International Journal of u- and e- Service, Science and Technology
Vol.8, No. 7 (2015), pp.25-34
http://dx.doi.org/10.14257/ijunesst.2015.8.7.03

Components Replenishment Model of ATO Supply Chain with Supplier Alliance

Li Yuyu[1] and Huang Bo[2*]

[1]*College of Computer and Information Science, Chongqing Normal University, Chongqing, China, 400047*

[2*]*School of Economics and Business Administration, Chongqing University, Chongqing, China, 400044*

**Corresponding Author: Huang Bo. Email: huangbo@cqu.edu.cn*

Abstract

Under the assumption that ATO (assemble-to-order) supply chain consists of an assembler and two suppliers, who respectively provide one component to the assembler and may form an alliance to negotiate with the assembler, we develop a components replenishment model to study how the ATO supply chain decides components replenishment policies and the suppliers' alliance policies. The optimal solution of the model is obtained, and the effects of the bargaining power on the optimal solution are theoretically and numerically analyzed. We found that with the enhancement of the bargaining power of the assembler to one supplier, the component selling price and profits of this supplier reduce, and those of another supplier increase. Simultaneously, the expected profit of the assembler increases if its bargaining power is stronger than the other supplier, otherwise, it decreases. If both suppliers' bargaining powers are stronger than assembler's, they should bargain with the assembler jointly; if both of their bargaining powers are weaker than the assembler's, they will bargain with the assembler individually.

Keywords: *ATO supply chain, components replenishment, supplier alliance, negotiation*

1. Introduction

As the market competition has changed into quick response to customers' diverse and individual demands, ATO (Assemble-to-Order) has become one of the most popular production modes in many industries, such as computer, automobile, due to its ability of quick satisfying customers' diverse and individual demands at lower costs [1, 2]. Under ATO mode, only after the customers' orders arrive, does the assembler assemble products according to the orders. Therefore, the timely components supply is the key for ATO mode's success [3, 4].

Many researches on the component replenishment policies for ATO supply chain have been done. Zhao derived exact expressions for key ATO supply chain's performance metrics under non-split orders or split orders, and developed an efficient sampling method to estimate these metrics, *e.g.*, the expected delivery lead-times and the order-based fill-rates [5]. Guo, Fu and Chen developed a profit maximization model to analyze the optimal inventory and production decisions in a single-period ATO system with subassembly and uncertain future demand [6]. Doğru, Reiman and Wang considering ATO inventory systems with identical component lead times, used a stochastic program to study the assembler's inventory strategy [7]. Dan, Li, Huang, *et al.*, developed a components replenishment model of ATO manufacturer under enterprise-driven demand substitution, and analyzed the optimal replenishment policy is presented through theoretical and numerical [8]. Bernstein, DeCroix and Wang proposed a collection of

ISSN: 2005-4246 IJUNESST

allocation mechanisms to explore the impact of the allocation scheme on sales, profits, and capacity decisions, including the degree of capacity imbalance [9]. Reiman and Wang introduce a multi-stage stochastic program that provides a lower bound on the long-run average inventory cost of a general class of ATO inventory systems. The stochastic program also motivates a replenishment policy for these systems [10]. Karaarslan, Kiesmüller and De Kok analyzed the system under pure base stock policy and balanced base stock policy, and show how to determine the policy parameters that minimize average holding and backorder costs [11].

However, the above researches are only on the replenishment policies of single enterprise, *i.e.*, the assembler. In reality, the market competition has changed into the competition between supply chains. Therefore, assemblers and suppliers improve cooperation to enhance the competiveness of their supply chain. In recent year, some researches on the replenishment policies of ATO supply chain have been done, for example, Li, *et al.,* developed a two-stage Stackelberg game model to study the production and replenishment policy of ATO system where one supplier makes pull supply contract with the assembler while the other supplier makes push supply contract [12]. In additional, suppliers may form alliance to cooperate with assemblers in order to improve their position, for example, suppliers of electronic component, suppliers of automobile component [13, 14]. Nagarajan and Sošić [15] and Sošić [16] studied dynamic supplier alliances in a decentralized assembly system under three modes of competition, *i.e.*, Supplier Stackelberg, Vertical Nash, and Assembler Stackelberg models.

However, Stackelberg game is non-cooperative, under which the optimal policies of ATO supply chains can not be reached, and the core enterprises own the absolute power on making these decisions, their counter participants can only receive or reject these decisions. Therefore, Stackelberg game runs counter to the reality where assemblers and suppliers negotiate to decide the optimal policies and the profit-sharing arrangement to maximize the total profit of ATO supply chain and their individual profit. In other words, bargaining game models are much more suitable for replenishment decision problems in ATO supply chains, for example, Nagarajan and Bassok [17] proposed a bargaining framework for supply chains based on the research of Nash [18] and Muthoo [19].

The main contributions of this paper are as follows. Firstly, we proposed a bargaining game model of ATO supply chain to study the components production and replenishment policies, as well as the supplier's alliance policies, under the assumption that the ATO supply chain consists of an assembler and two suppliers, who may form an alliance to bargain with the assembler. Secondly, we theoretically and numerically analyzed the impact of the bargaining powers on the components production quantities and selling prices and the profits of the assembler and the suppliers, as well as whether the suppliers should collaborate in negotiation.

2. The Model

An ATO supply chain consists of one assembler (labeled as a) and two component suppliers (labeled as s_1 and s_2), who respectively sells component 1 and 2 to the assembler. The assembler purchases component 1 and 2 respectively from supplier 1 and 2 at the beginning of the sales season, and assembles one component 1 and one component 2 into a seasonal final product only after the customer's orders arrive. The demand for the product is uncertain and follows a distribution with density function $f(x)$ and distribution function $F(x)$. As one product consists of one component 1 and one component 2, the quantities of the final product, component 1 and component 2 are the same, *i.e.*, $q = q_1 = q_2$.

In order to maximize total profit of the ATO supply chain, the assembler and the suppliers decide the quantity of the two components, q , together. Then, the assembler negotiates with supplier s_1 and s_2 to decide their profit shares and the corresponding component purchase prices.

There are two modes of negotiation, *i.e.,* individual negotiation and joint negotiation. Under individual negotiation mode, supplier s_1 and s_2 individually negotiate with the assembler to decide their profit shares and the corresponding component purchase prices, under the condition that the assembler owns the right to decide the negotiation sequence. Under joint negotiation mode, supplier s_1 and s_2 form an alliance to jointly negotiate with the assembler to decide the profit shares of the assembler and the supplier alliance. Then, supplier s_1 and s_2 decide their profit shares and the corresponding component selling (purchase) prices through negotiation.

After the negotiation, the suppliers respectively produce component 1 and 2 at the unit production cost C_1 and C_2, and sell to the assembler at price P_1 and P_2. The assembler assembles products at unit assembly cost C and sell to customers at price P only after receives their orders. The salvage values of the unused components are zero, and the assembler bears all the loss. All the above information is common knowledge.

Now, we can get the total profit of the ATO supply chain as

$$E\pi = (P - C)\left[\int_0^q xf(x)dx + \int_q^\infty qf(x)dx\right] - (C_1 + C_2)q \,. \tag{1}$$

As the assembler purchases components at the beginning of the sales season and bears all the loss of unused components, the suppliers' profits are deterministic.

$$\pi_{sj} = (P_j - C_j)q\,,\quad j = 1,2\,. \tag{2}$$

The expected profit of the assembler is

$$E\pi_a = (P - C)\left[\int_0^q xf(x)dx + \int_q^\infty qf(x)dx\right] - (P_1 + P_2)q \,. \tag{3}$$

3. The Optimal Solution

Solving $\partial E\pi / \partial q = 0$, we can get the optimal replenishment quantity of the components as

$$q^* = F^{-1}\left(\frac{P - C - C_1 - C_2}{P - C}\right). \tag{4}$$

Substituting (4) into (1), we can get the optimal expected total profit $E\pi^*$.

From (4), we can get proposition 1 as follows.

Proposition 1. The optimal component production quantity rises with the rise of the selling price of final product, and declines with the rise of the unit production cost of every enterprise.

Proof: Respectively solving the first order derivative of the optimal component production quantity q^* with respect to the selling price of the final product, the unit production cost of every enterprise, we can get $\frac{\partial q^*}{\partial P} = \frac{C_1 + C_2}{(P - C)^2} > 0$, $\frac{\partial q^*}{\partial C} = -\frac{C_1 + C_2}{(P - C)^2} < 0$, $\frac{\partial q^*}{\partial C_1} = \frac{\partial q^*}{\partial C_2} = \frac{1}{C - P} < 0$. Therefore, the optimal component production quantity q^* is the strictly increasing function of the final product's selling price, and the strictly decreasing function of the unit production cost of every enterprise.

Q.E.D.

3.1. Individual Negotiation

Under individual negotiation mode, the assembler and supplier s_1 and s_2 make sequential negotiation to share $E\pi^*$ to obtain their respective profit $E\pi_a^I$, π_{s1}^I and π_{s2}^I. As the assembler makes sequential negotiation with supplier s_1 and s_2, the cooperation of ATO supply chain consists of two basic negotiations, in which the assembler and one supplier negotiate to determine their allocation of the profit. The bargaining solution of the basic negotiation is obtained by solving the following optimization problem

$$\max \ (\pi_a - d_a)^{\alpha_{asj}} (\pi_j - d_j)^{\beta_{asj}}, \quad j = 1,2, \tag{5}$$

$$\text{s.t.} \ (\pi_a, \pi_{sj}) \geq (d_a, d_{sj})$$

$$\pi_a + \pi_{sj} \leq \pi_k, \quad k = 1,2.$$

π_k ($k = 1,2$) is the profit should be allocated in the k^{th} negotiation. d_a and d_{sj} are the profits obtained by the assembler and suppliers when they fail to reach an agreement. Without losing generality, we assume $d_a = d_{s1} = d_{s2} = 0$. α_{asj} and β_{asj} are loosely representative of the respective bargaining power of assembler and supplier j, which satisfies $\alpha_{asj} + \beta_{asj} = 1$.

The assembler and suppliers have their own expectation on their allocation of profit $\overline{\pi}_i$, $i = a, s_1, s_2$. They make commitments to receiving no less than the expectation $\overline{\pi}_i$ before negotiation. But, these commitments are partial, and they may receive an allocation of profit lower than their expectation, that is, they revoke their commitment at a certain cost c_i, which is mainly the loss of credibility. If the allocation of profit obtained by participant i is no less than its expectation, it is unnecessary for it to revoke its commitment, that is, $c_i = 0$ if $\overline{\pi}_i \leq \pi_i$. If the allocation of profit obtained by participant i is less than its expectation, it bear the cost of revoking its commitment $c_i = K_i(\overline{\pi}_i - \pi_i)$, where $K_i > 0$.

Now, we can get the cost of participant i ($i = a, s_1, s_2$) revoking its commitment as

$$c_i = \begin{cases} 0 & \overline{\pi}_i \leq \pi_i \\ K_i(\overline{\pi}_i - \pi_i) & \overline{\pi}_i > \pi_i \end{cases}, \quad i = a, s_1, s_2. \tag{6}$$

The unique Nash equilibrium in the k^{th} negotiation ($k = 1,2$) is

$$(\overline{\pi}_a^*, \overline{\pi}_{sj}^*) = (\pi_a^*, \pi_{sj}^*) = \left((1 + K_a)\pi_k / (2 + K_a + K_{sj}), (1 + K_{sj})\pi_k / (2 + K_a + K_{sj})\right), \quad j = 1,2. \tag{7}$$

From (6) and (7), we can get the relationships between K_i and α_{asj}, β_{asj} as

$$\alpha_{asj} = (1 + K_a)/(2 + K_a + K_{sj}), \quad j = 1,2, \tag{8}$$

$$\beta_{asj} = (1 + K_{sj})/(2 + K_a + K_{sj}), \quad j = 1,2. \tag{9}$$

Proposition 2. If the assembler bargains with supplier s_1 in the first negotiation and with s_2 in the second negotiation, the profits of these enterprises are respectively $E\pi_a^{I*} = \alpha_{as1}\alpha_{as2}E\pi^*$, $\pi_{s1}^{I*} = \beta_{as1}E\pi^*$ and $\pi_{s2}^{I*} = \alpha_{as1}\beta_{as2}E\pi^*$. On the contrary, their profits are respectively $E\pi_a^{I*} = \alpha_{as1}\alpha_{as2}E\pi^*$, $\pi_{s1}^{I*} = \alpha_{as2}\beta_{as1}E\pi^*$ and $\pi_{s2}^{I*} = \beta_{as2}E\pi^*$.

Proof: If the assembler bargains with supplier s_1 in the first negotiation, their profit shares are respectively $\alpha_{as1} E\pi^*$ and $\beta_{as1} E\pi^*$. Then, the assembler bargains with supplier s_2 to allocate $\alpha_{as1} E\pi^*$, and respectively get $\alpha_{as1}\alpha_{as2} E\pi^*$ and $\alpha_{as1}\beta_{as2} E\pi^*$. Therefore, their profits are respectively $E\pi_a^{I*} = \alpha_{as1}\alpha_{as2} E\pi^*$, $\pi_{s1}^{I*} = \beta_{as1} E\pi^*$ and $\pi_{s2}^{I*} = \alpha_{as1}\beta_{as2} E\pi^*$.

Similarly, if the assembler bargains with supplier s_2 in the first negotiation, their profits are respectively $E\pi_a^{I*} = \alpha_{as1}\alpha_{as2} E\pi^*$, $\pi_{s1}^{I*} = \alpha_{as2}\beta_{as1} E\pi^*$ and $\pi_{s2}^{I*} = \beta_{as2} E\pi^*$.

Q.E.D.

Proposition 3. The supplier who bargains with the assembler in the first negotiation pays the assembler $\prod_{j=1,2} (1-\alpha_{asj}) E\pi^*$, $j = 1,2$.

Proof: From proposition 2, we can find that the assembler always gets $E\pi_a^{I*} = \alpha_{as1}\alpha_{as2} E\pi^*$ in two negotiation sequences. Meanwhile, the supplier gets $\prod_{j=1,2} (1-\alpha_{asj}) E\pi^*$ more profits bargaining in the first negotiation than bargaining in the second negotiation. However, the suppliers have no right to decide the negotiation sequence, which is owned by the assembler. Therefore, they have to pay the whole extra profits $\prod_{j=1,2} (1-\alpha_{asj}) E\pi^*$ for bargaining in the first negotiation.

Q.E.D.

Proposition 4. The final profits of the assembler and the two suppliers are respectively $E\pi_a^{I*} = (1-\alpha_{as2}\beta_{as1} - \alpha_{as1}\beta_{as2}) E\pi^*$ and $\pi_{sj}^{I*} = \alpha_{as(3-j)}\beta_{asj} E\pi^*$, $j = 1,2$.

Proof: From proposition 2 and 3, we can get that the assembler gets a profit share $E\pi_a^{I*} = \alpha_{as1}\alpha_{as2} E\pi^*$ and a payment $\prod_{j=1,2} (1-\alpha_{asj}) E\pi^*$ from the supplier who bargains in the first negotiation, as a result, its final profits are $E\pi_a^{I*} = (1-\alpha_{as2}\beta_{as1} - \alpha_{as1}\beta_{as2}) E\pi^*$, while the supplier who bargains in the first negotiation gets a profit share $\pi_{sj}^{I*} = \beta_{asj} E\pi^*$ at the cost of $\prod_{j=1,2} (1-\alpha_{asj}) E\pi^*$, $j = 1,2$, as a result, its final profits are $\pi_{sj}^{I*} = \alpha_{as(3-j)}\beta_{asj} E\pi^*$, the same as its final profits bargaining in the second negotiation. Therefore, the final profits of the assembler and the two suppliers are respectively $E\pi_a^{I*} = (1-\alpha_{as2}\beta_{as1} - \alpha_{as1}\beta_{as2}) E\pi^*$ and $\pi_{sj}^{I*} = \alpha_{as(3-j)}\beta_{asj} E\pi^*$.

Q.E.D.

Substituting (2) and (4) into $\pi_{sj}^{I*} = \alpha_{as(3-j)}\beta_{asj} E\pi^*$, $j = 1,2$, we can get the optimal sales prices of their components are as follows

$$P_j^{I*} = \alpha_{as(3-j)}\beta_{asj} E\pi^* \Big/ F^{-1}\left(\frac{P - C - C_1 - C_2}{P - C}\right) + C_j, \quad j = 1,2. \tag{10}$$

Proposition 5. The optimal selling prices of the suppliers' components decrease with the rise of the bargaining powers of the assembler to them, and increase with the rise the bargaining powers of the assembler to the suppliers of the other components.

Proof: Respectively solving the first order derivative of the optimal selling prices of the suppliers' components P_j^{I*} ($j = 1,2$) with respect to the assembler's bargaining powers

α_{asj} and $\alpha_{as(3-j)}$, we can get $\frac{\partial P_j^{I*}}{\partial \alpha_{asj}} = -\frac{\alpha_{as(3-j)} E\pi^*}{q^*} < 0$ and $\frac{\partial P_j^{I*}}{\partial \alpha_{as(3-j)}} = \frac{\beta_{asj} E\pi^*}{q^*} > 0$. Therefore, the optimal selling prices of the suppliers' components are the strictly decreasing functions of the bargaining powers of the assembler to them, and the strictly increasing functions of the bargaining powers of the assembler to the suppliers of the other components.

Q.E.D.

Proposition 6. The stronger the bargaining power of the assembler to one supplier is, the smaller the profit of this supplier is, while the bigger the profit of the other supplier is. At the same time, if the bargaining power of the assembler is stronger than the other supplier, its profit increases, otherwise, its profit decreases.

Proof: Respectively solving the first order derivative of every enterprise's profits with respect to the assembler's bargaining powers α_{asj} and $\alpha_{as(3-j)}$, $j = 1,2$, we can get $\partial \pi_j^{I*} / \partial \alpha_{asj} = -\alpha_{as(3-j)} E\pi^* < 0$, $\partial \pi_j^{I*} / \partial \alpha_{as(3-j)} = (1 - \alpha_{asj}) E\pi^* > 0$ and $\partial \pi_a^{I*} / \partial \alpha_{asj} = (2\alpha_{as(3-j)} - 1) E\pi^*$. Therefore, the profits of one supplier are the strictly decreasing function of the assembler's bargaining power to it, while the increasing function of the assembler's bargaining powers to the other supplier. If the assembler's bargaining power is stronger than one supplier, its profits are the strictly increasing function of its bargaining power to the other supplier, and the strictly decreasing function of its bargaining power to the other supplier, otherwise.

Q.E.D.

3.2. Joint Negotiation

Under joint negotiation mode, supplier s_1 and s_2 form an alliance s to jointly negotiate with the assembler to decide the profit shares, and negotiate with each other to decide the profit shares of every enterprise and the corresponding component selling prices.

We further assume that the bargaining power of the alliance is the average of the suppliers' bargaining powers, *i.e.*,

$$\beta_{as} = (\beta_{as1} + \beta_{as2})/2 . \tag{11}$$

And the bargaining powers of the suppliers to each other are as follows.

$$\alpha_{sjs(3-j)} = \beta_{s(3-j)sj} = (1 + K_{sj})/(2 + K_{s1} + K_{s2}), \quad j = 1,2 . \tag{12}$$

Comparing (8), (9) and (12), we can get that the relationships among the bargaining powers of all enterprises to each other are as follows.

$$\frac{\alpha_{sjs(3-j)}}{\beta_{sjs(3-j)}} = \frac{\beta_{s(3-j)sj}}{\alpha_{s(3-j)sj}} = \frac{\beta_{asj} / \alpha_{asj}}{\beta_{as(3-j)} / \alpha_{as(3-j)}} . \tag{13}$$

Proposition 7. The suppliers bargain with the assembler jointly if their unit revoking costs satisfy $(k_{s1} + 1)(k_{s2} - k_a) + (k_{s2} + 1)(k_{s1} - k_a) > 0$, otherwise, they bargain with the assembler individually.

Proof: If the suppliers bargain with the assembler jointly, the supplier alliance gets profit share $\pi_s^{J*} = \beta_{as} E\pi^*$. Then, they get final profits $\pi_{sj}^{J*} = \alpha_{sjs(3-j)} \beta_{as} E\pi^*$, $j = 1,2$, through internal negotiation. From proposition 3, we know that suppliers' final profits under individual bargaining mode are $\pi_{sj}^{I*} = \alpha_{as(3-j)} \beta_{asj} E\pi^*$.

Obviously, only if the suppliers' profits obtained through joint negotiation are more than that through individual negotiation, will the suppliers bargain jointly. Solving $\pi_{sj}^{J*} - \pi_{sj}^{I*}$, $j = 1,2$, we can find that $\pi_{sj}^{J*} - \pi_{sj}^{I*} > 0$ if $(k_{s1}+1)(k_{s2}-k_a)+(k_{s2}+1)(k_{s1}-k_a) > 0$. Therefore, their profits obtained through joint negotiation are more than that through individual negotiation if $(k_{s1}+1)(k_{s2}-k_a)+(k_{s2}+1)(k_{s1}-k_a) > 0$, which means they will bargain with the assembler jointly, otherwise, they bargain with the assembler individually.

Q.E.D.

Obviously, $(k_{s1}+1)(k_{s2}-k_a)+(k_{s2}+1)(k_{s1}-k_a) > 0$ if $k_{s1}-k_a > 0$ and $k_{s2}-k_a > 0$ while $(k_{s1}+1)(k_{s2}-k_a)+(k_{s2}+1)(k_{s1}-k_a) < 0$ if $k_{s1}-k_a < 0$ and $k_{s2}-k_a < 0$. As mentioned above, the bigger the unit revoking cost is, the stronger the bargaining power is. Therefore, if both suppliers' bargaining powers are stronger than that of the assembler, they should bargain with the assembler jointly; if both of their bargaining powers are weaker than that of the assembler, they will bargain with the assembler individually.

Proposition 8. Under the joint negotiation mode, the profits of the assembler and the suppliers are respectively $\pi_a^{J*} = \alpha_{as} E\pi^*$ and $\pi_{sj}^{J*} = \alpha_{sjs(3-j)}\beta_{as} E\pi^*$, $j = 1,2$.

Proof: The proof of proposition 8 can be obtained by the proof of proposition 7.

Q.E.D.

Substituting (2) and (4) into $\pi_{sj}^{J*} = \beta_{sjs(3-j)}\beta_{as} E\pi^*$, $j = 1,2$, we can get the optimal sales prices of their components are as follows

$$P_j^{J*} = \alpha_{sjs(3-j)}\beta_{as} E\pi^* \Big/ F^{-1}\left(\frac{P - C - C_1 - C_2}{P - C}\right) + C_j, \quad j = 1,2. \tag{14}$$

Proposition 9. The optimal components selling price and profits of the supplier increase with the rise of its bargaining power to the other supplier, and decrease with the rise the bargaining power of the assembler to the supplier alliance and that of the other supplier to it. The assembler's optimal profits are the strictly increasing function of its bargaining power to the supplier alliance, while is irrelative to the suppliers' bargaining powers to each other.

Proof: Respectively solving the first order derivative of the optimal components selling prices and profits of the suppliers, P_j^{J*} and π_{sj}^{J*}, $j = 1,2$, as well as the assembler's profits with respect to the assembler's bargaining power to the supplier alliance α_{asj}, as well as the supplier's bargaining powers to each other α_{s1s2} and α_{s2s1}, we can get $\frac{\partial P_j^{J*}}{\partial \alpha_{as}} = -\frac{\alpha_{sjs(3-j)} E\pi^*}{q^*} < 0$, $\frac{\partial P_j^{J*}}{\partial \alpha_{sjs(3-j)}} = \frac{\beta_{as} E\pi^*}{q^*} > 0$, $\frac{\partial P_j^{J*}}{\partial \alpha_{s(3-j)sj}} = -\frac{\beta_{as} E\pi^*}{q^*} < 0$, $\frac{\partial \pi_{sj}^{J*}}{\partial \alpha_{as}} = -\alpha_{sjs(3-j)} E\pi^* < 0$, $\frac{\partial \pi_{sj}^{J*}}{\partial \alpha_{sjs(3-j)}} = \beta_{as} E\pi^* > 0$, $\frac{\partial \pi_{sj}^{J*}}{\partial \alpha_{s(3-j)sj}} = -\beta_{as} E\pi^* < 0$, $\frac{\partial \pi_a^{J*}}{\partial \alpha_{as}} = E\pi^* > 0$ and $\frac{\partial \pi_a^{J*}}{\partial \alpha_{sjs(3-j)}} = 0$. Therefore, the optimal components selling price and profits of the supplier are the strictly increasing functions of its bargaining power to the other supplier, and the strictly decreasing functions of the assembler's bargaining power to the supplier alliance and the other supplier's bargaining power to it. The assembler's optimal profits increase with rise of its bargaining power to the supplier alliance, while is irrelative to the suppliers' bargaining powers to each other.

Q.E.D.

4. Numerical Analysis

An assembler plans to manufacture a seasonal product with uncertain demand, which follows normal distribution with average of 500 and standard deviation of 5, *i.e.*, $N(500,25)$. One product consists of one component 1 and one component 2, which respectively replenished from supplier s_1 and supplier s_2. The unit producing costs of the assembler and supplier s_1 and s_2 are respectively 10, 8 and 9. The selling price of the product is 40. The salvage value of the unused components in the end of the selling season is 0.

The parameters of all enterprises' revoking costs are respectively $K_a = 1.2$, $K_{s1} = 0.6$ and $K_{s2} = 0.8$. From (8), (9), (11) and (12), we can get the bargaining powers of all enterprises and supplier alliance are respectively $\alpha_{as1} = 0.579$, $\alpha_{as2} = 0.55$, $\alpha_{as} = 0.564$, $\alpha_{s1s2} = 0.471$, $\beta_{as1} = 0.421$, $\beta_{as2} = 0.45$, $\beta_{as} = 0.436$ and $\beta_{s1s2} = 0.529$.

Firstly, we find the optimal solutions under individual negotiation mode. From (4), we can obtain the optimal quantity of the components as $q^* = 499$. Substituting the optimal quantity into (1), we can get the optimal profit of the ATO supply chain as $E\pi^* = 6441$. From proposition 4 and (10), we can obtain the optimal profits of all enterprises as $E\pi_a^{I*} = 3271.35$, $\pi_{s1}^{I*} = 1491.60$ and $\pi_{s2}^{I*} = 1678.05$, as well as the optimal selling prices of the components as $P_1^{I*} = 10.99$ and $P_2^{I*} = 12.36$.

Then, we find the optimal solutions under joint negotiation mode. From proposition 8 and (14), we can get the optimal profits of all enterprises as $E\pi_a^{J*} = 3635.77$, $\pi_{s1}^{J*} = 1320.11$ and $\pi_{s2}^{J*} = 1485.12$, as well as the optimal selling prices of the components as $P_1^{J*} = 10.64$ and $P_2^{J*} = 11.98$.

Comparing the optimal solutions of these two modes, we can find that the suppliers' profits under joint negotiation mode are less than that under individual mode. Therefore, they will bargain individually. The results illuminate proposition 7, *i.e.*, if both of the suppliers' bargaining powers are weaker than that of the assembler, they will bargain with the assembler individually.

Now, we make a sensitivity analysis of the main parameters. Taking the assembler's bargaining power to supplier s_1, α_{as1}, as example, we analyze the impact of the change of enterprise's bargaining power on their profits, components' selling prices and whether forming supplier alliance.

Table 1. The Effect of α_{as1} on the Optimal Solutions

α_{as1}	α_{as2}	q	Individual Negotiation			Joint Negotiation			Collaborate or Not
			$E\pi_a^{I*}$	π_{s1}^{I*}	$E\pi_{s2}^{I*}$	$E\pi_a^{J*}$	π_{s1}^{J*}	π_{s2}^{J*}	
0.4	0.55	499	3156.09	2125.53	1159.38	3059.47	2188.04	1193.48	Yes
0.5		499	3220.50	1771.27	1449.22	3381.52	1682.71	1376.76	No
0.6		499	3284.91	1417.02	1739.07	3703.57	1229.05	1508.38	No
0.4	0.45	499	3284.91	1739.07	1417.02	2737.42	2040.74	1662.83	Yes
0.5		499	3220.50	1449.22	1771.27	3059.47	1521.69	1859.64	Yes
0.6		499	3156.09	1159.38	2125.53	3381.52	1079.81	1979.66	No

From Table 1, we can find that as proposition 6 points out with the enhancement of the assembler's bargaining power to one supplier, this supplier's profits keep decreasing, while the other supplier's profits keep increasing. Meanwhile, if the assembler's bargaining power is stronger than the other supplier's, its profits keep increasing, otherwise, it keeps decreasing.

Besides, we can find from Table 1 that both of the suppliers' profits through joint negotiation are more than that through individual negotiation when both supplier's bargaining powers are stronger than that of the assembler, so they should bargain with the assembler jointly; when both of their bargaining powers are weaker than that of the assembler, at least one of their profits through joint negotiation are less than that through individual negotiation, which means they will bargain with the assembler individually. The results of sensitivity analysis illuminate proposition 7.

5. Conclusions

We proposed a bargaining game model between an assembler and two suppliers (or supplier alliance), who respectively provide one component to the assembler, to study their profits-sharing policies and suppliers' alliance policies. We found that the assembler can force the suppliers to pay for their position in the negotiation sequence due to its ability of deciding negotiation sequence; with the enhancement of the bargaining power of the assembler to one supplier, the component sales price and profit of this supplier reduce, and those of another supplier increase, simultaneously, the expected profit of the assembler increases if its bargaining power is stronger than the another supplier, otherwise, it decreases; if both suppliers' bargaining powers are stronger than assembler's, they should bargain with the assembler jointly; if both of their bargaining powers are weaker than the assembler's, they will bargain with the assembler individually.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (No. 71102178), Social Science Foundation of Ministry of Education of China (Grant no. 11YJC630070), and the Fundamental Research Funds for the Central Universities (Grant No. CDJSK100201 and CQDXWL-2014-025).

References

[1] M. Rabbani, M. Baghersad and R. Jafari, "A new hybrid GA-PSO method for solving multi-period inventory routing problem with considering financial decisions", Journal of Industrial Engineering and Management, vol. 6, no. 4, **(2013)**, pp. 909-929.

[2] C. Chackelson, A. Errasti and S. Martinez, "Supply strategy configuration in fragmented production systems: an empirical study", Journal of Industrial Engineering and Management, vol. 6, no. 3, **(2013)**, pp. 742-750.

[3] Y. Y. Li and B. Huang, "Product Selection and Components Replenishment Model of ATO Manufacturer under Heterogeneous Demand", Journal of Applied Mathematics, vol. 2013, **(2013)**, pp. 1-8.

[4] M. M. Gil, P. M. Espiñeira and J. M. L. Cerqueira, "Supply chain management in automotive international logistics: a scenario and its challenges", Journal of System and Management Sciences, vol. 1, no. 4, **(2011)**, pp. 83-95.

[5] Y. Zhao, "Analysis and evaluation of an assemble-to-order system with batch ordering policy and compound Poisson demand", European Journal of Operational Research, vol. 198, no. 3, **(2009)**, pp. 800-809.

[6] J. Guo, K. Fu and G. Y. Chen, "Optimal inventory and production decisions for an ATO system with variable capacity", Chinese Journal of management science, vol. 20, no. 3, **(2012)**, pp. 94-103.

[7] M. K. Doğru, M. I. Reiman and Q. Wang, "A stochastic programming based inventory policy for assemble-to-order systems with application to the W model", Operations research, vol. 58, no. 4-1, **(2010)**, pp. 849-864.

[8] B. Dan, Y. Y. Li and B. Hang, “Replenishment policy of ATO manufacturer under enterprise-driven demand substitution”, Computer Integrated Manufacturing Systems, vol. 18, no. 6, **(2012)**, pp. 1257-1262.
[9] F. Bernstein, G. A. DeCroix and Y. Wang, “The impact of demand aggregation through delayed component allocation in an assemble-to-order system”, Management Science, vol. 57, no. 6, **(2011)**, pp. 1154-1171.
[10] M. I. Reiman and Q. Wang, “A stochastic program based lower bound for assemble-to-order inventory systems”, Operations Research Letters, vol. 40, no. 2, **(2012)**, pp. 89-95.
[11] A. G. Karaarslan, G. P. Kiesmüller and A. G. De Kok, “Analysis of an assemble-to-order system with different review periods”, International Journal of Production Economics, vol. 143, no. 2, **(2013)**, pp. 335-341.
[12] Y. Y. Li, B. Dan and G. Guo, “Research on production and replenishment policy of ATO system under pull and push contract”, Journal of Industrial Engineering/Engineering Management, vol. 24, no. 3, **(2010)**, pp. 144-149.
[13] P. K. C. Lee, A. C. L. Yeung and E. T. C. Cheng, “Supplier alliances and environmental uncertainty: an empirical study”, International Journal of Production Economics, vol. 120, no. 1, **(2009)**, pp. 190-204.
[14] M. Nagarajan and G. Sošić, “Game-theoretic analysis of cooperation among supply chain agents: Review and extensions”, European Journal of Operational Research, vol. 187, no. 3, **(2008)**, pp. 719-745.
[15] M. Nagarajan and G. Sošić, “Coalition stability in assembly models”, Operations Research, vol. 57, no. 1, **(2009)**, pp. 131-145.
[16] G. Sošić, “Impact of demand uncertainty on stability of supplier alliances in assembly models”, Production and Operations Management, vol. 20, no. 6, **(2011)**, pp. 905-920.
[17] M. Nagarajan and Y. Bassok, “A bargaining framework in supply chains: The assembly problem”, Management Science, vol. 54, no. 8, **(2008)**, pp. 1482-1496.
[18] J. F. Nash, “The bargaining game”, Econometrica, vol. 18, no. 2, **(1950)**, pp. 155-162.
[19] A. Muthoo, “A bargaining model based on the commitment tactic”, Journal of Economic theory, vol. 69, no. 1, **(1996)**, pp. 134-152.

