

Properties of Monte Carlo and Its Application to Risk Management

Li Rui-mei

*Shaanxi University of Technology School of Management, Han thong, Shaanxi,
China, 723001
huweiscj@126.com*

Abstract

The complexity and turbulence of a project may lead to its unpredictability, so risk management is increasingly important. This paper focuses on the application of Monte Carlo method in project management as follow: first, deals with the estimated scale with Wide band Delphi method and gets the interval of prediction after making repeated samples in line with the generally adopted probability models; then figures out the workloads according to the management abilities and historical date; finally obtains the progress of the project with a method combining COCOMO II method and then improves the ability to predict and control management activities with uncertainties.

Keywords: *Monte Carlo method, Project management, Wide band Delphi method, COCOMO II method, Normal distribution*

1. Introduction

Modern projects are always characterized by high technological risks, large investments and long continuation. With the growth of uncertain factors, projects face more and more risks, thus the risk management [1-6] is playing an increasingly important role. In project management, how to deal with the collected date properly by taking various uncertainties into consideration so as to improve the accuracy of evaluation is a vital problem whose solution has certain significance. So evaluating the uncertainties in project progress is a crucial aim in modern risk management.

This paper focuses on the introduction and application of Monte Carlo method, which enables people to imitate and balance the influence of different variables on the target more scientifically and thus improve their abilities to deal with the problems. Firstly dealing with the scale of evaluation, the interval of prediction has ben gotten after making repeated samples in line with the generally adopted probability models, then the workloads has been figured out according to the management abilities and historical date; finally combining with COCOMO II method [7] the progress of the project has been obtained.

2. Monte Carlo Method

The modern version of the Monte Carlo method was invented in the late 1940s by Stanislaw Ulam, while he was working on nuclear weapon projects at the Los Alamos National Laboratory. It was named, by Nicholas Metropolis, after the Monte Carlo Casino, where Ulam's uncle often gambled [8]. Immediately after Ulam's breakthrough, John von Neumann understood its importance and programmed the ENIAC computer to carry out Monte Carlo calculations. Monte Carlo methods are mainly used in three distinct problems: optimization, numerical integration and generation of samples from a probability.

2.1 Introduction of Monte Carlo Method

Monte Carlo method [9-12] is a highly efficient calculation method with the basic idea being treating the frequency of an event approximately as its probability. It can make some parameters of an event coincide with the values to be calculated by experientially summarized probability model; then help to get the estimated statistical numbers of such parameters after repeated imitation experiments; finally regards the estimated figures as the approximate target values. The appearance of computers and advanced processing language (such as MATLAB) in the last two decades makes rapid and ample imitation of such experiments possible.

Among all the quantitative analyses Monte Carlo method is specially suggested by American Project Management Institute (PM I). As is known to all, any project management must face numerous uncertainties and there exist some unicity and particularity to certain degree. So when faced with various changing factors(from both inside and outside, active and passive, foreseeable and unforeseeable),we need to collect plentiful experiences and refer to organization abilities to make more reasonable and effective measures for next step after careful consideration of all the environmental influences so as to improve the possibility of success. Monte Carlo method is one of the best tools for a more scientific and effective management as well as better ability to foresee and control problems.

Monte Carlo method applied to approximating the value of π . After placing 30000 random points, the estimate for π is within 0.07% of the actual value. This happens with an approximate probability of 20%.

Monte Carlo methods vary, but tend to follow a particular pattern:

- i .Define a domain of possible inputs.
- ii .Generate inputs randomly from a probability distribution over the domain.
- iii .Perform a deterministic computation on the inputs.
- iv .Aggregate the results.

For example, consider a circle inscribed in a unit square. Given that the circle and the square have a ratio of areas that is $\pi/4$, the value of π can be approximated using a Monte Carlo method: [13]

- i .Draw a square on the ground, then inscribe a circle within it.
- ii .Uniformly scatter some objects of uniform size (grains of rice or sand) over the square.
- iii .Count the number of objects inside the circle and the total number of objects.
- iv .The ratio of the two counts is an estimate of the ratio of the two areas, which is $\pi/4$. Multiply the result by 4 to estimate π .

In this procedure the domain of inputs is the square that circumscribes our circle. We generate random inputs by scattering grains over the square then perform a computation on each input (test whether it falls within the circle). Finally, we aggregate the results to obtain our final result, the approximation of π .

If the grains are not uniformly distributed, then our approximation will be poor. Secondly, there should be a large number of inputs. The approximation is generally poor if only a few grains are randomly dropped into the whole square. On average, the approximation improves as more grains are dropped [14].

2.2 Definitions of Monte Carlo Method

There is no consensus on how Monte Carlo should be defined. For example, Ripley [15] defines most probabilistic modeling as stochastic simulation, with Monte Carlo being reserved for Monte Carlo integration and Monte Carlo statistical tests. Sawilowsky [16] distinguishes between a simulation, a Monte Carlo method, and a Monte Carlo simulation: a simulation is a fictitious representation of reality, a

Monte Carlo method is a technique that can be used to solve a mathematical or statistical problem, and a Monte Carlo simulation uses repeated sampling to determine the properties of some phenomenon (or behavior). Examples:

Simulation: Drawing one pseudo-random uniform variable from the interval (0, 1) can be used to simulate the tossing of a coin: If the value is less than or equal to 0.50 designate the outcome as heads, but if the value is greater than 0.50 designate the outcome as tails. This is a simulation, but not a Monte Carlo simulation.

Monte Carlo method: The area of an irregular figure inscribed in a unit square can be determined by throwing darts at the square and computing the ratio of hits within the irregular figure to the total number of darts thrown. This is a Monte Carlo method of determining area, but not a simulation.

Monte Carlo simulation: Drawing a large number of pseudo-random uniform variables from the interval (0, 1), and assigning values less than or equal to 0.50 as heads and greater than 0.50 as tails, is a Monte Carlo simulation of the behavior of repeatedly tossing a coin.

Kalos and Whitlock [7] point out that such distinction are not always easy to maintain. For example, the emission of radiation from atoms is a natural stochastic process. It can be simulated directly, or its average behavior can be described by stochastic equations that can themselves be solved using Monte Carlo methods. "Indeed, the same computer code can be viewed simultaneously as a 'natural simulation' or as a solution of the equations by natural sampling."

2.3 Calculation of Monte Carlo Method

Monte Carlo methods are useful in many areas of computational mathematics, where a "lucky choice" can find the correct result. A classic example is Rabin's algorithm for primality testing: for any n that is not prime, a random x has at least a 75% chance of proving that n is not prime. Hence, if n is not prime, but x says that it might be, we have observed at most a 1-in-4 event. If 10 different random x say that " n is probably prime" when it is not, we have observed a one-in-a-million event. In general a Monte Carlo algorithm of this kind produces one correct answer with a guarantee n is composite, and x proves it so, but another one without, but with a guarantee of not getting this answer when it is wrong too often—in this case at most 25% of the time. See also Las Vegas algorithm for a related, but different, idea.

In general, Monte Carlo methods are used in mathematics to solve various problems by generating suitable random numbers and observing that fraction of the numbers that obeys some property or properties. The method is useful for obtaining numerical solutions to problems too complicated to solve analytically. The most common application of the Monte Carlo method is Monte Carlo integration.

i . Monte Carlo and random numbers

Monte Carlo simulation methods do not always require truly random numbers to be useful — while for some applications, such as primality testing, unpredictability is vital.[22] Many of the most useful techniques use deterministic, pseudorandom sequences, making it easy to test and re-run simulations. The only quality usually necessary to make good simulations is for the pseudo-random sequence to appear "random enough" in a certain sense.

What this means depends on the application, but typically they should pass a series of statistical tests. Testing that the numbers are uniformly or follow another desired distribution when a large enough number of elements of the sequence are considered is one of the simplest and most common ones.

Sawilowsky lists the characteristics of a high quality Monte Carlo simulation: [17, 18]

①the (pseudo-random) number generator has certain characteristics (*e.g.*, a long

“period” before the sequence repeats)

② the (pseudo-random) number generator produces values that pass tests for randomness

③ there are enough samples to ensure accurate results

④ the proper sampling technique is used

⑤ the algorithm used is valid for what is being modeled

⑥ It simulates the phenomenon in question.

Pseudo-random number sampling algorithms are used to transform uniformly distributed pseudo-random numbers into numbers that are distributed according to a given probability distribution.

Low-discrepancy sequences are often used instead of random sampling from a space as they ensure even coverage and normally have a faster order of convergence than Monte Carlo simulations using random or pseudorandom sequences. Methods based on their use are called quasi-Monte Carlo methods.

ii. Calculation steps of Monte Carlo method in project management

① For each activity, the minimum value, maximum value and the most estimate data are input, and an appropriate prior distributions for the model selection;

② The computer according to the input, using some rules are given, rapid implementation of full random sampling;

③ The random sampling of the data necessary mathematical calculations, the results is obtained;

④ The calculated results were treated statistically; find the minimum value, maximum value and the mathematical expectation and standard deviation;

⑤ According to the statistical data obtained, let the computer automatically generates the probability distribution curve and cumulative probability curve (usually the cumulative probability of normal distribution based on S curve);

⑥ On the basis of project risk analysis of cumulative probability curve.

2.4 Matlab Introduction

Matlab [19-25] was both a powerful computational environment and a programming language that easily handles matrix and complex arithmetic. It was a large software package that has many advanced features built-in, and it had become a standard tool for many working in science or engineering disciplines. Among other things, it would allow easy plotting in both two and three dimensions.

Matlab had two different methods for executing commands: interactive mode and batch mode. In interactive mode, commands are typed (or cut-and-pasted) into the 'command window'. In batch mode, a series of commands were saved in a text file (either using Mat lab's built-in editor, or another text editor such as Emacs) with a '.m' extension. The batch commands in a file were then executed by typing the name of the file at the Matlab command prompt. The advantage to using a '.m' file was that you could make small changes to your code (even in different Matlab sessions) without having to remember and retype the entire set of commands. Also, when using Matlab's built-in editor, there were simple debugging tools that can come in handy when your programs start getting large and complicated. More on writing .m files later.

3. The Basic Steps OF Monte Carlo Method

About the classified software code modules, each expert gives the most likely estimated numbers by Wide Band Delphi method that is obtained by PERT [26] method and Delphi method, then choose a commonly used a priori distribution

model(in this paper a even distribution is adopted: $p(x) = \frac{1}{b-a}$;while a commonly used model is $\beta(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$);

② Take abundant Monte Carlo method random samples by using probability density function of the given model, then calculate the results statistically and get the expected mathematical value and standard deviations;

③ Make computerized probability distribution (such as the normal distribution) curves. When the curves don't correspond with the trend on which they are based, redo step2;

④ Get the sample mean and confidence interval of standard deviations by probability distribution curves.

3.2 Estimating the Workload

① Get the productivity by collecting historical experiences of the project and process ability data of the mobile institute.

② Make Monte Carlo method samples of production scale and productivity in accordance with normal distribution $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}(\frac{x-\mu}{\sigma})^2]$, get the confidence interval when confidence level of the workload is 90 % (significant level $\alpha = 0.10$) (workload =scale/productivity).

3.3 Estimate Progress of the Project

The estimating is done via COCOMO II method with the formula being $TDEV = [C \times (PM_{adjust})^F] \times \frac{SCED\%}{100}$,the adjust workload $PM_{adjust} = PM_{nominal} \times \prod_{i=1}^n EM_i$

Here $PM_{nominal}$ is set to be the estimated workload. All the parameters required are acquired from imitation samples of the historical experiences and actual situation of the project.

4. Analysis and Deduction

4.1 Estimating Scale of the Project

After the estimation meet hosted by project manager, the related staff get the following estimated figures in line with WBD.

Table 1. Scale Estimation Figures

Code Type	Activity Description	Code Scales of Project Software by WBD Method (unit: e Loc)				
		Staff 1	Staff 2	Staff 3	Staff 4	Staff 5
Background Software	RMCM detailed design code	7500	8000	8500	7500	9000
	RALM detailed design code	3000	3500	3800	4000	4000
	RCFG detailed design code	4000	4500	4300	5000	4500
	RTST detailed	3500	3300	3900	3800	3350

Code	design code					
Quantity	RVER detailed design code	6500	6000	5500	6000	5500
	RVME detailed design code	6500	700	7100	6500	6750
	RBOOT detailed design code	3200	2600	3000	3300	2700
	DSP detailed design code	6000	5000	6000	5500	6000
	RMCM unit test code	5500	5100	5750	5800	4800
	RALM unit test code	2000	2300	2200	2500	1700
	RCFG unit test code	2100	2500	2000	2400	1800
	RTST unit test code	3000	3300	2700	3200	2820
	RVER unit test code	2000	1800	2200	1800	1600
	DSP unit test code	2000	2000	2500	1800	2200
Underlying Hardware Code	HDTB board FPGA code	15000	16000	16000	17000	15000
Quantity	HDTB board EPLD code	900	800	1000	800	1000
	HDTB board BSP code	50000	50000	50000	55000	45000

Take the specific design code of Module I (RMCM) as an example The first Monte Carlo method sample is shown in Table 2.

Table 2. Monte Carlo Method Sample

maximal value	minimum value	Adjusting Maximal value	Adjust -ing Minimum value	random (U(0,1) distribution)	number	Analog sampling value
9000	7500	8850	7650	0.8421		8660.5

In Table 2, Adjusting maximal value and minimum value is the residual interval values of both experts estimate the truncated after 10%, in order to lessen the influence of the most optimistic or pessimistic attitude on the results.

Sample each software module 30 times imitatively and experiment 510 times (17*30), then get the results shown in Figure 1.

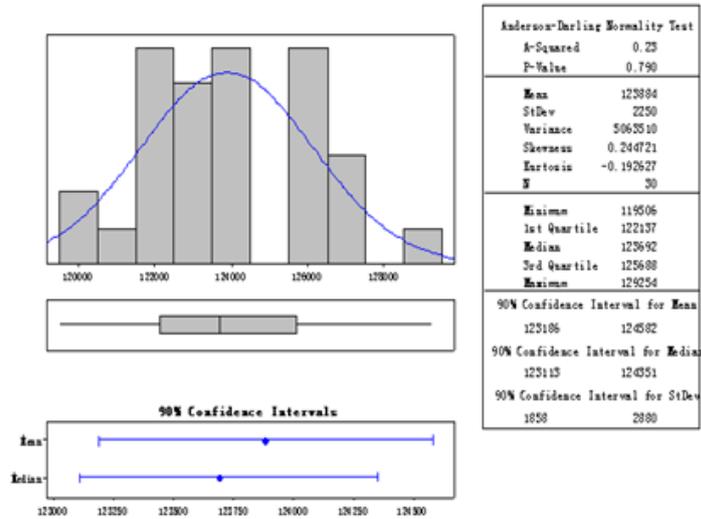


Figure 1. Summary for Estimate Scale

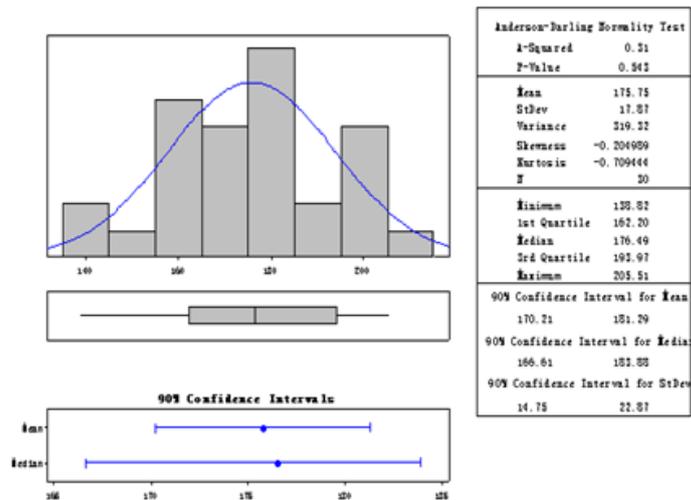


Figure 2. Summary for Workload

In Figure 1, since $P_VALUE = 0.79 > 0.05$, the hypothesis is accepted, and the estimate scale shows a normal distribution. When the confidence level is below 90%, confidence interval of the average of the estimated workload is [123190, 124580], with that of standard deviations being [1860, 2880] (effective lines of code).

4.2 Estimating the Workload

According to the newly finished counter offer report of the project, the workload of each person is 35 per day, namely, 0.77kiloc/person/month. When the main staff remains constant, and a supposed normal distribution $N(0.725, 0.072)$ of the new software productivity is acceptable.

Make Monte Carlo method samples of the scale and productivity respectively 30 times, then get the following figures shown in Figure 2 ($N(123884, 2250)$, workload=scale/productivity, $\alpha = 0.10$):

In Figure 2, since $P_VALUE = 0.543 > 0.05$, the hypothesis is accepted, and the estimate scale shows a normal distribution. When the confidence level is below 90%, confidence interval of the estimated workload is [170, 181] with that of standard deviations being [14.75, 22.87] (person per month).

4.3 Estimate Progress of the Project

Calculate all parameters in the formula COCOMO II method. For example,

$$E = 0.91 + 0.01 \times \sum_{j=1}^5 W_j$$
 in the scale of $W_1 \sim W_5$ ranges in accordance with the situation of the project and its historical experiences as shown in Table 3.

In Table 3, red arrows signify the ranges of $W_1 \sim W_5$. More than 20 factors such as EM (Product factor workload), C (time coefficient), SCED (Schedule Compression or Tensile percentages) are calculated in the same way with Monte Carlo method (by even distribution). Put them into the formula and get the results shown in Figure 3.

In Figure 3, progress of the project is estimated to average 15.13 and 15.39 with standard deviations being 0.35, 0.55. According to normal distribution, when the probability is 90%, progress of the project is less than 15.9 months.

5. Conclusions

① Monte Carlo method is a more realistic and comprehensive imitation of real situation. For example, in risk management, the probability of certain risk is not so definite, thus an imitation of its range is more meaningful;

② To complicated problems and evaluation of big systems, Monte Carlo method—to be understood as the control system's insensibility to us own characters or the change of original parameters). In situations where there are more uncertain and less controllable parameters, Monte Carlo method shows greater advantages over other methods.

③ Shortcomings: To get a high accuracy needs too much calculation and repeated consideration.

④ The core of Monte Carlo method is sampling the probability. So a high expected accuracy can be acquired only when the historical data are as many and as authentic as possible. Otherwise, this method will become a meaningless pure mathematical calculation.

⑤ When programming with Matlab, Statistics toolbox is mainly used, among which normpdf function (Normal distribution probability density function), uni function (continuous uniform distribution), norm function (Normal data for parameter estimation and confidence interval)

Table 3 W1~W5 ranges

Rating factor	VL (very low)	Lowness	Comm.-on	Height	VH (very high)	Particular high
PREC(W1) Precedential	Entirely without precedent (6.20)	Most without precedent (4.96)	Part without precedent (3.72)	Ordinary familiar (2.48)	Most familiar (1.24)	Fully familiar (0.00)
FLEX(W2) Development degree of freedom	Strict (5.07)	occasional loose (4.05)	Part loose (3.04)	General consistency (2.03)	Partial coherence (1.01)	Generic objectives (0.00)
RESL (W3) Factor associ-	Seldom	Some	Regular	Customarily	Majority	Total

ated with risk manage-ement	(20%)	(40%)	(60%)	(75%)	(90%)	(100%)
	(7.07)	(5.65)	(4.24)	(2.83)	(1.41)	(0.00)
TEAM (W4) Team cohe- sion factor	Very difficult communication	Difficult communication	Basic exchanges and cooperation	Most of the cooperative	Highly cooperation	Seamless communication
	(5.48)	(4.38)	(3.29)	(2.19)	(1.10)	(0.00)
PMAT (W5) grade of maturity	SW-CMM1 grade	SW-CMM1 grade	SW-CMM2 grade	SW-CMM3 grade	SW-CMM4 grade	SW-CMM5 grade
	(7.80)	(6.24)	(4.68)	(3.12)	(1.56)	(0.00)

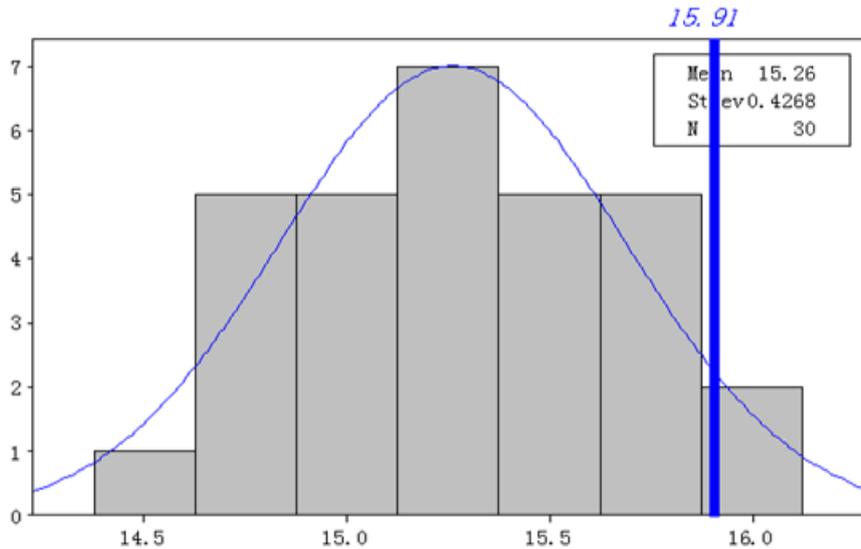


Figure 3. Histogram of Schedule

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Author



Li Rui-mei, Project: the Science and Technology Program of the Education Department of Shaanxi Province Government (number: 14JK1123). Shaanxi University of technology project "Research on multi project risk management of communication enterprise", no. SLGKY14-29). (1978-), male, born in Shaanxi weinan, lecturer, major: Research on risk management and business administration. E-mail: huweiscj@126.com.