

A Simply Carbon Pricing Model between Government and Enterprises based on Game Theory

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Abstract

The purpose of this paper is to give a novel carbon pricing model gamed between firms and government. Under non-gratuitous mode of emission rights for distribution adopted by the government, from their own profits of view, the both sides of the deal played the game of price, according to the demand and supply in the market, the transaction prices were decided by the market mechanisms.

Keywords: carbon emission trade, carbon price, game theory

1. Introduction

To mitigating climate change, some measures are taken by the countries in the world to reduce emission of the greenhouse gas (GHG). It is a crucial time to develop economic in China, the demand of energy keep rising, in the “Eleventh Five-Year Plan”, China government set the target of reducing its unit GDP energy consumption by 20% by the end of 2010. Before the Copenhagen Climate Change Conference 2009, China State Council declared its goal to reduce its unit GDP CO₂ emission by 40-45% by 2020, compared with 2005. In the long-run, the target about emission reduction will achieve mainly through progress in technology and optimization of the structure of energy consumption. Coal provides around 70% of the total primary energy consumption in China, which is the produces the most GHG and most serious pollution to environment, the proportion of coal will remain unchanged in short time, as shown in Figure 1.

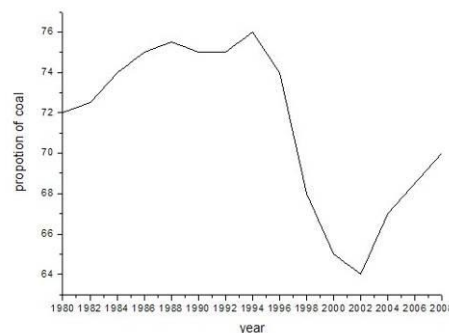


Figure 1. Trend of Proportion of the Coal

In global level, pollution discharge trade mechanism is mainly applied to carbon emission

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trade. From Kyoto Protocol in 1997, there were three valid mechanisms to reduce emission of GHG, clean development mechanism (CDM), joint implementation (JI) and emission trade (ET). Although there exist some defects to these mechanisms, they play an important role in reducing the GHG emission. In 2013, the gross of global carbon trade is 1.04 billion tons, total amount 5.5 billion dollars. It may to say 2013 is the milestone of China carbon market, in Twelfth Five-Year.

Plan, gradually setting up the carbon market was proposed explicitly by the government. So at the Primarily stage, five provinces and two cities were chosen as the locations to demonstrate in 2013.

As the largest emitter in the world, China doesn't have the compulsory obligation to reduce emission, but it means a lot to do this for this country. Before the Copenhagen Climate Change Conference 2009, China State Council declared its goal to reduce its unit GDP CO₂ emission by 40-45% by 2020, compared with 2005 (Table 1).

Table 1. Overview of Goals of Energy Conservation and Emission Reduction

Policy Focus		Detailed Target	Latest Development
Emission of GHG	Intensity of Carbon	Down to 17% in 2015 compared to 2010 Down to 40-45% in 2020 compared to 2005	
Consumption of the energy	Intensity of energy	Down to 16% in 2015 compared to 2010	
	Gross of energy	Less than 0.42 billion tons standard coal in 2015	3.62 billion tons standard coal in 2012
	Gross of electricity	Less than 6150 billion kwh in 2015	4208 billion kwh in 2010
	Share of renewable energy in primary energy consumption	Comprised 11.4% in 2015 Comprised 15% in 2020	Comprised 10.3% in 2009

The target of emission reductions of every demonstrated zones are presented in Table 2, these values are higher than the contemporary target of the country. Guangdong has the highest target of carbon intensity, should reduce 19.5% of carbon intensity, Hubei and Chongqing are lowest, just 17%.

Table 2. Target Parsing of Emission Reduction in Demonstrated Zones during Twelfth Five-Year Plan

Zone	Benchmark data in 2010			Target of carbon intensity(%)	Average growth rate per annum of GDP(%)	Emission reductions need to accomplish in 2015(million tons)
	Emission reductions (million tons)	GDP(billion yuan, based on price in 2006)	Carbon intensity(million tons/ billion yuan)			
Guangdong	529	4.1	130	-19.5	7.5	113
Hubei	337	1.3	263	-17	8.1	62.1
Shanghai	250	1.6	161	-19	6.9	12.2
Chongqing	151	0.6	245	-17	8.5	29.1
Tianjin	155	0.8	193	-19	9.1	7.5
Beijing	115	1.2	97	-18	7.5	4.6
Total	1537	9.6	162			219

The emission trade in China is just in the stage of start, the patterns of trade are different. According to the current state of the country, in view of lowering the cost and keeping the trade to be smooth, the completely system of the carbon emission trade should include legislation, public notification, monitoring system, cap and trade based on initial allocation of the right. China's emissions trading scheme is shown in chart 1.

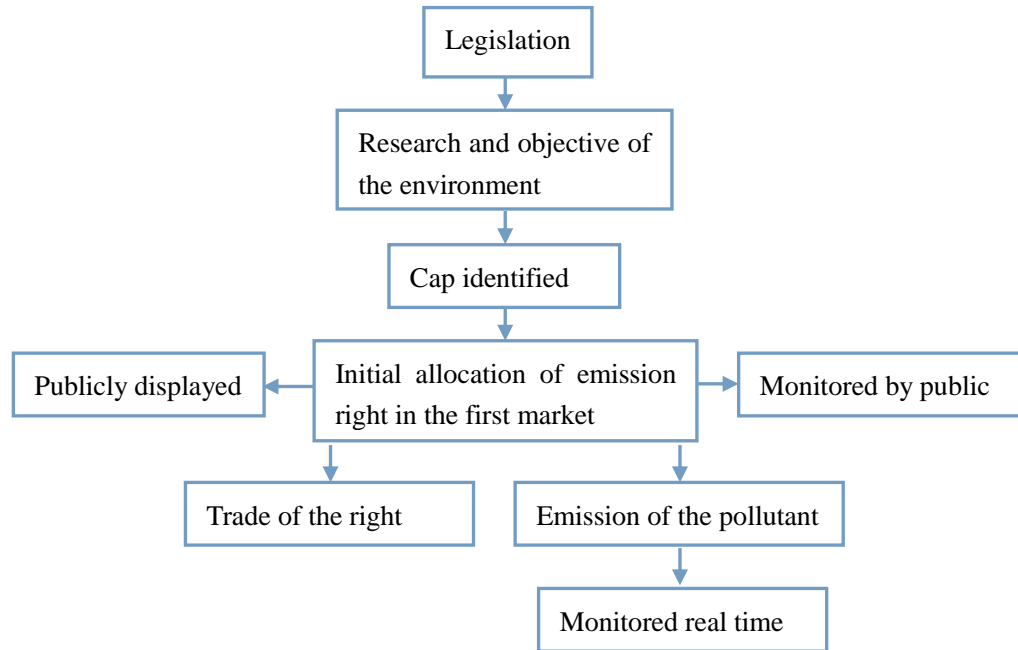


Chart 1. Design of the Regime of Pollutant Discharge Trade in China

Carbon emission trade is an effective method to reduce the emission of CO₂. To establish the market of pollutant discharge right, after solving the allocation successful, how to decide the initial price of the right is another problem left. It is very important for the government to give a basic price, which can stimulate the trade and enthusiasm of industries. The following rules must be obeyed when the initial price was given. First of all, the initial price should higher than the social optimal cost to manage the pollution, the optimal cost refer to the minimum expense to curb the pollution of some industries with advanced technologies. Under this circumstance, these enterprises will supply the extra right to the market. Second, the basic price should lower than the social worst cost to harness the pollution, the worst cost refer to the maximum expense to manage the pollution of some industries with out-dated technologies, then these enterprises will choose to buy the right and abandon to control the pollution themselves. Pollutant reduction is encouraged, which is the last rule.

The ways of allocation of right of pollution discharge mainly are: allocation freely and auction. Generally speaking, for a special pollutant, the nation will set a cap in a period of time, which is a gross control objective, the cap will be distributed between each province under some principles, eventually the quota will be distributed in enterprises in the province for free or paid by the local environmental organization. So under the distribution with fee, the two sides play the game for pollution discharge from their own profits. At last, the price will be decided by market mechanism according to the demand and supply in the market.

Many researchers had cared the issue of carbon pricing and a lot models related to carbon emission and trade had been constructed. Hahn R. W. [2] had discussed the market mechanisms and carbon emission rights. Athanasios K [3] had discussed the rules of allocation of rights in the agricultural sector. Zheng S. R. [4-6] had conducted a series of

research about structure of energy consumption, carbon tax and cost of reduction etc, some useful results had been proposed in these papers. Ai H.L. [9, 10] presented a remarkable model and effective solution of a kind of optimization problems. Luis M, Chamorro J M. [11] studied the issue of EU CO₂ emission and investment.

The rest of this paper is structured as follows. Section 2 presents basic hypotheses of the model, a gambling model. Section 3 presents the model and discusses the results respectively, then a case is analyzed through applying this model. Section 4 concludes the paper.

2. Basic Hypotheses of the Model

Allowing to the trading market of pollution discharge right, some assumptions should set as follows:

First of all, the information is incomplete, the buyer and seller have no knowledge about each other. Given the price estimated by the buyer and seller are V_1 and V_2 separately to the initial quantities of right of pollution discharge, then two sides play games around their estimated price.

Second, the strategy of the buyer is $P_1 = V_1(1 + C_1)$, this formula means choosing an appropriate C_1 ($C_1 \in [-1, 0]$) based on V_1 . The buyer tend to reduce the price, a high price than V_1 will not be bade and accepted, so set $C_1 \in [-1, 0]$ to satisfy $P_1 < V_1$.

Third, the strategy of the seller is $P_2 = V_2(1 + C_2)$, this formula means choosing an appropriate C_2 ($C_2 \in [-1, 1]$) based on V_2 . The seller tend to increase the price, but in the negotiations, considering to utilization of the buyer and the whole wealth of the society, the seller have to compromise and accept the price lower than V_2 , so set $C_2 \in [-1, 1]$.

At last, the two parameters C_1, C_2 have the normal distribution and extra profits are neglected generated by the trade.

3. The Model of Game and Solutions

3.1. Model Presentation

Case1 If $P_1 < P_2$, then the two sides can't attain a consistent price, so the negotiation fail, the utility of each side is zero.

Case 2 If $P_1 > P_2$, an approvable price can be attained by two sides, then

$$P = P_2 + \theta(P_1 - P_2) = V_2(1 + C_2) + \theta[V_1(1 + C_1) - V_2(1 + C_2)],$$

where $\theta \in [0, 1]$, the value of θ is decided by the two sides ultimately, so their payment function are $U_1 = V_1 - P, U_2 = V_2 - P$ separately.

To maximize their utility, the seller hope the price more higher than its estimated price, the buyer want more lower than its price. Allowing for all these assumptions, some expectations of the two sides as below:

1) The payment function expected by the buyer is

$$\begin{aligned} U_1 &= (V_1 - P)P(P_1 \geq P_2) \\ &= [V_1 - V_2(1 + C_2) - \theta[V_1(1 + C_1) - V_2(1 + C_2)]]P[V_1(1 + C_1) \geq V_2(1 + C_2)] \\ &= [V_1 - V_2(1 + C_2) - \theta[V_1(1 + C_1) - V_2(1 + C_2)]]P[C_2 \leq \frac{V_1(1 + C_1)}{V_2} - 1] \\ &= \frac{1}{2}[V_1 - V_2(1 + C_2) - \theta[V_1(1 + C_1) - V_2(1 + C_2)]]\left[\frac{V_1(1 + C_1)}{V_2}\right] \end{aligned}$$

2) The utility function expected by the seller is

$$\begin{aligned}
 U_2 &= (S + P - V_2)P (P_1 \geq P_2) \\
 &= [S + V_2(1 + C_2) + \theta[V_1(1 + C_1) - V_2(1 + C_2)] - V_2]P[V_1(1 + C_1) \geq V_2(1 + C_2)] \\
 &= [S + V_2(1 + C_2) + \theta[V_1(1 + C_1) - V_2(1 + C_2)] - V_2]P\left[C_1 \geq \frac{V_2(1 + C_2)}{V_1} - 1\right] \\
 &= [S + V_2(1 + C_2) + \theta[V_1(1 + C_1) - V_2(1 + C_2)] - V_2]\left[1 - \frac{V_2(1 + C_2)}{V_1}\right]
 \end{aligned}$$

Where S denotes other profits generated by the trade, in generally S is far more less than V_1 and V_2 .

If there exists a harmonious strategic price (P_1^*, P_2^*) ,

then there exists (C_1^*, C_2^*) satisfy

$$\begin{aligned}
 C_1^* &\in \max_{C_1} U_1 \\
 C_2^* &\in \max_{C_2} U_2
 \end{aligned}$$

So we can find (C_1^*, C_2^*) through this model, then the harmonious solution of price (P_1^*, P_2^*) can be found.

3.2. The Solutions of the Model

The model can be solved by partial derivative, so the (C_1^*, C_2^*) can be found.

$$\begin{aligned}
 \frac{\partial U_1}{\partial C_1} &= \frac{V_1}{2V_2} [V_1 - V_2(1 + C_2) + \theta V_1(1 + C_1) - \theta V_2(1 + C_2) - V_2] - \theta V_2 \left[1 - \frac{V_2(1 + C_2)}{V_1}\right] \\
 \frac{\partial U_2}{\partial C_2} &= -\frac{V_2}{V_1} [S + V_2(1 + C_2) + \theta V_1(1 + C_1) - \theta V_2(1 + C_2) - V_2] + \left[1 - \frac{V_2(1 + C_2)}{V_1}\right] (V_2 - \theta V_2)
 \end{aligned}$$

Let $\frac{\partial U_1}{\partial C_1} = 0$, $\frac{\partial U_2}{\partial C_2} = 0$ then

$$C_1 = \frac{1 - 2\theta}{2\theta} - \frac{V_2(1 + C_2 - \theta C_2 - \theta)}{2V_1\theta}, \quad C_2 = \frac{S + \theta V_1(1 + C_1) - V_2 - (1 - \theta)V_1}{2V_2(\theta - 1)} - 1$$

For the convenience of analysis, let $V_1 = V_2 = V$, then C_1, C_2 can be simplified as

$$C_1 = \frac{1 - 2\theta}{2\theta} - \frac{(1 + C_2 - \theta C_2 - \theta)}{2\theta}, \quad C_2 = \frac{S}{2V(\theta - 1)} + \frac{\theta(1 + C_1)}{2(\theta - 1)} - \frac{\theta - 2}{2(\theta - 1)} - 1$$

Combined these two equations, then

$$C_1^* = -\frac{2}{3} + \frac{S}{3\theta V}, \quad C_2^* = -\frac{4}{3} + \frac{2S + 2V}{3V(\theta - 1)} \quad (1)$$

1) The relation between parameters and the ultimate price (C_1^*, C_2^*)

From the formula of the C_1^* and C_2^* , we can get

$$\begin{aligned}
 \frac{\partial C_1^*}{\partial \theta} &= -\frac{S}{3V\theta^2} < 0, & \frac{\partial C_1^*}{\partial \theta^2} &= \frac{2S}{3V\theta^3} > 0 \\
 \frac{\partial C_2^*}{\partial \theta} &= \frac{2S + 2V}{3V(1 - \theta)^2} > 0, & \frac{\partial C_2^*}{\partial \theta^2} &= -\frac{2S + 2V}{3V(1 - \theta)^3} < 0
 \end{aligned} \quad (2)$$

So with the increase of θ ($\theta \in (0, 1)$), C_1^* monotonic decreases and C_2^* increases.

If the value of θ negotiated is large by two sides, then the buyer tend to lower the price P by a big margin, on the contrary, the seller want to more higher price P , each side want more profit in the trade, there is a conflict of profits in the negotiation.

2) The relations between θ and V, S

If $P_1 = P_2, V_1 = V_2$, then $C_1^* = C_2^*$, which imply that $2V\theta^2 - (4V - S)\theta - S = 0$,
 this equation has solutions because $\Delta = (4V - S)^2 + 4S \cdot 2V = 16V^2 + S^2 \geq 0$, and

$$\theta = 1 - \frac{S}{4V} \pm \sqrt{1 + \frac{S^2}{16V^2}}$$

From these solutions, we can get

$$\frac{\partial \theta}{\partial S} = \frac{S - \sqrt{16V^2 + S^2}}{4V \sqrt{16V^2 + S^2}} < 0, \quad \frac{\partial \theta}{\partial V} = \frac{S \sqrt{16V^2 + S^2} + 4S^2}{4V^2 \sqrt{16V^2 + S^2}} > 0.$$

So with increase of S , θ decreases and θ increase with the increase of V .

In sum, when S increases, θ increase, then C_1 decrease and C_2 increase, it indicates that if potential profit is relatively high for the seller, the buyer will lower the price, on the other hand, the social wealth is more obvious for the seller, he may wish the trade succeed and will give up some profit to urge the trade to success. So after a negotiation for each other, the trade of discharge right will bargain with a lower price at last.

When V increases, θ increase, then C_1 decrease and C_2 increase, it indicates that the price is high for the buyer, on one hand, the buyer has seen the potential profit brought by the trade, he expect the trade succeed and tend to buy with a high price. On the other hand, the buyer wants to lower the price to reduce payment. The seller think the value of pollution discharge right tends to increase and will more scarcity, so he will give a higher price. Both sides have seen the potential profit in future, a higher price will be given for the both. After a negotiation for both sides, the trade of discharge right will bargain with a relatively higher price in the end.

3.3. Case Study

After the equilibrium achieved for the enterprises in the competition who need the pollution discharge right, how to make a deal with the government is a problem faced by the enterprise, the ultimate price can be achieved through the game model.

Suppose θ be a moderate value, let $\theta = 0.5$, the extra profit from the trade S equal to 10.

To the buyer of the pollution discharge right, the estimated price is V_1 , which come from the competition between the enterprises when balance is attained, then the estimated price of the buyer and the balanced price are identical, so $V_1 = \lambda = 200$, the price offered is P_1 and $P_1 = V_1(1 + C_1) = 200(1 + C_1)$.

To the seller of the pollution discharge right, the estimated price is V_2 ,

which is the same with the buyer, so $V_2 = V_1 = 200$, the price offered is P_1 and $P_2 = V_2(1 + C_2) = 200(1 + C_2)$.

After the prices are given by each side, there are negotiations between them, an acceptable price P will be achieved,

$$\begin{aligned} P &= P_2 + \theta(P_1 - P_2) = V_2(1 + C_2) + \theta[V_1(1 + C_1) - V_2(1 + C_2)] \\ &= 200(1 + C_2) + 200\theta(C_1 - C_2) \end{aligned}$$

When $V = 200, S = 10, \theta = 0.5$ from the formula (1),

we get $C_1^* = -\frac{19}{30}, C_2^* = -\frac{41}{15}$, then the price of the trade $P = 74$, the buyer receives

the lower price than the estimated price, the seller makes a concession for the sake of consequences and the common wealth, the trade will succeed in the end.

4. Conclusions

As the carbon pricing concerned, it is still at a groping stage, the market price mechanism doesn't form yet. There exist several issues in the practice, in most cases the carbon price is lower than imputed treatment cost when a real deal is made, it was influenced by human factor and fluctuated wildly, so the price doesn't reflect the scarcity of environmental resources. It is very important to the government to give a suitable initial price of carbon emission rights, too high or too low are not helpful to the market. So in this paper, a simple carbon pricing model based on game theory was constructed, the solution was also attained by deduction, case study showed the rationality of the model.

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