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Short-Term Hydrothermal Scheduling Based on Lagrange Function and Determining Initial Hydrothermal Generations

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Abstract

The paper presents an effective method based on the Lagrange multiplier theory to solve optimal scheduling of hydrothermal power system. Optimal scheduling of hydrothermal power systems is a great important problem to electric utility systems, the main objective of the problem is to determine the generation for each plant during scheduling period of time such that the total system generation cost is minimum while satisfying the system constrains of the generating limits and available water. The problem of optimal economic operation of hydrothermal power systems with fixed head hydro plants is considered and has had many researches about this. Determining of water discharge at the first interval is performed in this paper and lead to a low number of iteration and short computation time for convergence. The proposed method is tested on one system consisting of one hydro and one thermal plant through two examples.

Keywords: *Initial water discharge, scheduling of hydrothermal system, minimizing fuel cost*

1. Introduction

A modern power system consists of a large number of thermal and hydro plants connected at various load centres through a transmission network. An important objective in the operation of such a power system is to generate and transmit power to meet the system load demand at minimum fuel cost by an optimal mix of various types of plants. The hydro resources being extremely limited, the worth of water is greatly increased. If optimum use is made of their limited resource in conjunction with the thermal sources, huge saving in fuel and the associated cost can be made. Hydrothermal scheduling is required in order to find the optimum allocation of hydro energy so that the annual operating cost of a mixed hydrothermal system is minimized. Over the last decade the hydrothermal scheduling problem has been the subject of considerable discussion [1-7]. Many methods presented in [8]. In scheduling of hydro-thermal problem, we have to consider time length of scheduling of hydro-thermal problem. It is separated into three types: long range scheduling [9, 11, 12], medium range scheduling [12], short range scheduling [1-3], [8, 10]. With different cared period of time, input data as well as constraints are different. Long range scheduling also involves metrological and statistical analysis. The benefit of this scheduling is to save the cost of generation, in addition to meeting the

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agricultural and irrigational requirements. The short range problem usually has an optimization interval of a day or a week. This period is normally divided in to sub-intervals for scheduled purposes. Here, the load, water inflows and unit availabilities are assumes to be known. A set of starting conditions (i.e. reservoirs levels) being given, the optimal hourly schedule can be prepared that minimizes a desired objective while meeting system constraints successfully [2]. With the short range scheduling problems, cost optimization of hydro stations can be achieved by assuming the water heads constants and converting the incremental water (i.e. fuel) rate characteristics in to incremental fuel cost curves by multiplying it with cost of water per cubic meter and applying the conventional technique of minimizing the cost function. The performance of each hydro plant is represented by Glimn-Kirchmayer's model [6].

In the short term scheduling of hydrothermal system problem, evolutionary programming method [2], power flow results and operating fuel cost have been found out with relatively low operating cost. The method, however, required many procedures, take long simulation time to get convergence [2]. Gradient method [13], the least cost operation during scheduling period of time at thermal plants is gained with a very small difference power between generation and load demand. An effective method, Lamda method [13], uses Lagrange multiple theory, based on assuming effectively used-water co-efficient. The simulation results of this are good solutions compared to method in [2]. These methods, however, also cancel determining start generation of each hydro plant, leading to take long time simulation and a large number of iteration to reach convergence.

The paper proposes a new method also based on Lagrange multiple theories accompany with determining initial generation of each hydro plant. The simulation results show that the method is effective compared to others.

2. Problem Formulation

Consider a system consisting of M thermal and N hydro plants. The work is to determine the generation of each plant during scheduling period of time. The objective of problem is to minimize total generation cost for the entire scheduled time period subject to allowable water in use at each reservoir, system demand and the individual plant constraints. The time unit is one hour and planning horizon is one day. In order to formulate the problem mathematically, the following notation is first introduced:

$C_T(P_{Tmi})$: *Fuel cost of thermal , in (Rs/h).*
P_{Tmi}, P_{Hni}: *Power generated by mth thermal plant and nth hydro plant at time i, in MW.*
i: *Index of time, i = 1, 2,...,I.*
I: *Number of times*
M: *Number of thermal plants in system.*
m: *Index of thermal plant, m=1, 2,...,M*
N: *Number of hydro plants in system.*
n: *Index of hydro plant, n = 1, 2,...,N.*
a_m, b_m, c_m: *Cost coefficients of mth thermal plant*
P_{Di}: *Load demand at time i, in MW.*
V_n^S , V_n: *Pre-specified available water and used water in day for hydro plant in* m^3
V_{ni}: *Water discharge at time i of nth hydro plant, in* m^3
P_{Li}: *Power loss at time i, in MW.*

$P_{Tm}^{\min}$, $P_{Tm}^{\max}$ *: Minimum and maximum generation level of mth thermal plant, in MW.*

$P_{Hn}^{\min}$, $P_{Hn}^{\max}$ *: Minimum and maximum generation level of nth hydro plant, in MW.*

λ_i, μ_n*: Lagrange multipliers.*

q_{ni}*: Rate of discharge in* m^3/MWh*.*

x_n, y_n, z_n*: Discharge coefficients of nth hydro plant.*

P_{Taver}*: Active power generated by all thermal plants per hour on average.*

P_{Daver}*: Load demand per hour on average .*

P_{Haver}*: Active power generated by all hydro plants per hour on average.*

P_{Hi}*: Active power generated by all hydro plants at time ith.*

q_{aver}*: Water discharge per hour on average.*

The main objective is to minimizing generation cost of thermal plants during entire scheduling time period. The objective function is as follow:

$$Min\sum_{i=1}^{I} T_i \sum_{m=1}^{M} C_T(P_{Tmi}) \qquad (1)$$

Subject to power balance, water balance for each reservoir:

$$\sum_{m=1}^{M} P_{Tmi} + \sum_{n=1}^{N} P_{Hni} - P_{Li} - P_{Di} = 0 \qquad (2)$$

$$V_n^S = \sum_{i=1}^{I} T_i . q_{ni} \qquad (3)$$

Subject to generation level for each hydro and thermal plant:

$$P_{Hn}^{\min} \le P_{Hni} \le P_{Hn}^{\max} \qquad (4)$$

$$P_{Tm}^{\min} \le P_{Tmi} \le P_{Tm}^{\max} \qquad (5)$$

In equation (1), $C_T(P_{Tmi})$ is a quadratic function in term of power generated:

$$C_T(P_{Tmi}) = a_m P_{Tmi}^2 + b_m P_{Tmi} + c_m \qquad (6)$$

Relationship between water discharge and power generated was proposed by Glimn-Kirchmayer [6].

$$q_{ni} = x_n P_{Hni}^2 + y_n P_{Hni} + z_n \qquad (7)$$

Water discharge per hour on average is below.

$$q_{aver} = \frac{V^S}{24} \qquad (8)$$

Obtain power generated by hydro plant per hour on average P_{Haver} by substituting (8) in (7).

Calculate average load demand for per hour by

$$P_{Daver} = \frac{\sum_{i=1}^{24} P_{Di}}{24} \qquad (9)$$

Average power generated by all thermal plants at per hour can be obtained as follows.

$$P_{Taver} = P_{Daver} - P_{Haver}, \qquad (10)$$

Power generated by all hydro plants at time ith is below.

$$P_{Hi}=P_{Di}-P_{Taver}. \quad (11)$$

Thermal plants should be run at constant incremental cost for the entire period it is on and During peak load it is economical to use hydro plants. Hence the thermal plant is preferred as a base load plants whereas the hydroelectric plant is run as a peak load plant [2].

Figure 1.b shows that thermal plants should generate at base load according to BCFE area, rest of power above base load generated by hydro plants. Hence, generations generated by all hydro plants per hour on average are obtained as follow:

$$P_{Hi}=P_{Di}-P_{Taver}. \quad (12)$$

If load demand at the first hour is lower than hourly average load demand, generation of hydro plants calculated from above equation will be less than zero. This is invalid, leading to take long time simulation to reach convergence. According to the daily five-order load cycle in figure 1, the fifth interval load demand is approximately equal to hourly average load demand. The interval load demand is chosen as the first interval load demand and start generation of each plant is determined. Finally, a new daily load cycle is completely built as Figure 1.c.

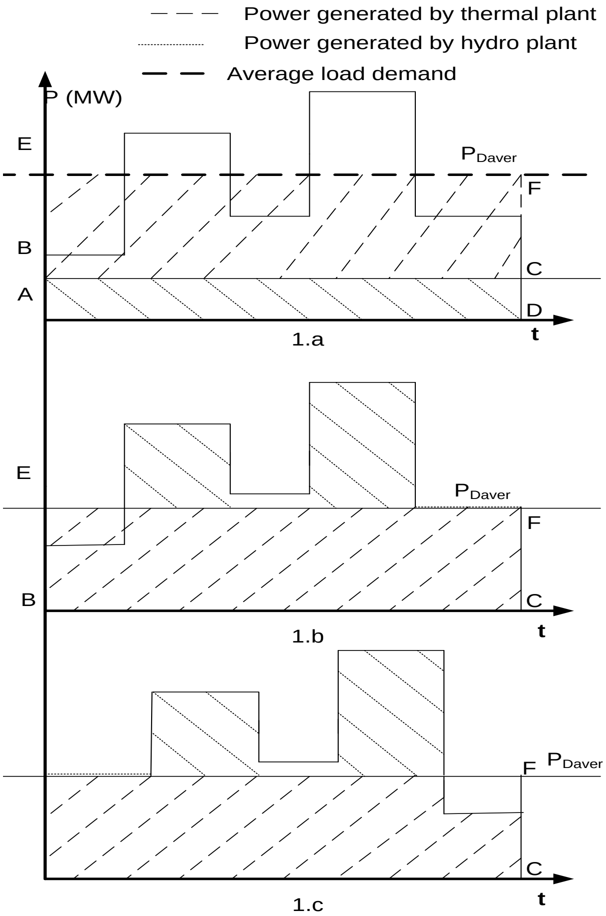

Figure 1. The Chart for Determining Initial Generation of Each Hydro Plant

Power generated by hydro plant at first interval:

$$P_{H1}=P_{D1}-P_{Taver} \tag{13}$$

Substituting P_{H1} in equation (7), obtains water discharge at the first interval. In the case of system having more than one hydro thermal plant, power output generated by nth hydro plant at time ith is obtained as below.

$$P_{Hni}=\frac{P_{H1}}{\sum_{n=1}^{N} P_{Hnaver}}.P_{Haver} \tag{14}$$

3. Proposed Method for Short-term Hydrothermal Scheduling Problem

The Lagrange function L is formulated by augmenting the objective function of eq. (1) with equality constraints of eqs. (2) and (3). Thus,

$$L=\sum_{i=1}^{I} T_i[\sum_{m=1}^{M} C_T(P_{Tmi})]+\sum_{i=1}^{I} \lambda_i.T_i(P_{Li}+P_{Di}-\sum_{m=1}^{M} P_{Tmi}-\sum_{n=1}^{N} P_{Hni})+\sum_{n=1}^{N} \mu_n[\sum_{i=1}^{I} q_{Hni}(P_{Hni}).T_i-V_n^S] \tag{15}$$

where P_{Tmi}, P_{Hni} must be satisfied inequality constraints of (4) and (5).

The partial derivatives of the Lagrange function (15) with respect to the dependent variables yield the following equations.

$$\frac{\partial L}{\partial P_{Tmi}}=T_i\frac{\partial C_T(P_{Tmi})}{\partial P_{Tmi}}+T_i\lambda_i(\frac{\partial P_{Li}}{\partial P_{Tmi}}-1)=0 \tag{16}$$

$$\frac{\partial L}{\partial P_{Hni}}=T_i.\lambda_i(\frac{\partial P_{Li}}{\partial P_{Hni}}-1)+\mu_n.T_i.\frac{\partial q_{xni}}{\partial P_{Hni}}=0 \tag{17}$$

$$\frac{\partial L}{\partial \lambda_i}=P_{Li}+P_{Di}-\sum_{m=1}^{M} P_{Tmi}-\sum_{n=1}^{N} P_{Hni}=0 \tag{18}$$

$$\frac{\partial L}{\partial \mu_n}=\sum_{i=1}^{I} q_{ni}.T_i-V_n^S=0 \tag{19}$$

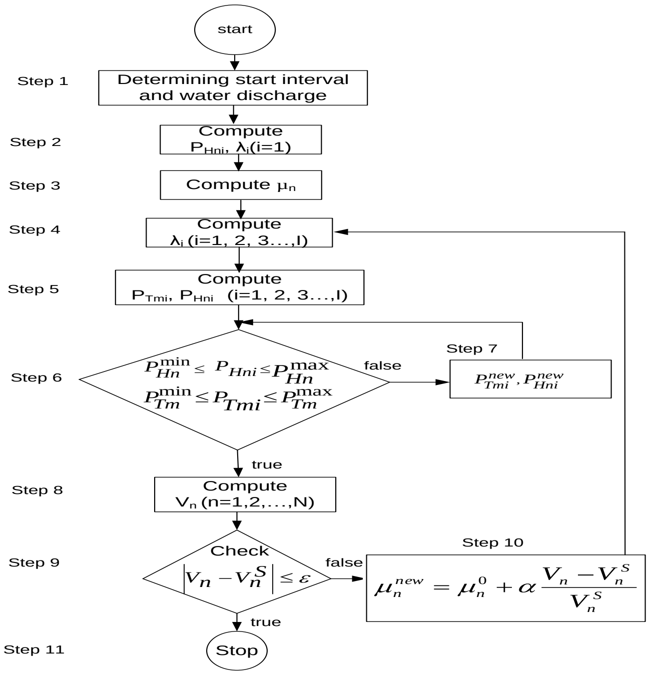

Figure 2. The Flow Chart for Solving Optimal Scheduling of Hydrothermal Problems

Figure 2 presents the flow chart for solving optimal scheduling of hydrothermal system. At step 2, obtain determining start interval and water discharge based on the chart for determining initial generation of each hydro plant and equations from (8) to (11).

4. Numerical Results

Consider a hydrothermal system consisting of one hydro plant and one thermal plant [2]. Determine initial generation at hydro plant in following two cases.

Case 1: V^S= 73.31217 Mm^3.

Case 2: V^S= 73.81 Mm^3.

The proposed algorithm has been implemented in Matlab 7.2 programming language and executed on Intel(R) Core (TM)2 Duo CPU T7250 @2.00GHZ (2CPU) laptop.

Developed program gives the thermal and hydro generations and total operating cost, computation time and number of iteration. The simulation results obtained by the proposed method and Lamda method in [13] for two cases are shown in Tables 1 and 2.

In Table 1, load demand of 580 MW at 13th interval is chosen as initial hour and initial generation for hydro plant is calculated at the interval. The initial generations for 2 cases are shown in Tables 3 and 4. At the case 1, initial generation of hydro plant anticipated is 239.4346 MW, different from convergence result (238.655 MW). At the case 2, initial generation anticipated has a small difference of 0.7802MW with convergence result (241.3592MW compared to 240.579 MW). In 2 cases, operation cost and total water discharge are nearly equal to Lamda method. For operation cost, the method obtain better solution (0,7019 less) at the case 1, 0.821005 RS at the case 2 than Lamda method. For total water discharge, two methods have a very small difference. The table 2 and 3 show that the method take a very short simulation time, 0.02 seconds at the cases 1 and 2 and very low number of iterations, 3 iterations at both case 1 and 2 whereas the Lamda method spends 36 iteration for convergence, approximately 7.0 seconds. Note both 2 methods in this research use the same programming software and the same computer.

Table 1. Obtained Results for Case 1

P_{Di}	i	P_{Hi} (MW)	P_{Ti} (MW)	q_i m^3	Cost (10^3 RS)
580	**13**	**238.6547**	341.3453	305.3459	3.8846
455	1	228.4606	226.5394	294.635	2.652
425	2	226.014	198.986	292.1323	2.364
415	3	225.1985	189.8015	291.3039	2.2687
407	4	224.546	182.454	290.6433	2.1926
400	5	223.9752	176.0248	290.0668	2.1263
420	6	225.6062	194.3938	291.7177	2.3163
487	7	231.0703	255.9297	297.3336	2.9626
604	8	240.612	363.388	307.4545	4.1273
665	9	245.5867	419.4133	312.8897	4.7528
675	10	246.4022	428.5978	313.7911	4.8565
695	11	248.0333	446.9667	315.6026	5.065
705	12	248.8488	456.1512	316.5127	5.1698
605	14	240.6935	364.3065	307.5427	4.1375
616	15	241.5906	374.4094	308.5151	4.2494
653	16	244.6081	408.3919	311.8119	4.6288

721	17	250.1536	470.8464	317.975	5.338
740	18	251.7031	488.2969	319.7212	5.539
700	19	248.441	451.559	316.0573	5.1174
678	20	246.6469	431.3531	314.0621	4.8877
630	21	242.7323	387.2677	309.7579	4.3924
585	22	239.0625	345.9375	305.7838	3.935
540	23	235.3926	304.6074	301.8688	3.4845
503	24	232.3751	270.6249	298.6941	3.1191
Total				$7.3312.10^3$	93.5674109

Table 2. Obtained Results for Case 2

P_{Di}	i	P_{Hi} (MW)	P_{Ti} (MW)	q_i (m^3)	Cost (10^3 RS)
580	**13**	**240.5788**	339.4212	307.4186	339.4212
455	1	230.3044	224.6956	296.5386	224.6956
425	2	227.8386	197.1614	293.9963	197.1614
415	3	227.0167	187.9833	293.1548	187.9833
407	4	226.3591	180.6409	292.4838	180.6409
400	5	225.7837	174.2163	291.8981	174.2163
420	6	227.4276	192.5724	293.5752	192.5724
487	7	232.9347	254.0653	299.2798	254.0653
604	8	242.5514	361.4486	309.5606	361.4486
665	9	247.5653	417.4347	315.0817	417.4347
675	10	248.3873	426.6127	315.9973	426.6127
695	11	250.0312	444.9688	317.8374	444.9688
705	12	250.8531	454.1469	318.7619	454.1469
605	14	242.6336	362.3664	309.6502	362.3664
616	15	243.5378	372.4622	310.638	372.4622
653	16	246.579	406.421	313.9868	406.421
721	17	252.1682	468.8318	320.2473	468.8318
740	18	253.7299	486.2701	322.0211	486.2701
700	19	250.4421	449.5579	318.2993	449.5579
678	20	248.6338	429.3662	316.2725	429.3662
630	21	244.6885	385.3115	311.9003	385.3115
585	22	240.9897	344.0103	307.8635	344.0103
540	23	237.291	302.709	303.8866	302.709
503	24	234.2498	268.7502	300.6617	268.7502
Total				$7.381.10^3$	93.061

Table 3. The Result Comparison for Case 1

Method	Total cost (RS)	Initial gen. P_{H1} (MW)	V_S (Mm3)	Iterations	CPU time (s)
The proposed method	93,567.4109	239.4346	$7.3312.10^3$	3	0.02
Lamda method	93,568.1128	~	73.31149	36	7.695291

Table 4. The Result Comparison for Case 2

Method	Total cost (RS)	Initial gen. P_{H1} (MW)	V_S (Mm3)	Iterations	CPU time (s)
The proposed method	93,060.756259	241.3592	73.810113	3	0.02
Lamda method	93,061.577264	~	73.809132	36	6.962007

5. Conclusion

The paper presented an effective method based on Lagrange multiplier theories and determining initial generation for both hydro and thermal plants for solving optimal scheduling of hydrothermal power systems. The method tested on one system consisting of one hydro and thermal plant with two different cases. The simulation results has indicated that the determination of initial power output for each plant is very efficient since it enables the proposed method to be faster for getting convergence with three iterations. Consequently, it can be sated that the proposed method is one of effective methods for solving short-term hydro thermal scheduling problem.

References

[1] C.E. Zoumas, A. G. Bakirtzis, J. B. Theocharis and Vasilios Petridis, “A Genetic Algorithm Solution Approach to the Hydrothermal Coordination Problem”, IEEE transactions on power systems, vol. 19, no. 2, **(2004)**.

[2] A. K. Sharma, “Short term hydrothermal scheduling using evolutionary programming”, Thesis submitted in partial fulfillment of the requirements for the award of degree of Master of Engineering in Power Systems & Electric Drives, Thapar University, Patiala, **(2009)**.

[3] N. Jiménez Redondo, A.J. Conejo, “Short-term hydro-thermal coordination by Lagrangian rslaxation: solution of the dual problem”, IEEE transactions on power systems, **(1999)**, vol.14, no.1.

[4] H. Habibollahzadeh, G.X. Luo, A. Semlyen, “Hydrothermal optimal power flow based on a combined linear and nonlinear programming methodology”, IEEE Transactions on Power Systems, **(1989),** vol. 4, no. 2, pp. 530- 537.

[5] M.Pereira, N. Campodónico, R. Kelman, “Application of stochastic dual dp and extensions to hydrothermalschedulin*g*”, psri technical report 012/99, **(1999)**.

[6] J. S. Dhisillon and DP.kothari, “Multi-objective short-term hydro thermal scheduling based on heuristic search techniqu*e*, “Asian Journal of information technology”, vol. 6, no. 4, **(2007)**, pp. 447-454.

[7] S.M. Xaiomin, Shahidehpour, “Hydro-thermal scheduling by Tabu search and decomposition method. IEEE Transactions on Power Systems, vol. 11, no. 2, **(1996)**.

[8] I.A. Farhat, M.E. El-Hawary, “Optimization methods applied for solving the short-term hydrothermal coordination problem”, Electrical power system researchs, vol. 79, no.9, **(2009),** pp 1308-1320.

[9] T. G. Siqueira, M. Zambelli, M. Cicogna, M. Andrade, S. Soares, “Stochastic Dynamic Programming for Long Term Hydrothermal Scheduling Considering Different Streamflow Models”, 9th International Conference on Probabilistic Methods Applied to Power Systems KTH, Stockholm, **(2006);** Sweden.

[10] S.M. Amado, C.C. Rebeiro, "Short-Term Generation Scheduling of Hydraulic Multi Reservoir Multi Area Interconnected Systems", IEEE Trans. on power systems, vol. 2, no.3, **(1987)**, pp.758-763.
[11] R.W. Ferrero, J.F. Rivera, S.M. Shahidehpour, "A dynamic programming two-stage algorithm for long-term hydro-thermal scheduling of multireservoir systems", IEEE transactions on Power systems, **(1998)** vol.13, no.4.
[12] S. Sivanagaraju, G. Sreenivasan, "Power systems operation and control", Dorking Kindersley, **(2010).**
[13] A. Wood, B. Wollenberg, "Power Generation, Operation and Control", Wiley, **(1996)**.

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