rate drying, the effective moisture diffusivity can be calculated by Fick's second law [14]. The effective moisture diffusivity of materials D_{eff} can be by calculated formula (9).

$$\ln MR = \ln \frac{8}{\pi^2} - \frac{\pi^2 D_{eff} t}{L^2} \quad (9)$$

In the formula, D_{eff} is the effective moisture diffusivity of materials in drying ($m^2 \cdot s^{-1}$); L is the average diameter of grape, and its value is $1.8 \times 10^{-2} m$, t is the drying time (s).

$$\ln D_{eff} = \ln D_0 - \frac{E_a}{R(T + 273.15)} \quad (10)$$

In the formula, D_0 is the frequency factor of effective diffusion coefficient, is a constant value ($m^2 \cdot s^{-1}$); E_a is drying activation energy of materials ($kJ \cdot mol^{-1}$); R is molar gas constant and its value is $8.314 (J \cdot mol^{-1} \cdot K^{-1})$; T is drying temperature of materials ($^{\circ}C$).

3. Results and Analysis

3.1. The Influence of Different Temperature on Drying Rate of Grapes under the Same Drying Method

The drying moisture ratio curves and drying rate curves of temperature controlled air blast drying and temperature controlled infrared drying showed the similar rule, the time that grapes needed to the targeted moisture content gradually decreased with the increase of drying temperature, and to raise the drying temperature could effectively shorten the drying time of grapes. The whole process of grapes drying belonged to the process of falling rate, the higher the drying temperature was, the more quickly the drying rate

dropped. The drying rate gradually decreased with the prolonging of drying time, see Figure 2.

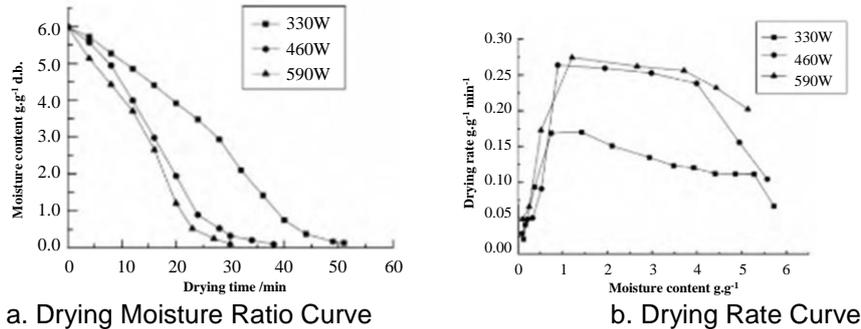


Figure 2. Drying Curve of Grape in Different Drying Methods and Drying Temperature

3.2. The Influence of Drying Method on Grape Drying Rate

Under the same temperature condition, the grapes drying rate of temperature controlled infrared drying was larger that of the temperature controlled air blast drying. The trend of grape drying rate in the whole drying process was in a significant falling-rate period, showing the rule that higher the initial rate of drying was, the more obvious falling rate was Figure 1.

3.3. Use Weibull Distribution Function to Simulate the Drying Curve

Adopted Weibull distribution function to simulate the grape drying curve after different drying methods and temperature treatment. The results showed that the determination coefficient R^2 was at the interval 0.994~0.999, the root-mean-square error RMSE was at 0.0085~0.0242, sum of deviation square χ^2 was at $0.965 \times 10^{-4} \sim 7.791 \times 10^{-4}$, see Table 2. This showed that the Weibull distribution function can be used well to simulate the drying curves of grapes with different drying methods and different drying conditions, to provide the basic conditions for the further analysis of the drying process by using Weibull distribution function.

Table 2. Parameter Values of Angelicae Sinensis Radix Drying Curves Modeled by Weibull Distribution Function

Dry processing methods	Model parameter		R^2	RMSE	χ^2
	a/min	β			
1	869.4	0.700	0.997	0.0135	1.997×10^{-4}
2	453.3	0.878	0.997	0.0128	2.724×10^{-4}
3	263.9	0.866	0.998	0.0084	0.945×10^{-4}
4	506.7	0.751	0.994	0.0214	5.075×10^{-4}
5	132.6	0.732	0.993	0.0253	7.792×10^{-4}
6	61.1	0.756	0.997	0.0137	2.111×10^{-4}

3.3.1. Physical Meaning and Influence Factors of Parameter: Under the same drying method, with the drying temperature increased from 50°C to 70°C, the scale parameter under the temperature controlled air blast drying was reduced from 968.4 min to 264.9 min, and the scale parameter under the temperature controlled infrared drying decreased from 507.1 min to 60 min Table 2. It can be seen that the scale parameter α was affected by temperature, the temperature increased, α decreased, and it showed that the elevated temperature can shorten the drying time and improve the drying efficiency. Moreover,

under the same drying temperature, the corresponding α was also different under the different drying methods. Under the temperature of 50, 60, 70 °C the parameter value α under the method of temperature controlled air blast drying was larger than that of the temperature controlled infrared drying, which was related to the special the heat and quality transferring law of infrared drying: infrared drying belongs to the radiation dryness, part of energy absorbed by the materials is converted into vibrated and rotational kinetic energy, to generate heat from the inside, so infrared heat is the concurrent heating process of the surface and inner, the direction of transfer of heat and mass is identical, to greatly speed up the drying speed; while the method of temperature controlled air blast drying heats the materials from outside to inside only by heat transfer, the moisture is diffusing from the inside to the outside, the direction of transfer of heat and mass is opposite. Therefore, under the same temperature, drying rate of temperature controlled air blast drying is lower than that of temperature controlled infrared drying.

For the same drying material, the drying rate constant α is related to the drying temperature and the drying method. Under the same drying method, the drying rate constant α decreases with the raising temperature of drying.

3.3.2. Physical Meaning and Influence Factors of Parameter β : From the Table 2 we can see that the form parameter β of temperature controlled air blast drying and temperature controlled infrared drying were all in 0.3~1, and showed the characteristics of falling rate drying, which was consistent with the conclusion of figure 1B. The range of form parameter β of temperature controlled air blast drying was in 0.800~0.978, the range of form parameter β of temperature controlled infrared drying was in 0.713~0.851, without significant change, showing that drying temperatures has little influence on it. It is reported [8] that form parameter β is a parameter relevant to the form of drying materials, will make a difference with the change of materials under the same drying conditions. Similarly, if the change of material state is small during the process of drying, the effect on the form parameter β is also small. In the drying process of grapes, there is no marked change in the characteristics of grape under different drying methods, so the form parameter β doesn't have remarkable difference.

3.3.3. The Effective Diffusion Coefficient of Water in the Drying Process of Grape: The effective diffusion coefficient of water is a parameter to represent the rate of water migration in the drying process. From the formula (9) it can be seen that the natural logarithm of moisture ratio $\ln MR$ in the drying process has a linear relationship with drying time t . Calculate the effective diffusion coefficient of water D_{eff} through linear regression, see table 3. The effective diffusion coefficient of D_{eff} was calculated by linear regression. The results showed that the effective diffusion coefficient of grape moisture increased with the increase of drying temperature. When the drying temperature of temperature controlled air blast drying and temperature controlled infrared drying were respectively 50, 60, 70 °C, the effective diffusion coefficients of grape moisture were respectively 0.425×10^{-9} , 1.251×10^{-9} , 1.881×10^{-9} , 0.811×10^{-9} , 1.544×10^{-9} and $2.260 \times 10^{-9} \text{ m}^2 \cdot \text{s}^{-1}$. Thus, the effect of temperature on the effective diffusion coefficient of grape moisture is greater, the higher the temperature is, the greater the diffusion coefficient is. Under the same drying temperature condition, the effective diffusion coefficient of water of different drying methods is also different, and the temperature controlled air blast drying is less than temperature controlled infrared drying. The results are related with mechanisms of heat and mass transfer of different drying methods, the temperature controlled air blast drying is to heat the materials from the inside to the outside, opposite to the direction of moisture diffusion, while temperature controlled infrared drying is to heat the inside and outside of materials, identical with the direction of moisture diffusion,

so the effective diffusion coefficient of water of temperature controlled air blast drying is less than that of temperature controlled infrared drying.

Table 3. Moisture Effective Diffusion Coefficients of Angelicae Sinensis Radix Drying

The treatment method of drying	Fitting formula linear regression	R ²	D _{eff} /m ² •s ⁻¹
1	lnMR=-0.0466t-0.1797	0.9795	0.425×10 ⁻⁹
2	lnMR=-0.1371t-0.0718	0.9947	1.251×10 ⁻⁹
3	lnMR=-0.2061t-0.0814	0.9957	1.881×10 ⁻⁹
4	lnMR=-0.0889t-0.1983	0.9749	0.811×10 ⁻⁹
5	lnMR=-0.1691t-0.8181	0.9079	1.544×10 ⁻⁹
6	lnMR=-0.2476t-1.7938	0.9321	2.260×10 ⁻⁹

3.4. Drying Activation Energy in Drying Process of Grape

Drying activation energy refers to the activated energy that materials need to eliminate one mole of moisture in the drying process. Through drying activation energy the difficulty level of drying for materials can be seen and the energy consumption for drying can be estimated. The larger drying activation energy shows that it is much more difficult to dry and will waste greater energy. From the formula (10) to know the natural logarithm of effective diffusion coefficient of water lnD_{eff} and 1/(T+273.15) show a linear relationship, and its slope is (-E_a/R), see Figure 3.

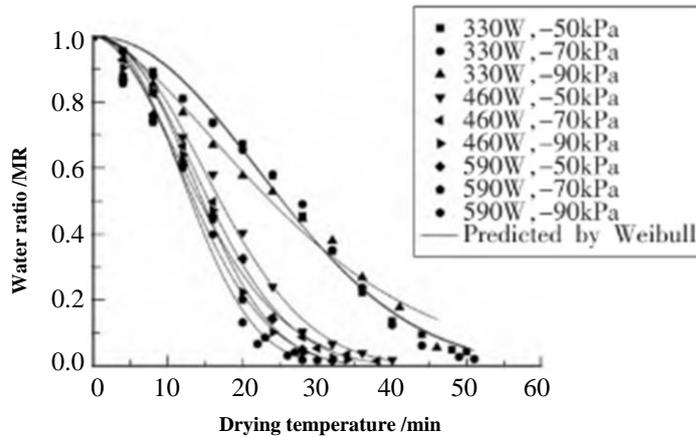


Figure 3. Relation Curves of Moisture Effective Diffusion Coefficients and Drying Temperatures

From the linear regression equation in Figure 2, the drying activation energy of the grapes after the method of temperature controlled air blast drying and temperature controlled infrared drying were calculated respectively 68.82, 29.60 kJ·mol⁻¹, and it showed that temperature controlled infrared drying was much easier than temperature controlled air blast drying in removing material moisture. Generally speaking, activation energy is related with variety, internal composition and structural states [8], and the different drying methods may change the internal structural state and organizations of grapes in the drying process, to indirectly affect drying activation energy. This showed that the drying method had certain effects on the drying activation energy.

4. Calculation of Moisture Effective Diffusion Coefficient of Grape

Moisture transfer in the process of drying is a very complicated process, which may be the coefficient results of many phenomena including the capillary flow, vapor diffusion and liquid diffusion. In order to synthetically consider these factors, using the testing method to measure and calculate the moisture effective diffusion coefficient has great significance for describing the characteristics and optimizing drying technology in drying process. As a common Fick's second law, its defects are that the whole process of drying must be in the process of falling-rate of drying, but the application characteristic of Weibull distribution function in the drying process is that it could calculate moisture effective diffusivity D_{cal} and ignore the characteristics of moisture migration, its calculation formula is listed as follows:

$$D_{cal} = \frac{r^2}{\alpha} \quad (11)$$

In the formula, D_{cal} --the estimated moisture effective diffusion coefficient, m^2/s ; r —— equivalent radius of grape dimension. Its value is about 0.025 m in this experiment; α —— scale parameter min in Weibull distribution function

$$D_{eff} = \frac{D_{cal}}{R_g} \quad (12)$$

In the formula, D_{eff} —— — moisture effective diffusion coefficient in the drying process, m^2/s ; R_g —— a constant related with geometric dimensioning

The results of moisture effective diffusion coefficient of grape slices under different drying conditions was shown in Table 4. From Table 4, it can be seen that the estimated moisture effective diffusion coefficient D_{cal} in drying process varied among $3.45078 \times 10^{-7} \sim 6.74613 \times 10^{-7} m^2/s$. In addition, it can be summarized that the estimated moisture effective diffusion coefficient increased with the increase of drying power and vacuum degree. By using Fick's second law to calculate the moisture effective diffusion coefficient and its value can be seen in Table 2, and it can be concluded that moisture effective diffusion coefficient of grape slices in drying process increased with the increase of microwave power and vacuum degree. It can also be found that the value of geometric parameter R_g changed with the effective diffusion coefficient, but the research of Marabi finds that the value of R_g has nothing to do with moisture effective diffusion coefficient. The reason for such difference is that the internal state and organizational structure changed when dry in microwave vacuum drying.

Table 4. Moisture Effective Diffusion Coefficients of Kiwifruit Slices under Different Drying Conditions

Microwave power	Vacuum degree	Estimated moisture effective diffusion coefficient	DeffMoisture effective diffusion coefficient	Geometric parameter
330	-50	3.45078×10^{-7}	2.24204×10^{-8}	15.39123
330	-70	3.45430×10^{-7}	3.15924×10^{-8}	10.93394
330	-90	3.46948×10^{-7}	4.11040×10^{-8}	8.440708
460	-50	5.13420×10^{-7}	2.47983×10^{-8}	20.70387
460	-70	5.63688×10^{-7}	3.80467×10^{-8}	14.81565
460	-90	6.05170×10^{-7}	4.48408×10^{-8}	13.49601
590	-50	5.97053×10^{-7}	2.75159×10^{-8}	21.69842
590	-70	6.41860×10^{-7}	4.31423×10^{-8}	14.87775
590	-90	6.74613×10^{-7}	4.95966×10^{-8}	13.60199

5. Calculation of Drying Activation Energy

Drying activation energy refers to the activated energy that materials need to eliminate one mole of water in the drying process. The larger drying activation energy shows that it is much more difficult to dry. The relation between moisture effective diffusion coefficient of materials and microwave power can be represented through Arrhenius equation as:

$$D_{eff} = D_0 \exp\left[\frac{E_a \cdot m}{p}\right] \quad (13)$$

In the formula, D_0 —diffusion coefficient in the material, a constant, m^2/s ; E_a —drying activation energy of materials, kJ/mol ; m — — quality of material, g ; p — — microwave power, W .

Put the formula(12)into the formula(13)into, it can be reached

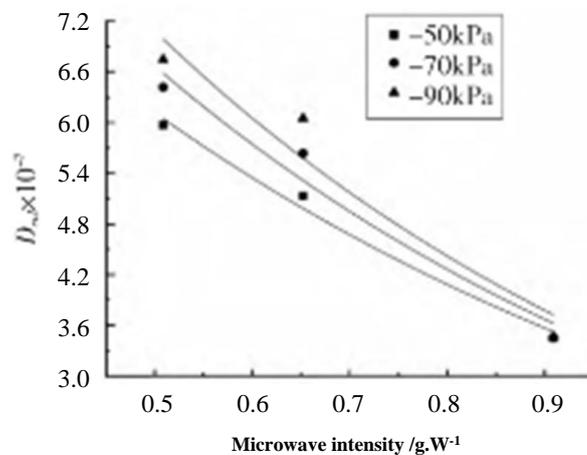


Figure 4. The Relation Curves of Moisture Effective Diffusion Coefficients and mMicrowave Intensity

$$D_{cal} = R_g D_0 \exp\left[\frac{E_a \cdot m}{p}\right] \quad (14)$$

Its drying activation energy were 1.34701, 1.49099 and 1.57108W/g under the vacuum degree of -50, -70 and -90kPa.

6. Conclusion

1. Weibull distribution function can well describe the vacuum drying process of grapes.
2. Scale parameter α was related with both microwave power and vacuum degree, and scale parameter α decreased with the increase of microwave power and vacuum degree; while scale parameter β changed little with the change of drying parameter.
3. The value of estimated moisture effective diffusion coefficient of grapes in the process of microwave vacuum drying was in $3.45078 \times 10^{-7} \sim 6.74613 \times 10^{-7} m^2/s$ by using scale parameter α ; through Arrhenius its drying activation energy were 1.34701, 1.49099 and 1.57108W/g under the vacuum degree of -50, -70 and -90kPa.

References

- [1] D. R. Pangavhane and R. L. Sawhney, "Review of research and development work on solar dryers for grape drying", *Energy Conversion and Management*, vol. 43, no. 1, (2002), pp. 45–61.
- [2] W. Weibull, "A staistical distribution function of wide applicability", *Appl Mech*, vol. 18, no. 3, (1951), pp. 293.
- [3] O. Corzo, N. Bracho and A. Pereira, "Weibull distribution for modeling air drying of coroba slices", *Food Sci Technol*, vol. 41, no. 10, (2008), pp. 2023.
- [4] M. Miranda, A. V.Gálvez and P. García, "Effect of temperature on structural properties of Aloe vera (Aloe barbadensis Miller) gel and Weibull distribution for modelling drying process", *Food Bioprod Process*, vol. 88, no. 2, (2010), pp. 138.
- [5] O. Corzo, N. Bracho and C. Alvarez, "Weibull model for thin-layer drying of mango slices at different maturity stages", *Journal of Food Processing and Preservation*, vol. 34, no. 6, (2010), pp. 993-1008.
- [6] M. Miranda, A. V. Gálvez and P. García, "Effect of temperature on structural properties of Aloe vera (Aloe barbadensis Miller) gel and Weibull distribution for modelling drying process", *Food and Bioproducts Processing*, vol. 88, no. 2, (2010), pp. 138-144.
- [7] M. Bantle, K. Kolsaker and T. M. Eikevik, "Modification of the weibull distribution for modeling atmospheric freeze-drying of food", *Drying Technology*, vol. 29, no. 10, (2011), pp. 1161-1169.
- [8] Y. Geng, J. Chen, R. Fu, G. Bao and K. Pahlavan, "Enlighten wearable physiological monitoring systems: On-body rf characteristics based human motion classification using a support vector machine", *IEEE transactions on mobile computing*, vol. 1, no. 1, (2015) April, pp. 1-15.
- [9] Z. Lv, A. Halawani and S. Feng, "Multimodal hand and foot gesture interaction for handheld devices", *ACM Transactions on Multimedia Computing, Communications, and Applications (TOMM)*, vol. 11, no. 1, (2014), pp. 10.
- [10] G. Liu, Y. Geng and K. Pahlavan, "Effects of calibration RFID tags on performance of inertial navigation in indoor environment", 2015 International Conference on Computing, Networking and Communications (ICNC), (2015) Febuary.
- [11] J. He, Y. Geng, Y. Wan, S. Li and K. Pahlavan, "A cyber physical test-bed for virtualization of RF access environment for body sensor network", *IEEE Sensor Journal*, vol. 13, no. 10, (2013) October, pp. 3826-3836.
- [12] W. Huang and Y. Geng, "Identification Method of Attack Path Based on Immune Intrusion Detection", *Journal of Networks*, vol. 9, no. 4, (2014) January, pp. 964-971.

Author



Cong Wang, she is currently an undergraduate student in Fisher College of Business at the Ohio State University in the U.S.. Her research interest is mainly in the area of Mathematics and Finance. She has published several research papers in scholarly journals in above research areas.