

Robust Optimization Model of Reverse Logistics Network with Demand Uncertainty

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Abstract

A dynamic robust optimization model is established based on the objective of profit maximization to deal with the uncertainty of the problem through formulating a robust linear programming model. An equivalent robust optimization model using duality theory and auxiliary problem principle is obtained which allows the solutions to be derived more efficiently. Following the numerical simulation and sensitivity analysis is presented. The numerical simulation shows that the robust optimization mode can reduce the risk and uncertainty of reverse logistics operation process and the environment of demand uncertainty, which verifies the effectiveness of the approach.

Keywords: *Reverse logistics; demand uncertainty; robust optimization*

1. Introduction

Due to the circular economy, sustainable development as well as ecological protection, waste recycling and remanufacturing as a protection of the environment and the business and technology innovation has been focused on. Reverse logistics, which is simply the recovery of the used products, becomes more important. Reverse logistics can not only enclose the material flow and improve the resource utilization, but also reduce the discharge of pollutants and slow down the production costs of enterprises, which has a good social benefits and ecological benefits. Halabi, Montoya-Torres, Pirachicán, and Mejía [1] studied the negative impact on the environment. The paper is a research approach of reverse logistics practices in Colombian enterprises, with a particular focus on the plastic sector. Marwede, Berger, Schlummer, Mäurer, and Reller [2] studied recycling of thin film chalcogenide photovoltaic. They developed feasible recycling paths for chalcogenide photovoltaic modules. Liu, Xing, Mei, and Zhang [3] studied a tolerance grading allocation method. They presented a method of tolerance grading allocation for remanufactured parts based on uncertainty analysis of the remanufacturing assembly.

Generally, in a manufacturing and remanufacturing hybrid system, it is required to determine the quantities of new product to be manufactured, the quantities of returned product to be remanufactured and appropriate inventories of the sellable and returned products. Such a problem is hard to solve efficiently, especially in a multi-period environment [4]. It is much complicated to forecast the returned products, because the used products are usually returned through different channels, from different customers in different circumstances.

As it is well known, robust optimization is particularly powerful in dealing with uncertain variables with only known intervals, which is different from stochastic optimization which needs distributional information on the random variables. We obtain robust solutions by applying the concept of robust optimization as presented by J. Mulvey, R. Vanderbei and S. Zenios [5] on our capacitated facility location problem. Over the last years robust optimization has evolved significantly, especially the works of Ben-Tal and Nemirovski [6] and Bertsimas [7] contributed to this evolution. D.R.

Vincenzo, H. Evi, G. Marina and W. Jens [8] contributed a new model formulation for the robust capacitated facility location problem is presented to cope with uncertainty in planning with the aim of minimizing the expectation of the relative regrets across scenarios over multiple periods is the objective. M.I.Salema, A.P.Barbosa and A.Q.Novais [9] researched the network design of reverse logistics and constructed mixed integer programming model under the uncertainty of recycling types and quantities. R.L.Pati, P.Vrat and P.Kumar [10] presented a mixed integer programming model considering the operation cost minimization and recycling rate maximization of the reverse logistics system. M.G. Bardossy and S. Raghavan [11] built a logistics facility location problem using robust optimization model and verified the effectiveness of the model. C.Wei, Y.Li and X.Cai [12] optimized the reverse logistics using robust optimization model taking the quantity of new product and returned product as well as the rate of return as decision variables.

In this paper we will apply a robust optimization approach to tackle a production planning problem in which the product return process is integrated into the manufacturing process over a finite planning horizon. The model treats both the quantity of returns and the market demand as uncertain variables, with only upper and lower bounds being known. The optimal solution for our robust model can meet the constraints with a high probability. Numerical study shows the usefulness of robust approach.

2. Problem Description

A multi-period product planning and demand system with used product returns and remanufacturing is considered in this paper, as it is shown in Figure 1. In this system the returned product is used for remanufacturing product and sold with new product in market.

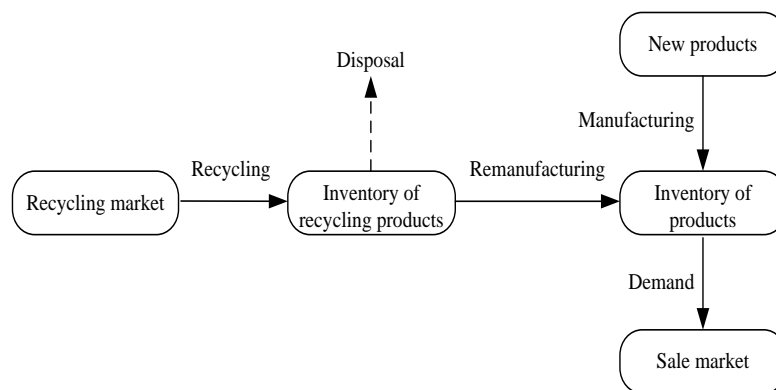


Figure 1. Structure of Reverse Logistics System

The procedures and assumption to run the system are as follows:

(1) The problem is considered over a finite planning horizon with T periods. Define v_m^t and v_r^t as the inventory levels of the new and returned products at the beginning of period t. At the beginning of the planning horizon, we assume that $v_m^0 = v_r^0 = 0$.

(2) At the beginning of every period, we first decide how many products will be remanufactured, which is noted as r_r^t . Then we decide whether and how many new products will be manufactured, noted as q_m^t . No setup costs are considered for the manufacturing, remanufacturing and the finished product demand can be backlogged, but returns cannot. A shortage cost will occur if the demand is unsatisfied.

(3) We consider the situations where there is limited information on the quantity of the returned product and the market demand for the new product.

(4) The quantity of the returned product and the demand for the new product fall in lower and upper bounds, respectively, which can be estimated in advance. D^t is demand function of new product, $D^t = \alpha^t - \beta^t p^t$.

3. The Nominal Model

In the nominal problem, the returns and demand are realized with the probability one. The notations are as follows.

Parameters:

- c_m^t : unit production cost of a new product
- c_r^t : unit cost of remanufacturing a returned product
- h_m^t : unit holding cost of the new product
- h_r^t : unit holding cost of the returned product
- p_r^t : unit returned cost of the returned product
- p^t : unit sales price of the new product

Decision variables:

- q_m^t : quantity of the new product
- γ_r^t : quantity of the returned product

The inventory equations are formulated as follows :

$$V_r^{t+1} = V_r^0 + \sum_{i=1}^t (\gamma_r^i - q_r^i)$$

$$V_m^{t+1} = V_m^0 + \sum_{i=1}^t (q_r^i + q_m^i - D^i)$$

The initial storage level $V_m^0 = V_r^0 = 0$ is pre-specified constants and $V_m^{t+1} \geq 0$, $V_r^{t+1} \geq 0$.

The revenue of the reverse logistics system is:

$$TR = \sum_{t=1}^T p^t \times D^t = \sum_{t=1}^T p^t \times (\alpha^t - \beta^t p^t)$$

and the cost of the reverse logistics system is:

$$TC = TC_1 + TC_2$$

$$TC_1 = \sum_{t=1}^T c_m^t q_m^t + c_r^t q_r^t + p_r^t \gamma_r^t$$

$$TC_2 = \sum_{t=1}^T h_m^t (V_m^t + q_m^t + q_r^t - D^t) + h_r^t (V_r^t + b_r^t - q_r^t)$$

TC_1 is the costs products of new product, returned product and remanufactured cost of returned product. TC_2 is the storage costs of new product and returned product. Thus the nominal model of maximizing the revenue of the reverse logistics system is as follows:

$$I : \max f = TR - TC$$

$$st \begin{cases} V_r^{t+1} = V_r^0 + \sum_{i=1}^t (\gamma_r^i - q_r^i) \\ V_m^{t+1} = V_m^0 + \sum_{i=1}^t (q_r^i + q_m^i - D^i) \\ V_m^{t+1} \geq 0, V_r^{t+1} \geq 0 \\ V_m^0 = V_r^0 = 0, q_m^t, q_r^t \geq 0 \end{cases}$$

4. Robust Optimization Model

4.1. Lemma 1:

Robust model

$$\begin{aligned} \max f &= c^T x \\ \text{st} \quad &\begin{cases} Ax \leq b \\ u \leq x \leq v \end{cases} \end{aligned}$$

We define J_i is the set combined with the line i column subscript j of uncertain parameter a_{ij} in coefficient matrix A . For every nominal value a_{ij} ($j \in J_i$), $\hat{\alpha}_{ij}$ is the interval scaled deviation from its nominal value and $\tilde{a}_{ij} \in [a_{ij} - \hat{\alpha}_{ij}, a_{ij} + \hat{\alpha}_{ij}]$. We give a budget of uncertainty Γ_i takes values in an interval $[0, |J_i|]$ which means that the increase in the uncertainty budgets is slower than the increase in the periods. In this way the robust optimization model is:

$$\begin{aligned} \max f &= c^T x \\ \text{st} \quad &\begin{cases} \sum_j a_{ij} x_j + \max_{\substack{S_i \cup \{t_i\} | S_i \in J_i \\ |S_i| = [\Gamma_i] \\ t_i \in J_i \setminus S_i}} \left\{ \sum_{j \in S_i} \hat{\alpha}_{ij} y_j + (\Gamma_i - [\Gamma_i]) \hat{\alpha}_{ij} y_{j_i} \right\} \leq b_i \\ -y_j \leq x_j \leq y_j, u \leq x \leq v, y \geq 0 \end{cases} \end{aligned}$$

Set:

$$\beta_i(x^*, \Gamma_i) = \max_{\substack{S_i \cup \{t_i\} | S_i \in J_i \\ |S_i| = [\Gamma_i] \\ t_i \in J_i \setminus S_i}} \left\{ \sum_{j \in S_i} \hat{\alpha}_{ij} y_j + (\Gamma_i - [\Gamma_i]) \hat{\alpha}_{ij} y_{j_i} \right\}$$

So $\beta_i(x^*, \Gamma_i)$ is equivalent to the following programming:

$$\begin{aligned} \max \beta_i(x^*, \Gamma_i) &= \sum_{j \in J} \hat{\alpha}_{ij} |x_j| z_{ij} \\ \text{st} \quad &\begin{cases} \sum_{j \in J} z_{ij} \leq \Gamma_i \\ 0 \leq z_{ij} \leq 1, \forall j \in J_i \end{cases} \end{aligned}$$

Thus the robust optimization model is as follows:

$$\begin{aligned} \max c^T x \\ \text{st} \quad &\begin{cases} \sum_j a_{ij} x_j + z_i \Gamma_i + \sum_{j \in J} p_{ij} \leq b_{ij} \\ z_i + p_{ij} \geq \hat{\alpha}_{ij} y_j \\ -y_j \leq x_j \leq y_j, u_j \leq x_j \leq v_j \\ p_{ij}, y_j, z_i \geq 0, \forall i, j \in J_i \end{cases} \end{aligned}$$

4.2. Robust Optimization

In model we assume α^t is nominal value and certain. But in fact the quantity α^t of is uncertain in every period t by taking a value in a support interval. $\hat{\alpha}^t$ is the interval scaled deviation from its nominal value and $|\tilde{\alpha}^t - \alpha^t| \leq \hat{\alpha}^t$. Set $z = \frac{\tilde{\alpha}^t - \alpha^t}{\hat{\alpha}^t}$, so $\tilde{\alpha}^t = \alpha^t + \hat{\alpha}^t z^t$.

We give a budget of uncertainty at each period t to restrict the cumulatively scaled deviation as $\sum_{i=1}^t |z_i| \leq \Gamma_t$, where the budget of uncertainty Γ_t takes values in an interval

$[0, |J_t|]$ which means that the increase in the uncertainty budgets is slower than the increase in the periods [13].

Similarly we present the definition of uncertain β_t and γ_t^i as follows:

$$\sum_{i=1}^t |y_i| \leq \Phi_t, \quad \text{which } \tilde{\beta}^t = \beta^t + \hat{\beta}^t y^t \quad (\Phi_t \in [0, |\Phi_t|])$$

$$\sum_{i=1}^t |w_i| \leq \Psi_t, \quad \text{which } \tilde{\gamma}^t = \gamma^t + \hat{\gamma}^t w^t \quad (\Psi_t \in [0, |\Psi_t|])$$

Thus the robust formulation of nominal model I is as follows :

$$\text{II : } \max_{\substack{p^t, p_r^t, \\ q_m^t, q_r^t}} \min_{\substack{z^t, y^t, w^t}} = \sum_{t=1}^T \left\{ (p^t + h_m^t) \alpha^t - \left[(p^t)^2 + h_m^t p^t \right] \beta^t + (p^t + h_m^t) \hat{\alpha}^t z^t - \left[(p^t)^2 + h_m^t p^t \right] \hat{\beta}^t y^t \right. \\ \left. - (p_r^t + h_r^t) (\gamma_r^t + \hat{\gamma}_r^t w^t) - (c_m^t + h_m^t) q_m^t - (c_r^t + h_r^t - h_r^t) q_r^t - h_m^t V_m^t - h_r^t V_r^t \right\}$$

$$\text{s.t. } \begin{cases} \sum_{i=1}^t (\gamma_r^i - q_r^i) + \sum_{i=1}^t \hat{\gamma}_r^i w^i \geq 0, \forall t \in T & (1) \\ \sum_{i=1}^t (q_m^i + q_r^i - D^i) + \sum_{i=1}^t (-\hat{\alpha}^i z^i) + \sum_{i=1}^t (-\hat{\beta}^i y^i p^i) \geq 0 & (2) \\ q_m^t + q_r^t \leq C^t, \forall t \in T & (3) \\ p^t \leq \frac{\alpha^t + \hat{\alpha}^t z^t}{\beta^t + \hat{\beta}^t y^t}, \forall t \in T & (4) \\ p^t, p_r^t, q_m^t, q_r^t \geq 0, \forall t \in T & (5) \\ 0 \leq z^t, y^t, w^t \leq 1 & (6) \end{cases}$$

Because of the influence of uncertainty on the constraints, we have to deal with these effects by reconstructing constraints (1) and (2). This auxiliary problem comes from minimizing $\sum_{i=1}^t \hat{\gamma}_r^i w^i$ in constraints (1) and $-\sum_{i=1}^t \hat{\alpha}^i z^i$ as well as $-\sum_{i=1}^t \hat{\beta}^i y^i p^i$ in (2). According to the robust optimization approach given by Bertsimas and Thiele [14], we need to maximize the right-hand side of constraints (1) and (2) over the set $\beta_i(x^*, \Gamma_i)$ to protect the feasibility of all the possible realizations of the uncertain returns and demand. When t is given, we then need to solve an auxiliary linear programming problem:

$$\min \sum_{i=1}^t \hat{\gamma}_r^i w^i$$

$$\text{s.t. } \begin{cases} \sum_{i=1}^t |w^i| \leq \Psi_t \\ -1 \leq w^i \leq 0, \forall i \leq t \end{cases} \quad (1)$$

$$-\min \sum_{i=1}^t \hat{\alpha}^i z^i$$

$$\text{s.t. } \begin{cases} \sum_{i=1}^t |z^i| \leq \Gamma_t \\ -1 \leq z^i \leq 0, \forall i \leq t \end{cases} \quad (2)$$

$$-\min \sum_{i=1}^t \hat{\beta}^i y^i p^i$$

$$\text{s.t. } \begin{cases} \sum_{i=1}^t |y^i| \leq \Phi_t \\ -1 \leq y^i \leq 0, \forall i \leq t \end{cases} \quad (3)$$

According to strong duality theory, we can get a linear programming problem by using duality while introducing deterministic parameters [15]. Then the duality problems of the auxiliary linear in constraint (1) and (2) are given as follows:

$$\begin{aligned} \min & s_t \Psi_t + \sum_{i=1}^t u(i, t) \\ \text{st} & \begin{cases} s_t + u(i, t) \geq \hat{\gamma}_r^i \\ s_t \geq 0, u(i, t) \geq 0, \forall i \leq t \end{cases} \end{aligned} \quad (4)$$

$$\begin{aligned} \min & m_t \Gamma_t + \sum_{i=1}^t n(i, t) \\ \text{st} & \begin{cases} m_t + n(i, t) \geq \hat{\alpha}^i \\ m_t \geq 0, n(i, t) \geq 0, \forall i \leq t \end{cases} \end{aligned} \quad (5)$$

$$\begin{aligned} \min & o_t \Phi_t + \sum_{i=1}^t k(i, t) \\ \text{st} & \begin{cases} o_t + k(i, t) \geq \hat{\beta}^i p^i \\ o_t \geq 0, k(i, t) \geq 0, \forall i \leq t \end{cases} \end{aligned} \quad (6)$$

Furthermore, by observing the objective function, the uncertainty such as (7), (8) and (9) also has an influence on the feasible solutions and optimization.

$$\begin{aligned} \min & \sum_{i=1}^t (p^i + h_m^i) \hat{\alpha}^i z^i \\ \text{st} & \begin{cases} \sum_{i=1}^t |z^i| \leq \Gamma_t \\ -1 \leq z^i \leq 0, \forall t \leq T \end{cases} \end{aligned} \quad (7)$$

$$\begin{aligned} \min & \sum_{i=1}^t \left[(p^i)^2 + h_m^i p^i \right] \hat{\beta}^i y^i \\ \text{st} & \begin{cases} \sum_{i=1}^t |y^i| \leq \Phi_t \\ -1 \leq y^i \leq 0, \forall T \leq T \end{cases} \end{aligned} \quad (8)$$

$$\begin{aligned} \min & \sum_{i=1}^t (p_r^i + h_r^i) \hat{\gamma}_r^i w^i \\ \text{st} & \begin{cases} \sum_{i=1}^t |w^i| \leq \Psi_t \\ -1 \leq w^i \leq 0, \forall t \leq T \end{cases} \end{aligned} \quad (9)$$

Also the auxiliary problem to deal with the impact of uncertainty on the objective is given as follows (10), (11)and(12):

$$\begin{aligned} \min & m_1^t \Gamma_t + \sum_{t=1}^T n_1(t, T) \\ \text{st} & \begin{cases} m_1^t + n_1(t, T) \geq \hat{\alpha}^t (p^t + h_m^t) \\ m_1^t \geq 0, n_1(t, T) \geq 0, \forall t \in T \end{cases} \end{aligned} \quad (10)$$

$$\begin{aligned} \min & s_1^t \Phi_t + \sum_{t=1}^T u_1(t, T) \\ \text{st} & \begin{cases} s_1^t + u_1(t, T) \geq \hat{\beta}^t \left[(p^t)^2 + h_m^t p^t \right] \\ s_1^t \geq 0, u_1(t, T) \geq 0, \forall t \in T \end{cases} \end{aligned} \quad (11)$$

$$\begin{aligned}
 & - \min o_1^t \Psi_t + \sum_{t=1}^T k_1(t, T) \\
 & \text{st. } \begin{cases} o_1^t + k_1(t, T) \geq \hat{\gamma}^t (p_r^t + h_r^t) \\ o_1^t \geq 0, k_1(t, T) \geq 0, \forall t \in T \end{cases} \quad (12)
 \end{aligned}$$

According to the duality relationships between (4),(5),(6) and (10),(11), (12), we formulate the robust counterpart model as follows:

$$\begin{aligned}
 \text{III : } \max f = & \sum_{t=1}^T \left\{ (p^t + h_m^t) \alpha^t - \left[(p^t)^2 + h_m^t p^t \right] \beta^t - (p_r^t + h_r^t) \gamma_r^t - (c_m^t + h_m^t) q_m^t - (c_r^t + h_m^t - h_r^t) q_r^t \right. \\
 & \left. - h_m^t V_m^t - h_r^t V_r^t - m_1^t \Gamma_t - o_1^t \Phi_t - s_1^t \Psi_t - n_1(i, t) - u_1(i, t) - k_1(i, t) \right\} \\
 \text{st. } & \begin{cases} \sum_{i=1}^t (\gamma_r^i - q_r^i) \geq s_t \Psi_t + \sum_{i=1}^t u(i, t), \forall t \in T \\ \sum_{i=1}^t (q_r^i + q_m^i - D^i) \geq m_t \Gamma_t + \sum_{i=1}^t n(i, t) + o_t \Phi_t + \sum_{i=1}^t k(i, t) \\ m_t + n(i, t) \geq \hat{\alpha}^t, \forall i \leq t, t \in T \\ o_t + k(i, t) \geq \hat{\beta}^t p^t, \forall i \leq t, t \in T \\ s_t + u(i, t) \geq \hat{\gamma}^t, \forall i \leq t, t \in T \\ m_1^t + n_1(t, T) \geq \hat{\alpha}^t (p^t + h_m^t), \forall t \in T \\ s_1^t + u_1(t, T) \geq \hat{\beta}^t \left[(p^t)^2 + h_m^t p^t \right], \forall t \in T \\ o_1^t + k_1(t, T) \geq \hat{\gamma}^t (p_r^t + h_r^t), \forall t \in T \\ q_m^t, q_r^t, m_t, o_t, s_t, m_1^t, s_1^t, o_1^t \geq 0, \forall t \in T \\ n(i, t), k(i, t), u(i, t), n_1(t, T), u_1(t, T), k_1(t, T) \geq 0 \end{cases}
 \end{aligned}$$

Constrains from (1) to (4) are the auxiliary linear programming problem, from (5) to (10) are constrains of uncertainty, from (11) to (12) are the value range of parameters. The main work of this section is to formulate the robust counterpart model. The model is formulated as a linear program and we can solve it easily by using optimization software for linear programming such as Lingo, Matlab and GA. In our paper, we solve our model in Lingo 12.0 and the numerical example of the model solution is given in the next section.

4. Numerical Study

The parameters in our numerical examples were determined as follows. $h_m^t > h_r^t$, $c_m^t > c_r^t$ means that a returned product holds a lower value than a new product. The parameters were selected as $h_m^t = 2$, $h_r^t = 1$, $c_m^t = 30$, $c_r^t = 10$, $\alpha^t \in [60, 80]$, $\beta^t \in [15, 20]$, $\gamma^t \in [80, 100]$, $0.05\alpha^t \leq \hat{\alpha}^t \leq 0.2\alpha^t$, $0.05\beta^t \leq \hat{\beta}^t \leq 0.2\beta^t$, $0.05\gamma^t \leq \hat{\gamma}^t \leq 0.2\gamma^t$. The uncertainty budgets Γ , Φ , Ψ is the random number generator among $[0.8, 1]$, $\Gamma(0) = 0$, $\Phi(0) = 0$, $\Psi(0) = 0$.

We use Lingo 12.0 in the following environment: Windows XP (Professional SP3), Intel Core i7-4510U, CPU 2.80 GHz, 1 GB memory. The optimal result of the robust model is 8847.389.

Table 1. Computational Requirements

Period	Calculating times	Optimal cost
T=10	2	8847.389
T=20	5	13894.624
T=50	9	31613.665
T=100	12	70419.811

Table 1 shows that the time requirements of our robust optimization approach, which suggests that the proposed approach is computationally efficient, which means the solutions from the robust optimization model is feasible with a high probability especially when dealing with a long-period problem.

(1) Uncertain parameters $\hat{\alpha}'$, $\hat{\beta}'$ and $\hat{\gamma}'$

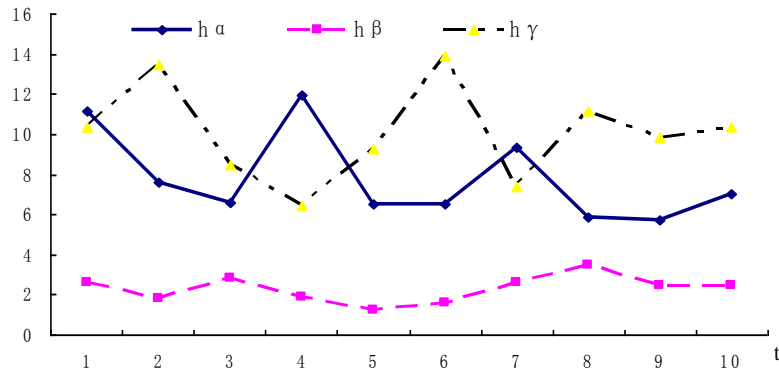


Figure 2. $h\alpha$, $h\beta$ and $h\gamma$ with Different T

Figure 2 shows the changes of uncertain parameter $\hat{\alpha}'$, $\hat{\beta}'$ and $\hat{\gamma}'$. In Figure 2, we replace $\hat{\alpha}'$, $\hat{\beta}'$ and $\hat{\gamma}'$ for $h\alpha$, $h\beta$ and $h\gamma$. From Figure 2 we see that the change interval of uncertain parameter is more and more small, which means that the stability of reverse logistics system is better. The operation risk of reverse logistics system decreases with the increase of stability. That is, the longer the reverse logistics system run, the more returns the system remanufactures and sales, also the more stable the reverse logistics system operates. The uncertain parameters can balance the robustness and optimality of the solution by changing their values to control the conservatism degree.

(2) q'_m with different T

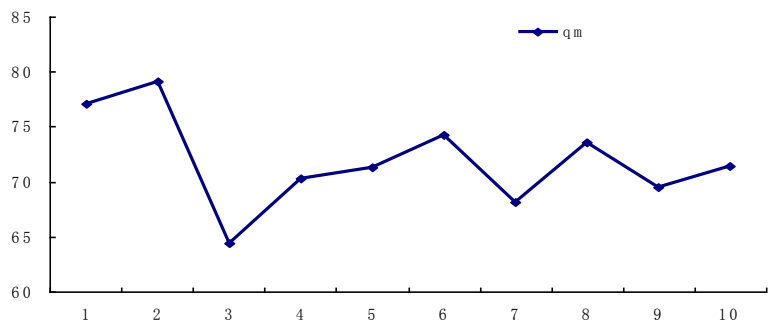


Figure 3. q'_m with Different T

Figure 3 shows that with the increases operation time of reverse logistics system, the quantity of new product became more stable gradually; the changing of market demand is concentrated in a relatively small fixed interval. That is, the market demand has tended to be stable and the market shocks and market risk of reverse logistics system operation decreases. The dynamic price/quantity robust optimization model can increase the robustness and reduce uncertainty risks of system operation, which means that the decrease of market shocks will lead to cost savings.

(3) γ_r^t and α with different T

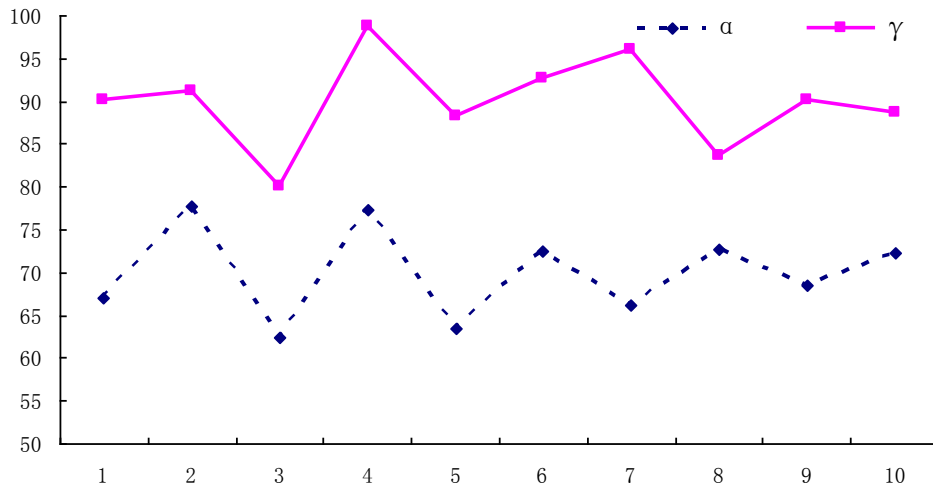


Figure 4. γ_r^t and α with Different T

Figure 4 shows that the shocks of the quantity of returned product is decreased with the different periods, which means the number and size of returned product is in a stable and control status with the increasing circular operation of reverse logistics system. The running of reverse logistics system is more and more normal and stable, which reduces the recovery fluctuation impact on reverse logistics system operation. The stable and control status of reverse logistics system can also lead to cost savings. The robust optimization model is an effective method to deal with the situations when returns of the product are very uncertain.

(4) β_r with different T

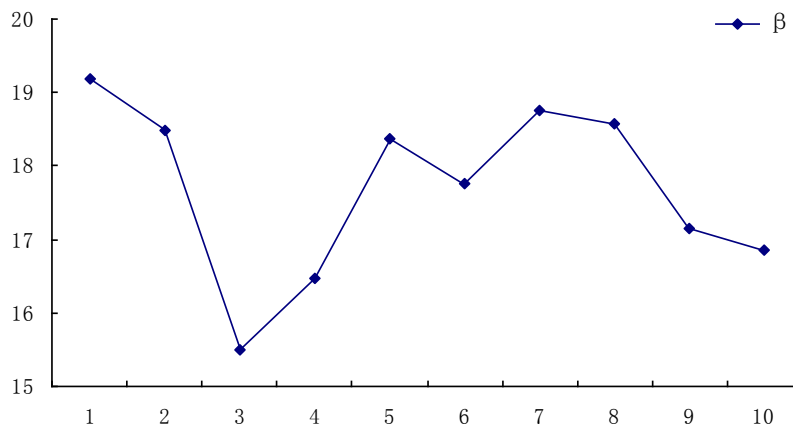


Figure 5. β_r with Different T

As it can be seen in Figure 5, during the increasing of operation periods, market demand of new product tends to be stable, which means the robust optimization model can be slowed down the risk of market demand. The dynamic price/quantity can be run in a suitable and specific interval during the operation of reverse logistics system so the reverse logistics system can be easily controlled to reduce the risk and improve the operational stability.

4. Conclusion

In this paper a hybrid robust manufacturing/remanufactured reverse logistics network optimization model to tackle a dynamic multi-period production planning problem with uncertain market demand and uncertain quantity of the returned product is presented. The robust optimization model is studied and an equivalent robust optimization model based on the duality theory and auxiliary problem principle which allows the solutions to be derived more efficiently is obtained. Numerical study is provided by using the Lingo 12.0 to validate the effectiveness of the robust model. The result shows that the robust optimization model combined both robust optimization and stochastic programming is an effective method to deal with the situations when returned product is uncertain.

This paper is a novel contribution to the hybrid robust manufacturing/remanufactured network research area as it is to consider both the forward and reverse supply chain. Several problem extensions also seem valuable such as to consider the capacity problems of the returned storage, or to assume that the remanufactured products need to be reintegrated into the forward supply chain. Future work can be considered multi-product multi-market reverse logistics system network design and planning.

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