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Strategy of Production and Ordering in Closed-loop Supply Chain under Stochastic Yields and Stochastic Demands

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Abstract

A closed-loop supply chain system consisting of one manufacturer with random yield and one retailer with random demand is discussed in this paper. The models of expected profit of supply chain system are founded under centralized decision and decentralized decision, respectively. The optimal strategies of yield and order are obtained in two modes. It is proved that the order of retailer is not affected by yield randomness. That is showed by numerical example, the results show: the scheduled production of new product for manufacture will decrease and the total profit and each participant of supply chain will increase as recovery price increases, when the cost of remanufacturing production is lower than the new product.

Keywords: *Close-loop supply chain; Remanufacturing; Random yield; Random demand*

1. Introduction

Due to the resource shortage and environment deterioration, manufacturing enterprises are facing the challenge of the coordinated development of production and environment. These enterprises are required to make the integrated optimization for the forward logistics of new products and reverse logistics of waste products and carry out the closed-loop supply chain management. Compared with the traditional forward supply chain, the closed-loop supply chain could reduce environment pollution, improve the utilization rate of resources, and extend the service life of the products and minimize the resource consumption and environmental influence on sustainable development. Therefore, it attracts extensive attention from academia and enterprises [1-3].

At present, the closed-loop supply chain has been successfully applied in many industries, such as electronics, automobile and machinery and a lot of related theoretical achievements have been made. From the point of maximizing enterprises profit and minimizing expected total costs, various models were established with game theory or contract theory to explore the product recovery, pricing, supply-demand decision, interest coordination and channel design problems [4-6]. Previous studies were mostly based on given conditions, while enterprises were under uncertain conditions, which mainly included demand uncertainty, supply uncertainty and uncertainty of recovery amount of waste products, etc. Yajun Guo *et al.* studied the supply chain of sales and recovery which was responsible by retailers under random demands, analyzed the pricing decision problems among node enterprises, proposed the revenue and expense sharing contract and realized the coordinated closed-loop supply chain [7]. Keyong Zhang *et al.* established a closed-loop supply chain model of single manufacturer and single retailer under random

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demands with game theory and studied the pricing strategy under centralized decision and decentralized decision [8]. Fuan Zhang *et al.* established the model of centralized decision and decentralized decision of the closed-loop supply chain in which random demands and recovery amount were sensitive to recovery price, provided optimal recovery price of different decision models and optimal outputs of new and remanufactured products, and made contracts to coordinate interests of all parties to provide optimal contract parameters [9]. Junhua Guo *et al.* considered the payment willingness differences among consumers under random demands of new and remanufactured products, constructed the sharing mechanism of extended responsibility of producers in the wholesale price agreement of remanufactured products, and provided the optimal conditions of the model [10]. The above scholars only discussed the problems of demand uncertainty in the closed-loop and obtained some conclusions, but the uncertainty of supply and demand in closed-loop chain was not studied.

In the traditional supply chain, random outputs and demands are one of the most common and important aspects in supply and demand uncertainty. Many previous studies were based on random outputs or random outputs and demands. Ismail et al. had studied the influence of different pricing and ordering decision on enterprise profit in the condition of random remanufactured product outputs [11]. He et al. pointed out that those random outputs might lead to the risk of overproduction and insufficient production in the supply chain, established 4 different collaboration contracts based on risk sharing, and compared these 4 contracts based on numerical analysis [12]. On the basis of random supplier outputs and distributor demand, Guohong Shi *et al.* assumed that there was a secondary market and studied the production and ordering strategy for centralized and decentralized decision [13]. Daozhi Zhao *et al.* established a supply chain model consisting of an output-randomized producer and a supply-randomized retailer and proposed that VMI coordination contracts could optimize the overall performance of the supply chain [14]. Xiaomin Zhao *et al.* studied the two-echelon closed-loop supply chain consisting of single manufacturer and single retailer with the game theory and analyzed the pricing strategy and system performance of the supply chain under three different market forces and structures [15].

This paper studied a two-echelon closed-loop supply chain consisting of single manufacturer and single retailer. Firstly, the random outputs were introduced into the closed-loop supply chain and three game structures of supply and demand sides were studied under random outputs and demands. Secondly, we analyzed optimal decisions and profits of each member under these three game structures and compared profit relationships among manufacturers, retailers and the whole supply chain. Thirdly, we interpreted the influence of recovery price on the profits of each member in the supply chain through numerical examples.

2. Model Descriptions and Assumptions

For a closed-loop supply chain consisting of one manufacturer and one retailer, we assumed that the production yield of new materials was random and consumer demand is uncertain. Figure 1 shows the closed-loop supply chain system.

2.1 Notation Description:

The symbols used in the paper were introduced as follows:

c_m —The marginal cost of a new product manufactured with raw materials, a constant;

c_r — The marginal cost of remanufacturing a used-product into a new one, a constant;

$c_m > c_r$;

ω —The wholesale price;

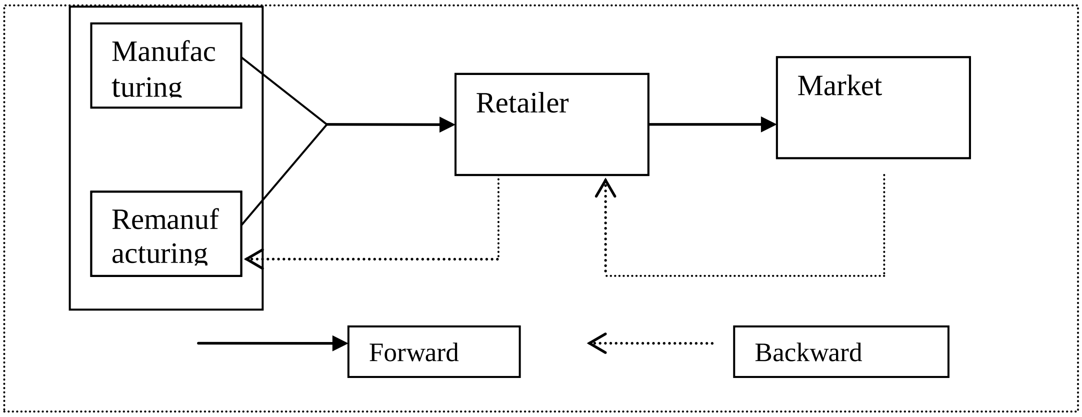

Figure 1. Closed-loop Supply Chain System

p —The sale price;
h —Inventory holding cost per unit;
b —shortage cost per unit;
q —The order amount;
R —The acquisition price of used-product;
p_r —The transfer payment price which the manufacturer paid to the retailer.

2.2 Assumptions

To facilitate this discussion, it is assumed that:

1) Both the manufacturer and retailer are the completely rational and neutral risk decision-maker, and market information is available to both sides.

2) Q (decision variable) is the planned production of new products of the manufacturer; U is the random output factor of new products; uQ is the actual yield; u is a random variable in the range $[0,1]$ with the density function of $g(u)$ and the distribution function of $G(u)$; $Eu = \mu$ is the mean value; $G(u)$ is the differentiable and strictly increasing function and u is not related to Q; p is the same price of the new products of manufacturer and remanufactured products sold in the market.

3) The retailers are facing the random market demand X, whose distribution function and density function are respectively $F(x)$ and $f(x)$. The mean value is $E(x) = d$, and here X and U are independent of each other.

4) Waste recovery amount is an increasing function of market recovery price and set as $\alpha + \beta R$; if $\alpha > 0$, the amount of waste product that consumers volunteer to return when the recovery price per unit paid to consumers by retailers is zero and these consumers have the completely environmental awareness. Therefore, α can be considered an index to measure social environmental awareness; $\beta > 0$ represents the levels that consumers are sensitive to the recovery price [8]; in addition, all the waste products from recovery will be used to remanufacture new products.

5) If $p > \omega > c_m$ and $(p+b)\mu - c_m > 0$, the manufacture can be ensured and non-infinite remanufacture can be avoided.

6) The manufacturer is a leader of the supply chain while the retailer is a follower. All members' decisions in the supply chain are aimed to achieve maximized profit.

3. Production Ordering Strategies of Closed-loop Supply Chain under Centralized Decision Making

Under centralized decision making, the supply chain is an integral whole and the target of decision making is to make the whole supply chain profit maximum.

$$\begin{aligned}\Pi^c = & pE_{UX}[\min\{uQ+\alpha+\beta R, x\}]-hE_{UX}[(uQ+\alpha+\beta R-x)^+] \\ & -bE_{UX}[(x-uQ-\alpha-\beta R)^+]-c_mQ-c_r(\alpha+\beta R)-R(\alpha+\beta R)\end{aligned} \quad (1)$$

Theorem 1 Under centralized decision making, the total expected profit of supply chain is a concave function about Q and the optimal planned production Q^* of new products is expressed as

$$\int_0^1\int_{uQ^*+\alpha+\beta R}^{+\infty} uf(x)g(u)dxdu = \frac{h\mu+c_m}{p+h+b} \quad (2)$$

Proof: According to Eq. (1), we can derive the process of Π^c as follows:

$$\begin{aligned}\Pi^c = & (p+h)d - h(\mu Q+\alpha+\beta R)-c_mQ-c_r(\alpha+\beta R)-f(\alpha+\beta R) \\ & -(p+h+b)\int_0^1\int_{uQ+\alpha+\beta R}^{+\infty}(x-uQ-\alpha-\beta R)f(x)g(u)dxdu\end{aligned}$$

$$\frac{\partial \Pi^c}{\partial Q} = -h\mu - c + (p+h+b)\int_0^1\int_{uQ+\alpha+\beta R}^{+\infty} uf(x)g(u)dxdu$$

$$\frac{\partial^2 \Pi^c}{\partial Q^2} = -(p+b+h)\int_0^1 u^2 f(uQ+\alpha+\beta R)g(u)du < 0$$

Therefore, Π^c is a concave function of Q,

Equation $\max_Q \Pi^c$ have unit optimal solution Q^*,

Making $\frac{\partial \Pi^c}{\partial Q} = 0$, we an obtain Eq. (2) .

According to Eq. (2), we know that the optimal planned production Q^* of new products will decrease with increasing per unit product cost c_m of a new product and per unit inventory holding cost h, will increase with increasing the b .

4. Production Ordering Strategies of Closed-loop Supply Chain under Decentralized Decision Making

When manufacturers dominate the supply chain, the game steps are interpreted as follows. Firstly, the manufacturers will determine the planned production Q of new products. Secondly, the retailers will determine the order amount of the products. Since the game is a dynamic game under complete information, whose equilibrium is sub-game perfect Nash Equilibrium, we can use the backward induction for analysis and consider the optimal decision of retailers under the given Q value.

Π^R—The expected profit functions of the retailer. The function is:

$$\begin{aligned}\Pi^R = & pE_{UX}[\min\{UQ+\alpha+\beta R, X, q\}^+] - \omega E_U[\min\{UQ+\alpha+\beta R, q\}] \\ & +(\alpha+\beta R)(p_r - R) - hE_U[\min\{UQ+\alpha+\beta R, q\} - X]^+ \\ & -bE_{UX}[(X - \min\{UQ+\alpha+\beta R, X\})^+]\end{aligned} \quad (3)$$

Theorem 2 In the model of Stackelberg Game dominated by manufacturers, under the designated planned production Q of new products of manufacturers, the expected profit of retailers Π^R have only one maximum value for its order amount q and the optimal order amount q^{**} of retailers is

$$q^{**} = F^{-1}\left(\frac{p+b-\omega}{p+b+h}\right) \quad (4)$$

Proof: due to

$$\frac{\partial \Pi^R}{\partial q} = \int_{\frac{q-\alpha-\beta R}{Q}}^{1}\int_0^q hf(x)g(u)dxdu + \int_{\frac{q-\alpha-\beta R}{Q}}^{1}\int_q^{+\infty}(p+b)f(x)g(u)dxdu$$

$$-\int_{\frac{q-\alpha-\beta R}{Q}}^{1}\omega g(u)du$$

$$= \int_{\frac{q-\alpha-\beta R}{Q}}^{1} -hF(q)g(u)du + \int_{\frac{q-\alpha-\beta R}{Q}}^{1}(1-F(q))(p+b)g(u)du$$

$$-\omega\left(1-G\left(\frac{q-\alpha-\beta R}{Q}\right)\right)$$

$$= [(p+b-\omega)-(p+b+h)][1-G\left(\frac{q-\alpha-\beta R}{Q}\right)]$$

Let $\frac{\partial \Pi^R}{\partial q} = 0$, then we obtained

$$q_1 = F^{-1}\left(\frac{p+b-\omega}{p+b+h}\right) \text{ Or } q_2 = Q+\alpha+\beta R$$

If $q^{**} = q_2$, manufactures will not produce or infinitely produce re-used products and the situation contradicts Assumption 5.

So there is the only reasonable stagnation point of Π^R,

Considering $F(x)$ is an increasing function,

When $q < F^{-1}\left(\frac{p+b-\omega}{p+b+h}\right)$,

We can get $\frac{\partial \Pi^R}{\partial q} > 0$, and expected profit of retailers Π^R increase as its order amount q increase;

When $q < F^{-1}\left(\frac{p+b-\omega}{p+b+h}\right)$,

We can get $\frac{\partial \Pi^R}{\partial q} < 0$ and expected profit of retailer Π^R is decreased when its order amount q increases.

Therefore, Π^R obtains the maximum value if $q^{**} = F^{-1}(\frac{p+b-\omega}{p+b+h})$

According to Eq. (4), we know that when the manufacturers dominate the market, the order amount of retailers will be not influenced by output randomness. It will increase with the increasing net profit per unit $p-\omega$ and will decrease with the increasing b and h.

Next, Manufacturers will determine the optimal planned Q of the new product, Π^M — The expected profit functions of the manufacturer. The function is:

$$\Pi^M = \omega E_U[\min\{UQ+\alpha+\beta R, q\}] - hE_U[(UQ+\alpha+\beta R-q)^+] \\ -c_m Q - c_r(\alpha+\beta R) - p_r(\alpha+\beta R) \tag{5}$$

Theorem 3 In the model of Stackelberg Game dominated by manufacturers, the expected profit of manufacturers is a concave function about Q and the optimal planned production Q^{**} of new products of manufacturers is expressed as

$$\int_0^{\frac{q-\alpha-\beta R}{Q^{**}}} ug(u)du = \frac{h\mu + c_m}{\omega + h} \tag{6}$$

Proof: According to Eq. (5), we can derive the process of Π^M as follows:

$$\frac{\partial \Pi^M}{\partial Q} = (\omega+h)\int_0^{\frac{q-\alpha-\beta R}{Q}} ug(u)du - h\mu - c_m$$

$$\frac{\partial^2 \Pi^M}{\partial Q^2} = -\frac{1}{Q^2}(\omega+h)\frac{q-\alpha-\beta R}{Q} g(\frac{q-\alpha-\beta R}{Q}) < 0$$

Therefore, Π^M is a concave function of Q,

If $\frac{\partial \Pi^M}{\partial Q} = 0$, one can obtain Eq. (6).

5. Comparison of Profits

Given below under the centralized decision-making and decentralized decision-making total expected profit comparison results

Theorem 4 In the closed-loop supply chain of random outputs and demands, the whole expected profit of the supply chain under centralized decision making is larger than that under decentralized decision making. $\Pi^c \geq \Pi^R + \Pi^M$.

Proof: Taking into account Eq. (3) and (5), we have

$$\Pi^R + \Pi^M = (p+h)d + \int_{\frac{q-\alpha-\beta R}{Q}}^1 \int_q^{+\infty} [(q-x)(p+b+h)] f(x)g(u)dxdu \\ -c_m Q - (c_r + R)(\alpha+\beta R)$$

Let $h(q) = \int_q^{+\infty} (p+b+h)(q-x) f(x)dx$

Sine $\frac{\partial h(q)}{\partial q} = (p+b+h)\int_q^{+\infty} f(x)dx > 0$, Therefore, $h(q)$ increase as q increase

When $q < uQ + \alpha + \beta R$, $h(q) < h(uQ + \alpha + \beta R)$, then

$$\int_{\frac{q-\alpha-\beta R}{Q}}^{1}\int_{q}^{+\infty}[(q-x)(p+b+h)]f(x)g(u)dxdu$$

$$< \int_{\frac{q-\alpha-\beta R}{Q}}^{1}\int_{uQ+\alpha+\beta R}^{+\infty}[(uQ+\alpha+\beta R-x)(p+b+h)]f(x)g(u)dxdu$$

$$\Pi^R + \Pi^M \leq \Pi^c = (p+h)d + \int_0^1[\int_{uQ+\alpha+\beta R}^{+\infty}[(p+b+h)(uQ+\alpha+\beta R-x)]f(x)dx]g(u)du$$

Due to $\Pi^R(Q) + \Pi^M(Q) \leq \Pi^c(Q)$, then

$$\Pi^R(Q^{**}) + \Pi^M(Q^{**}) \leq \Pi^c(Q^{**}) \leq \Pi^c(Q^*)$$

Theorem 4 shows that when manufacturer outputs are randomized, retailers may take initiative to increase the order amount to reduce the shortage risk as a result of output uncertainty.

6. Numerical Analysis

In order to validate the influence of recovery price on the closed-loop supply chain, we make a corresponding numerical calculation. Assuming all parameters in the model is given as:

$$p = 15, \omega = 6, h = 3, b = 2, \alpha = 10, \ \beta = 4, \ c_m = 2, \ c_r = 0.5, \ p_r = 1.2, \ d = 100, \ \mu = 0.5,$$

X The uniform distribution in the range [80,120] and U follows the uniform distribution in the range [0, 1]

The calculation results obtained with Theorem 1-3 are provided in Table 2:

Table 2. The Equilibrium Results under the Recovery Price Variation

R	Under centralized decision making		Under decentralized decision making			
	Q^*	Π^c	Q^{**}	Π^M	Π^R	$\Pi^M + \Pi^R$
1	138	515	100	153.72	245.48	399.2
0.75	139.5	504.8	101	150.47	234.93	385.4
0.5	141	493.75	102	147.25	221.14	368.39
0.25	142.5	470.94	103.4	144	211.6	355.6

From Table 2, we can know that under centralized decision making, with recovery price increasing the recovery amount and the quantity of remanufactured products increase, then the manufacturers will reduce the planned production of new products, and expected profits of each member in the supply chain will increase. Under decentralized decision making, In the Stackelberg Game dominated by manufactures, the output randomness has no effect on the order amount of retailers and the optimal ordering amount is a constant. For the manufacturers are not familiar with the market demands, they can not make a reasonable production plan.

7. Conclusion

In the paper, we established a closed-loop supply chain model under random outputs and demands and are focused on the study of production order decision of the manufacturers and retailers, profits of members in the supply chain and overall profits in the supply chain under decentralized and centralized decision making. Then we made a comparative analysis for these two models. Theoretical researches shows: the manufacturers profit and retailers profit are respectively the concave functions of planned production of new products and order amount, indicating that there is unique optimal planned production of new products and order amount; The numerical examples indicated that when the recovery price increased, the amount of remanufactured products will increases and the optimal planned production of new products will decrease. At the same time, the optimal order amount, expected profits of each member in supply chain and overall profits of system will increase.

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