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Sale Mode Choice of Product Extended Warranty based on the Service Level

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Abstract

Based on master-slave two-echelon supply chain, set a separate price for product extended warranty, and form both a product reflecting the risk avoidance by the customer and the demand function for product extended warranty. Study how service level tends to influence double marginal effect and complementary effect of the supply chain. And further analyze how service level tends to influence the sale mode choice of product extended warranty by the manufacturer and provide theories on the decision making for the manufacturer.

Keywords: *Sale mode choice, Product extended warranty, Service level, Game theory*

1. Introduction

Under the acceleration of global economic integration process, manufacturers, to seek new profit, have developed a new service mode: Product extended warranty (Cohen & Agrawal, 2006). Product extended warranty refers to the service which extends the product warranty. Product extended warranty is the service contract made by and among the consumer, the manufacturer and the retailer or third-party provider, which can be purchased by the consumer. It provides consumers with the opportunity of attaining the service out of the product warranty period, such as the maintenance of the product; besides it broadens the market for the service provider. From the above aspect, product extended warranty is considered the new profit source for the provider (Murthy & Djamaludin (2002); Lam & Kwok (2001); Albaum & Wiley (2010)).

Currently, one of the main research areas of product extended warranty lies in its sale mode choice and its improvement. Xia etc.(2007), by using Game Theory, illustrates how the sale mode choice of after-sale service influences the performance of the supply chain system. Xia concludes that sub-game equilibrium by the two parties of Game Theory (manufacturer and retailer) depends on who is tougher than the other. In the supply chain system dominated by the manufacturer, the manufacturer chooses the retailer to sell service only when the marginal cost of the service is relatively low; while in the supply chain system dominated by the retailer, the retailer is actively willing to provide the distribution service for the manufacturer in order to increase its profit. Wang Su-Juan (2010), by defining the attractiveness index of product extended warranty, quantitatively explores the effect caused by the index to the sale mode choice of product extended warranty. Wang concludes that among the consumers who have bought the product, when only a few consumers buy product extended warranty, the manufacturer is likely to choose the mode of letting the retailer provide product extended warranty; When all the consumers buy product extended warranty, the manufacturer, by using the attractiveness index range of product extended warranty for reference, choose one favorable mode on whether product extended warranty is provided by the manufacturer or by the retailer.

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Desai(2004), by applying Game Theory, studies the channel coordination of product extended warranty provided by the manufacturer, which finds out that it is good for the manufacturer to sell product extended warranty by means of Double Channel. Among the literature on the sale mode of product extended warranty, Wang (2010) doesn't consider the effect caused by the consumer's risk avoidance to Product Extended Warranty. In fact, however, as the after-sale service which decreases the risk for the consumer, product extended warranty closely relates to the degree of consumer's risk avoidance. Desai (2004) doesn't concern the fault rate constraint of the product, and he doesn't analyze the effect to the manufacturer's marketing channel caused by the standard of product extended warranty made by the manufacturer, which includes the scope and the quality of product extended warranty. While service standard directly reflects product extended warranty, which greatly influence the manufacturer's decision on the sale mode choice of product extended warranty. Li Jie & Liu Jian (2013) from the perspective of supply chain management, Used the game theory to construct models for four modes of warranty service and the service index was also introduced to the analysis.

On the issue of the direct-selling mode or the traditional retail mode of product extended warranty, there is the double marginal effect in the traditional retail channel, and there is the complementary effect between product extended warranty and the product itself, both of the effects have impact on the profit that the manufacturer gains under the two sale modes. Complementary effect lies in the influence caused by the service standard to both the market demand for the product and the demand for product extended warranty; Double marginal effect lies in the influence caused by the cost of product extended warranty, since the service standard is dominated by the cost of product extended warranty. Therefore, the adjustment of the service standard tends to influence the degree of both double marginal effect and complementary effect of the supply chain for product extended warranty, thus will affect the manufacturer on the sales mode choice of product extended warranty.

This article, basing on the above research, by considering the degree of the risk avoidance by the consumer, will form the demand function for the product and product extended warranty. It will also study how the service level of product extended warranty influences both the double marginal effect and the complementary effect of the supply chain for product extended warranty. And it will further put forward the choice strategy for the sale mode of product extended warranty by the manufacturer and provide the theories for the decision making in the supply chain.

2. Model

2.1. Problem Description and Assumptions

We will study the master slave two-echelon supply chain system which contains single manufacturers and single retailers, in which the product is provided by the manufacturers and is sold by the retailers, furthermore the product extended warranty is provided by the manufacturers which can be sold by the retail channel or the direct-selling channel.

Based on presupposition that the product retailed price is p, the product extended warranty price is p_e, when the consumer only buy the product (meanwhile the consumer did not buy the product extended warranty), the consumer's mean-variance utility function is

$$\begin{aligned} u_0 &= E(U_0) - rVar(U_0) \\ &= u(1-\lambda) - p - ru^2\lambda(1-\lambda) \end{aligned} . \quad (1)$$

u is the basic utility of the unit product, $\lambda \in (0,1)$ is malfunction rate during the product life cycle, U_0 is the random utility when the when the consumer only buy the product (meanwhile the consumer did not buy the Product extended warranty). Supposed the random utility U_0 follow two-point distribution, that is $U_0 \sim \begin{matrix} 0-p & u-p \\ \lambda & 1-\lambda \end{matrix}$, then in equation (1),

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$E(U_0) = u(1-\lambda) - p$ is the utility which the neutral risk consumer gain when he buy the unit product. $rVar(U_0) = ru^2\lambda(1-\lambda)$ in equation (1) is the risk aversion consumer risk cost(Xiao & Yang 2009; Tsay 2002), r is the consumer aversion degree, in general $r > 0$,when $r = 0$,it means the consumer is neutral risk; the more bigger the r is, the higher the consumer aversion degree is. If the r is bigger than the critical point, that is to say, the consumer aversion degree is so high that they did not choose to buy product easily let alone to buy corresponding product extended warranty.

When the consumer risk aversion degree is not high, the consumer will choose to buy product extended warranty to counteract some risk which is brought by the product malfunction. Supposed $q_e \in (0,1)$ is the manufacture product service level such as product extended warranty range and quality will differ with the product extended service length and its price. After the consumer buy the product extended warranty, he can gain the utility compensation by the product extended service provided by the manufacturer during the product life circle in which the product goes wrong. So when the consumer buys the product and product extended service at the same time, the consumer's mean-variance utility function is

$$\begin{aligned} u_1 &= E(U_1) - rVar(U_1) \\ &= u(1-\lambda) - p - p_e + q_e u\lambda - ru^2(1-q_e)^2\lambda(1-\lambda) \end{aligned} \tag{2}$$

U_1 is the consumer random utility who not only buy the product but also the Product extended warranty. Supposed the random utility U_1 follow two-point distribution, that is

$$U_1 \sim \begin{matrix} 0-p-p_e+q_e u & u-p-p_e \\ \lambda & 1-\lambda \end{matrix}.$$

Similarly, in (2) $E(U_1) = u(1-\lambda) - p - p_e + q_e u\lambda$ is the utility of the risk neutral consumers who buy the unity product and its product extended warranty. $rVar(U_1) = ru^2(1-q_e)^2\lambda(1-\lambda)$ in equation (2) is the risk cost of the risk aversion consumer who buy the product and extended warranty. In order to make the discussion more meaningful, we suppose the relevant parameter must meet $u(1-\lambda) - p > 0$ and $-p_e + q_e u\lambda < 0$, *i.e.*,: $q_e u\lambda < p_e < 1$.

Supposed the manufacture's production cost is c, for product extended warranty cost is related to the product extended warranty level and the malfunction rate, so the reference Heese (2008), supposed the unit product extended warranty average cost provided by manufacture is $k_m q_e^2 \lambda$, k_m is the unit product extended warranty cost efficiency coefficient provided by the manufactures. The more smaller k_m is, the higher the product extended warranty efficiency which is provided by the manufactures is.

2.2. Product Extended Warranty Supply Chain Model and the Equilibrium Strategies

2.2.1. Product Extended Warranty Demands In The Case Of Consumer Risk Aversion

For the different consumer risk aversion degree, the consumer demands of the product and extended warranty is different. So we can divide the consumer into three types: One is the consumers who only buy the product; the second is the consumers who buy the product and the extended warranty at the same time; the third is the consume who give up buy product showing as the Figure 1.

(1) The consumers who only buy the product. When the consumer's risk aversion degree is not high, the utility is bigger when the consumers only buy the product. The consumer will choose to buy product instead of buying product extended warranty to counteract some risk which is brought by the product malfunction. Showing as the Figure 1, utility relationship which meet the consumer is: $u_0 > u_1$, supposed the consumer's risk aversion degree of this type is $r \in [0, r^*]$.

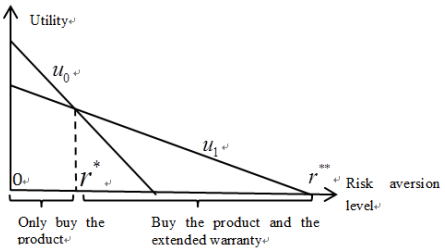

Figure 1. The Consumer Demands Analyses of the Product and Extended Warranty

(2) The consumers who buy the product and the extended warranty at the same time. When the consumer risk aversion degree goes up gradually, the consumer will choose to buy product extended warranty to counteract some risk which is brought by the product malfunction. Showing as the Figure 1, utility relationship which meet the consumer is $u_0 > u_1$, supposed the consumer's risk aversion degree of this type is $r \in (r^*, r^{**}]$.

(3) The consumes who give up to buy product, considering the product risk and the negative utility. When the consumer risk aversion degree is too big, the consumer will choose to give up buying the product and the corresponding extended warranty for choosing buying corresponding extended warranty just only reduce part of the negative utility brought by the risk, which cannot make the general utility positive. Utility relationship which meet the consumer is: $u_1 < 0$, supposed the consumer's risk aversion degree of this type is $r \in (r^{**}, +\infty]$.

For the product extended warranty is introduced in this chapter and the consumers are divided into three types according to the risk aversion level. We continue to build product demand function by the consumer risk aversion degree.

Showing as the figure 1, if the consumer's risk aversion degree is the critical point $r = r^*$,it is no difference for consumers who only buy the product or buy the extended warranty at the same time, i.e. $u_0 = u_1$. so from equation (1) and (2) can conclude

$$r^* = \frac{p_e - q_e u\lambda}{q_e(2-q_e)\lambda(1-\lambda)u^2} \tag{3}$$

Similarly, if the consumers, risk aversion degree is $r = r^{**}$, the utility is no difference for consumers who buy the product and the extended warranty at the same time, or give up to buy product, that is, $u_1 = 0$. So from the equation (2) , we conclude:

$$r^{**} = \frac{u(1-\lambda) - p - p_e + q_e u\lambda}{(1-q_e)^2\lambda(1-\lambda)u^2} \tag{4}$$

Apparently, the utility of the consumer whose risk aversion is at $[0, r^{**}]$, is greater than zero, they will choose to buy the product. According to the Mitra & Webster (2008) demand function building mode, then the product demand function is:

$$D = r^{**} = \frac{u(1-\lambda) - p - p_e + q_e u\lambda}{(1-q_e)^2\lambda(1-\lambda)u^2} \tag{5}$$

The utility of the consumer whose risk aversion is at $[r^*, r^{**}]$can meet $u_1 > u_0$, they not only choose to buy the product but also buy the extended warranty. So the product extended warranty demand function is

$$D_e = r^{**} - r^* = \frac{u(1-\lambda) - p - p_e + q_e u\lambda}{(1-q_e)^2\lambda(1-\lambda)u^2} - \frac{p_e - q_e u\lambda}{q_e(2-q_e)\lambda(1-\lambda)u^2} \tag{6}$$

2.2.2. The R mode

Under the R mode, the product extended service is provided by the manufactures and are sold with the products by retailers, which is used by Ford Motor, Apple etc. During the

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Stackelberg process among the two-echelon supply chain member, the manufacturers as the leader, maximize own profit by deciding the product wholesale price w and the product extended service wholesale price w_e in the first stage. In the second stage, according to the wholesale price which is made by the manufacture, the retailers as the follower, maximize own profit by deciding the product retail price p and product extended service retail price p_e.

Adopt backward introduction, the solution process as following:

The first step, in the second stage, the profit function of the retailer $\pi_r^R(p, p_e)$ is

$$\pi_r^R(p, p_e) = (p - w)D + (p_e - w_e)D_e \quad (7)$$

Used the equation (5) D and (6) D_e put into (7),we can find:

$$\pi_r^R(p, p_e) = (p - w)\frac{u(1-\lambda) - p - p_e + q_e u\lambda}{(1-q_e)^2\lambda(1-\lambda)u^2} + (p_e - w_e)[\frac{u(1-\lambda) - p - p_e + q_e u\lambda}{(1-q_e)^2\lambda(1-\lambda)u^2} - \frac{p_e - q_e u\lambda}{q_e(2-q_e)\lambda(1-\lambda)u^2}]$$

and $\frac{\partial^2 \pi_r^R}{\partial p^2} = \frac{-2}{(1-q_e)^2\lambda(1-\lambda)u^2} < 0$;

$\frac{\partial^2 \pi_r^R}{\partial p_e^2} = -2[\frac{1}{(1-q_e)^2\lambda(1-\lambda)u^2} + \frac{1}{q_e(2-q_e)\lambda(1-\lambda)u^2}] < 0$.So that, $\pi_r^R(p, p_e)$ is concave function of p and p_e, the retailer have the optimal price strategy, set $\frac{\partial \pi_r^R(p, p_e)}{\partial p} = 0$ and $\frac{\partial \pi_r^R(p, p_e)}{\partial p_e} = 0$, have:

$$p^{R*} = \frac{(1-\lambda)u + w}{2} \quad (8)$$

$$p_e^{R*} = \frac{q_e\lambda u + w_e}{2} \quad (9)$$

Second, in the first stage, the manufacturer profit function $\pi_m^R(w, w_e)$ is:

$$\pi_m^R(w, w_e) = (w - c)D + (w_e - k_m q_e^{\,2}\lambda)D_e \quad (10)$$

Use equation (8) and (9) are substituted in (10), the manufacturer have the optimal price strategy, set $\frac{\partial \pi_m^R(w, w_e)}{\partial w} = 0$ and $\frac{\partial \pi_m^R(w, w_e)}{\partial w_e} = 0$, can find:

$$w^{R*} = \frac{(1-\lambda)u + c}{2} \quad (11)$$

$$w_e^{R*} = \frac{q_e\lambda u + k_m q_e^2\lambda}{2} \quad (12)$$

Finally， Use equation (11) and (12) substituted in (8) and (9), can find:

$$p^{R*} = \frac{3(1-\lambda)u + c}{4} \quad (13)$$

$$p_e^{R*} = \frac{3q_e\lambda u + k_m q_e^2\lambda}{4} \quad (14)$$

According to hypothesis, and the equation (13), must $p^{R*} = \frac{3(1-\lambda)u + c}{4} < (1-\lambda)u$, that is, related parameter must meet： $c < (1-\lambda)u$;Then according to equation (11), if $p^{R*} \geq w^{R*}$, so $\lambda \leq 1 - \frac{c}{u}$. If the rate of product malfunction is too high (for example $\lambda = 1$), it will cause the product quality decrease, which will reduce the product demand. So the rate of product malfunction λ which meet this interval demand is reasonable. According to the hypothesis,

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related parameter must meet $q_e u\lambda < p_e < 1$, if equation (14), then $\frac{u}{k_m} \le q_e \le 1$, but, according to the equation (12), then $p_e^{R*} \le w_e^{R*}$.

Apparently, the product extended service price made by retailers must remain lower in order to guarantee the service level of the product extended service. The retailers will have minor loss during the process of selling the product extended service.

In addition, according to the equation (6), for $D_e = r^{**} - r^* \ge 0$, then $0 \le q_e \le \frac{2(u-c)-u\lambda}{u-c-u\lambda+k_m\lambda}$. Obviously, if the service level of the product extended service appear incredibly high, it will result in the opposite influence on the demand of the product extended service even on the product demand.

So, we can know the interval of product extended service level (q_e) under the R mode: $q_e \in (\frac{u}{k_m}, \frac{2(u-c)-u\lambda}{u-c-u\lambda+k_m\lambda})$.

At the moment, the profit of the manufacturer under the R mode is:

$$\pi_m^{R*} = \frac{[2(1-q_e)^2u^2 + 2k_m q_e^2(1-q_e)u + k_m^2 q_e^3]\lambda^2 - 2(2-q_e)(u-c)[(1-q_e)u + k_m q_e^2]\lambda + (2-q_e)(u-c)^2}{8\lambda(1-\lambda)(2-q_e)(1-q_e)^2u^2} \quad (15)$$

In addition, the profit of the retailer is:

$$\pi_r^{R*} = -\frac{(1-\lambda)u + c + q_e\lambda u + k_m q_e{}^2\lambda}{2u(1-q_e)^2\lambda} + [-\frac{(1-\lambda)uq_e + q_e c - q_e^2\lambda u - k_m q_e^3\lambda}{2u(1-q_e)^2(1-\lambda)} - \frac{q_e\lambda u + k_m q_e{}^2\lambda}{2(2-q_e)u(1-\lambda)}]$$

2.2.3. The M mode

Under the M mode, the product extended service is provided by the manufacturer and are sold with the products by retailer, which is used by Ford Motor, Apple etc. During the Stackelberg process among the two-echelon supply chain member, the manufacturer, as the leader, maximize own profit by deciding the product wholesale price w and the product extended service wholesale price w_e , in the first stage. In the second stage, according to the wholesale price which is made by the manufacturer, the retailer as the follower, maximize own profit by deciding the product retail price p and product extended service retail price p_e .

Adopted backward introduction, the solution process is as following:

The first step, in the second stage, the profit function of the retailer $\pi_r^M(p)$ is:

$$\pi_r^M(p) = (p-w)D \quad (16)$$

If D in equation (5) is substituted in (16), the result is:

$$\pi_r^M(p) = (p-w)\frac{u(1-\lambda) - p - p_e + q_e u\lambda}{(1-q_e)^2\lambda(1-\lambda)u^2} .$$

Easy to know: $\frac{\partial^2\pi_r^M}{\partial p^2} = \frac{-2}{u^2(1-q_e)^2\lambda(1-\lambda)} < 0$, $\pi_r^M(p)$ is the concave function of p , the retailers have the best pricing strategy. Meanwhile, in the first stage, the profit function of the manufacturer $\pi_m^M(p_e)$ is:

$$\pi_m^M(p_e) = (w-c)D + (p_e - k_m q_e^2\lambda)D_e \quad (17)$$

If D in equation (5) and D_e in (6) are substituted in equation (18), the result is:

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$$\pi_m^M(p_e) = (w-c)\frac{u(1-\lambda)-p-p_e+q_e u\lambda}{(1-q_e)^2\lambda(1-\lambda)u^2} + (p_e - k_m q_e^2\lambda)[\frac{u(1-\lambda)-p-p_e+q_e u\lambda}{(1-q_e)^2\lambda(1-\lambda)u^2} - \frac{p_e - q_e u\lambda}{q_e(2-q_e)\lambda(1-\lambda)u^2}]$$

Easy to know, $\pi_m^M(p_e)$ is the concave function of p_e ,the manufacturer have the best pricing strategy for the product extended warranty retail price (p_e). By $\frac{\partial \pi_r^M}{\partial p} = 0$ and $\frac{\partial \pi_m^M(p_e)}{\partial p_e} = 0$, the solution is:

$$p^{M*} = \frac{2(w+u-u\lambda)+q_e u\lambda-(2-q_e)q_e(w+c+u-u\lambda)-k_m q_e^2\lambda}{4-(2-q_e)q_e} \tag{18}$$

$$p_e^{M*} = \frac{(2-q_e)q_e(2c-3w+u)+2k_m q_e^2\lambda-(1-q_e)q_e^2 u\lambda}{4-(2-q_e)q_e} \tag{19}$$

The second step, in the first stage, the manufacturer decide the wholesale, if equation (18) and (19) are substituted in (17), the manufacturer have the best pricing strategy for wholesale w ,if $\frac{\partial \pi_m^M}{\partial p_e} = 0$, the solution is:

$$w^{M*} = \frac{(1-\lambda)u+c}{2} + \frac{(k_m q_e^2\lambda - q_e u\lambda)+[c-(1-\lambda)u]}{2(q_e+2)(4-q_e)}q_e(2-q_e) \tag{20}$$

If equation (20) is substituted in equation (18) and equation (19), the solution is:

$$p^{M*} = \frac{3(1-\lambda)u+c}{4} - \frac{2(k_m q_e^2\lambda - q_e u\lambda)[4+(2-q_e)q_e]+[c-(1-\lambda)u](2-q_e)q_e}{4(q_e+2)(4-q_e)} \tag{21}$$

$$p_e^{M*} = \frac{3q_e\lambda u + k_m q_e^2\lambda}{4} + \frac{(k_m q_e^2\lambda - q_e u\lambda)[8+5(2-q_e)q_e]+4[c-(1-\lambda)u](2-q_e)q_e}{4(q_e+2)(4-q_e)} \tag{22}$$

On the basis of equation (20) and (21), according to the assumption， must $p^{M*} \geq w^{M*}$, *i.e.*,

$$p^{M*} - w^{M*} = \frac{(2+2q_e-q_e^2)[(u(1-\lambda)-c)-(k_m q_e^2\lambda - q_e u\lambda)]}{(q_e+2)(4-q_e)} \geq 0$$

Because $q_e \in (0,1)$, $2+2q_e-q_e^2 > 0$,to make the above formula established, the related parameter must meet:

$$u(1-\lambda)-c \geq k_m q_e^2\lambda - q_e u\lambda \geq 0$$

In this case, q_e in $(-\infty,0)\bigcup(\frac{u}{k_m},+\infty)$, combined practical significance of the product extended warranty level, $q_e \in (0,1)$. So the condition that meet q_e is: $q_e \in (\frac{u}{k_m},1)$. If set $\eta = \frac{u(1-\lambda)-c}{k_m q_e^2\lambda - q_e u\lambda} \geq \frac{u(1-\lambda)-c}{(k_m-u)\lambda} \geq 2$, simplification: $\lambda \leq \frac{u-c}{k_m}$, obtain $\lambda \leq 1-\frac{c}{u}$,then if $\frac{u-c}{k_m} > 1-\frac{c}{u}$, the range of λ is $[0,1-\frac{c}{u}]$; If $\frac{u-c}{k_m} \leq 1-\frac{c}{u}$, the range of λ is $[0,\frac{u-c}{k_m}]$.

Product demands:

$$D^{M*} = (2+2q_e-q_e^2)\frac{[(u(1-\lambda)-c)-(k_m q_e^2\lambda - q_e u\lambda)]}{(q_e+2)(4-q_e)(1-q_e)^2 u^2(1-\lambda)\lambda}.$$

Product extended warranty demands:

$$D_e^{M*} = 3\frac{[(u(1-\lambda)-c)-(k_m q_e^2\lambda - q_e u\lambda)]}{(q_e+2)(4-q_e)(1-q_e)^2 u^2(1-\lambda)\lambda} - \frac{4(k_m q_e^2\lambda - q_e u\lambda)}{(4-q_e)(2+q_e)(2q_e-q_e^2)u^2(1-\lambda)\lambda} - \frac{(k_m q_e^2\lambda - q_e u\lambda)}{2(4-q_e)(2+q_e)u^2(1-\lambda)\lambda}.$$

If $\eta \geq 2$, $D^{M*} - D_e^{M*} = \frac{[(k_m q_e^2 \lambda - q_e u\lambda)(1+\frac{8+(2-q_e)q_e}{2(2-q_e)q_e}) - ((1-\lambda)u - c)]}{(q_e+2)(4-q_e)u^2(1-\lambda)\lambda} \geq 0$, So the solution:

$$q_e \leq \frac{3-2\eta+\sqrt{33+4\eta+4\eta^2}}{3+2\eta}.$$

The following is general rule, if $\eta = 3$, the condition that meet q_e is: $q_e \in (\frac{u}{k_m}, \frac{2}{3})$

The profit of the manufacturer is:

$$\pi_m^{M*} = \frac{M_1\lambda^2 + M_2\lambda + (2-q_e)(u-c)^2}{\lambda(1-\lambda)(4-q_e)(2+q_e)(2-q_e)(1-q_e)^2u^2}. \quad (23)$$

$M_1 = N_1u^2 + N_2u + N_3$, $N_1 = \frac{q_e^5}{4} - q_e^4 + \frac{q_e^3}{4} - \frac{q_e^2}{2} + 3q_e - 2$

$N_2 = (-\frac{q_e^6}{2} + 2q_e^5 - \frac{q_e^4}{2} - q_e)k_m$, $N_3 = (\frac{q_e^5}{4} - q_e^4 + \frac{q_e^3}{4} + \frac{3q_e^2}{2} - 2q_e)k_m^2q_e^2$,

$M_2 = (-2q_e^2 + 6q_e - 4)u^2 + [6q_ec + 2(c-2k_m)q_e^2 + 2k_mq_e^3 + 4c]u - 2k_mq_e^3c + 42k_mq_e^2c$.

3. Analytical Results

The manufacturer extend the product market but also gain more profit by selling products and providing corresponding extended warranty, but also result in profits differential among the manufacturer under the different sale mode of extended warranty. Selling products and corresponding extended warranty, affect the consumer product demands and utility and cause the ultimate profit differential among retailers. Comparing the profit differential under different sale mode, this section discusses the mode choice of extended warranty from the manufacturer, retailer and supply chain aspects.

Obviously, from equation (15) and (23), we can derive the manufacturer profit differential $\Delta\pi_m^* = \pi_m^{R*} - \pi_m^{M*}$ under R mode and M mode. But $\Delta\pi_m^*$ is the high order polynomial about q_e, so it is difficult to certain $\Delta\pi_m^*$ analytical formula about q_e. Similarly, the retailer profit and the supply chain profit are difficult to analyze directly. Using numerical example, we discuss how the extended warranty level affect the manufacturer and the supply chain profit under the R mode and the M mode.

3.1. Product Extended Warranty Sale Mode Selection in the Manufacturer

Supposed the product basic utility $u = 200$, the manufacture cost efficiency factor $k_m = 1000$, the fixed cost of production for manufacture $c = 40$, the condition that meet q_e (q_e is product extended warranty level) is

$$q_e \in [\frac{u}{k_m}, \frac{2(u-c)-u\lambda}{u-c-u\lambda+k_m\lambda}] = [0.2, \frac{8-5\lambda}{4+20\lambda}].$$

The condition that meet $q_e \in (\frac{u}{k_m}, \frac{2}{3}) = (0.2, 0.67)$. We discuss the corresponding conclusion within the meaningful range of the both above. Supposed $\lambda = 0.2$, discuss the change of q_e within the range $[0.4, 0.65]$. Supposed $\lambda = 0.2$, we can derive $p^{R*} = 130$ and $w^{R*} = 100$. The other results under R mode show as Table 1.

Table 1. The Equilibrium Results under the R Mode ($\lambda = 0.2$)

q_e	D^{R*}	D_e^{R*}	p_e^{R*}	w_e^{R*}	π_m^{R*}	π_r^{R*}	π_{sc}^{R*}
0.4	113	103	20	24	5946	2973	8919
0.45	126	113	23.6	29.3	6280	3140	9420

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0. 5	141	125	27. 5	35	6563	3281	9844
0. 55	157	138	31. 6	41. 3	6770	3385	10155
0. 6	176	153	36	48	6864	3432	10296
0. 65	196	170	40. 6	55. 3	6792	3396	10188

The other results under M mode show as table 2.

Table 2. The Equilibrium Results under the M Mode ($\lambda = 0.2$)

q_e	D^{M*}	D_e^{M*}	p^{M*}	w^{M*}	p_e^{M*}	π_m^{M*}	π_r^{M*}	π_{sc}^{M*}
0. 4	138	137	127. 9	96. 2	16. 3	5590	4383	9973
0. 45	156	149	126. 3	96. 1	21. 4	5929	4723	10652
0. 5	177	162	124. 4	96. 1	27. 3	6254	5001	11255
0. 55	200	177	122	96	33. 9	6552	5182	11734
0. 6	226	194	119. 7	96. 6	41. 2	6798	5225	12023
0. 65	254	213	116. 9	97	49. 2	6958	5069	12027

From the Table 1 and Table 2, we can derive the product extended warranty level curve graph (Figure 2) under the R mode and the M mode, which changes with the profit.

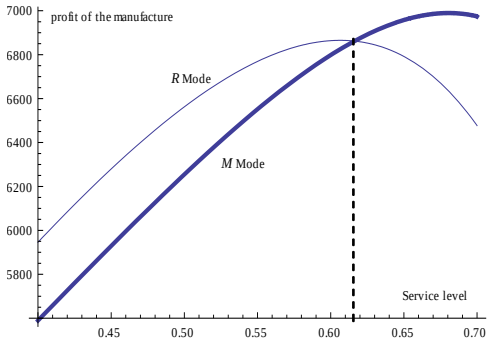

Figure 2. The Changes with the Profit of the Manufacturer in the R, M Mode

From the Figure 2, as to the manufacturer profit under the R mode, with the range $q_e \in (0.4, 0.61)$,the manufacturer profit increases with the improvement of the product warranty service level; if $q_e > 0.61$,the profit of the manufacturer goes down with the product warranty service level decline. As to the manufacture profit under the M mode, with the range $q_e \in (0.4, 0.65)$, the manufacturer profit increases with the warranty service level improvement. It demonstrate the influence of the warranty level which affect the manufacture profit meet the law of diminishing marginal utility.

When the warranty level is low, it is useful to improve product warranty service level to attract more consumers to buy the product and warranty service. When the level is to a certain extent, at that time, to continue to provide the level of service can only increase the warranty service costs, thereby reducing the profit of the manufacturer.

Meanwhile, within the range $q_e \in (0.4, 0.61)$, manufacturer profit under the R mode is higher than the profit under M mode. On the contrary, manufacturer profit under the M mode is higher than the profit under R mode. It illustrate that the influence that warranty level affect selection strategy of the manufacture extended sales mode is segmental.

When the warranty level is not high, the sales mode under R mode is the best selection strategy for manufacturer. While the warranty level is comparatively high, the sales mode under M mode is the best selection strategy for manufacturers. Because when the manufacturer sell the extended warranty under R mode, the retailer under the mode will not consider the complementary effect between the product and the extended warranty, only pursue maximizing its profit although there is not double marginal effect under this mode.

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When the warranty level and the cost are comparatively low, the double marginal effect is weak. The retailer will consider the complementary effect between the product and the extended warranty, appropriately reduce the warranty price, expand product and warranty service demand to increase the profit from the product and the extended warranty. In this case, Complementary effect is larger than the double marginal effect, it is beneficial for manufacturers to sell extended warranty under R mode for Product warranty service needs increase with the warranty service price decrease. Accompany the extended level going up gradually, the extended cost rise which reinforce the double marginal effect, which reduce the retailer profit from the extended warranty.

Raising extended warranty price is the only way to maintain profit which weak the complementary effect and reduce the product demands, which damages manufacturer profit. In that case, it is beneficial for the manufacturer to select sales mode under M mode to reduce the double marginal effect on the supply chain.

3.2 Product Extended Warranty Sale Mode Selection in the Supply Chain

In the same way, from the Table 1 and Table 2, we can derive the product extended warranty level curve graph (Figure 3) under the R mode and the M mode, which changes with the supply chain profit.

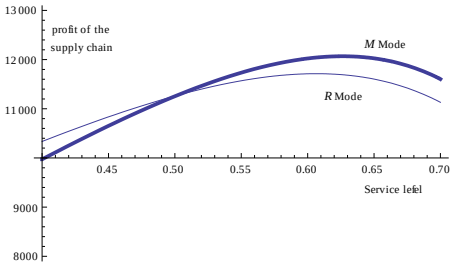

Figure 3. The Changes with the Profit of the Supply Chain in the R, M Mode

From the Figure 3, as to the supply chain system profit under the R mode, with the range $q_e \in (0.4, 0.6)$, the supply chain system profit increases with the improvement of the product warranty service level ; If $q_e > 0.6$,the supply chain system profit goes down with the product warranty service level decline.

As to the supply chain system profit under the M mode, with the range $q_e \in (0.4, 0.62)$, the supply chain system profit increases with the warranty service level improvement. It demonstrate the influence of the warranty level which affect the supply chain system profit meet the law of diminishing marginal utility.

Meanwhile, within the range $q_e \in (0.4, 0.5)$, the supply chain system profit under the R mode is higher than the profit under M mode. On the contrary, supply chain system profit under the M mode is higher than the profit under R mode.

The warranty service level is not high, the sales mode under the R mode is the best selection strategy for the supply system while the warranty level is high the sales mode under the M mode is the best selection strategy for the supply system.

4. Conclusion

This article analyze influence of the extended warranty level which affect manufacturer and supply chain profit in different sale mode and the best selection strategy. The study demonstrate: The influence of the warranty service level which affect the manufacturer profit under the M mode and the R mode meet the law of diminishing marginal utility. The influence that warranty level affect selection strategy of the manufacturer extended sales mode is segmental. The warranty level is not high, it is beneficial for the

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manufacturer to select sales mode under R mode while he warranty level is high, it is beneficial for the manufacturer to select sales mode under M mode.

So the manufacturer should consider fully the above cathartic to provide the proper extended warranty level instead of blindly raising costs which reduce profits. As to the mode selection, based on a variety of factors such as own ability and the quality of the products, the manufacturer shall select the appropriate mode to sell their extended warranty within the scope they can provide to maximize profit.

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