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The Real Estate Enterprise Supply Chain Logistics Model Research

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Abstract

To overcome the deficiencies of the existing literatures to describe the function of recovery quantity, it was necessary to construct a new function, in which recovery quantity connected with buy-back price and sale quantity. This paper analyzed the optimal decision of the manufacturer and competing retailers under the following three cases: two retailers participated in the recycle, one retailer participated in the recycle and the manufacturer collected directly. The results show that: when the saving cost (cost between new products and reused products) is large enough, the manufacturer tends to collect used products in the consumer market through the competing retailers. And when the saving cost is small, the manufacturer collects herself or chooses one of the competing retailers to collect the used products, which depends on the market size, the attract coefficient and the switching coefficient. Moreover, the improvement of the consumers' environmental awareness is beneficial to the manufacturer, consumers and social environment. The results of this paper provide a basis for the manufacturer on its channel selection.

Keywords: *Closed-loop supply chain, function of recovery quantity, channel selection, remanufacturing*

1. Introduction

With the rapid development of economy, a single enterprise is difficult to obtain larger market share in the fierce market competition. The supply chain theory, first put forward in the 1980s, well portrayed the relationship between the enterprises in reality. However with the increasingly highlight of the resource and environment problems, human's environmental awareness increased, and traditional supply chain theory cannot describe the new resource-production-consumption- renewable resources circulation patterns any more. In 1992, Shock first put forward the reverse logistics, which promoted the development of the reverse supply chain and organically integrated traditional supply chain theory and closed-loop supply chain theory. Based on the perspective of product life cycle, it well described the whole product procedure from production, sale, recycle to reproduce and resale.

In recent years, Number of multinational companies increased year by year. China has been a large manufacturing country. So environmental problems become increasingly serious and implementation of closed-loop supply chain management has positive social and environmental benefits.

Geyer (2007) demonstrates that by coordinating the production cost structure, collection rate, product life cycle, and component durability, we can create or maximize production cost savings from remanufacturing. Bayındır (2007) presents the profitability of remanufacturing option under substitution policy subject to a capacity constraint of the joint manufacturing/remanufacturing facility. Ferguson (2005) proposes a target rebate contract to decrease the number of false failures and (potentially) increase the net sales.

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Debo (2005) studies the joint pricing and production technology selection problem faced by a manufacturer. He thinks that controlling remanufacturing levels and adjusting remanufacturing costs could minimize the production costs. Based on the three methods of recycling (abandon, re-sale and re-manufacture), Karaer (2007) studies the manufacturers' inventory model about recycling products. Majumder (2001) builds a two-period model of remanufacturing in the face of competition between the original equipment manufacturer (OEM) and local remanufacturer (L). Hammond (2007) constructs a closed-loop supply chain network equilibrium model. Calcott (2005) and Ma (2012) focus on how the government replacement-subsidy influence the different models of closed-loop supply chain. Wang (2013) studies the production strategies and coordination strategies which closed-loop supply chain responds to disturbance of market demand and cost. Yi (2012) considers the advertising effect in the closed-loop supply chain.

Savaskan (2004) addresses the problem of choosing the appropriate reverse channel structure for the collection of used products from customers. By comparing the retailer and the supply chain profits of the three options, we find that ceteris paribus, the agent, who is closer to the customer (i.e., the retailer), is the most effective undertaker of product collection activity for the manufacturer. Then Savaskan (2006) expands the recycling model to closed-loop supply chain with competing retailers. Gu (2008) constructs a closed-loop supply chain model about recycle quantities and buy-back price to analyze the pricing of the buy-back price, the wholesale price and the retail price under three recycling options. We obtain the manufactures' optimal recycling decision. The results are different because the recovery quantity assumptions are different. Savaskan thinks recovery quantity is a decision variable and buy-back price is endogenous variable. However Qiaolun thinks recovery quantity is a function of buy-back price.

The existing literature mainly focus on the pricing, coordinating, network equilibrium, inventory model and recycling channel options of the closed-loop supply chain. The function of recovery quantity are roughly divided into two categories: first, the recovery rate stands for recovery quantity to construct the relationship between new products quantity and re-manufacturing products quantity. Second they assume that recovery quantity and buy-back price have a relationship, in which buy-back price is decision variable. However in reality, recovery quantity is not only related to sales quantity, but also to buy-back price. So this paper first constructs the function of recovery quantity both related to sales quantity and buy-back price. Then we further consider in which recycling channel competition is beneficial to manufacturers in a closed-loop supply chain with competing retailers. This paper analyzes retailers' pricing decision in the closed-loop supply chain under the three recycling options and how the coefficient influence the pricing decision. By comparing manufacturers' profits, we provide a basis for the manufacturer on its channel selection.

2. Model and Assumptions

2.1. Model Description

We construct a supply chain consisting of a manufacturer and two competing retailers. Due to the government's legislative provisions and their responsibility consciousness of the social environment, manufacturers as a stackelberg leader should recycle used products and remanufacture. Manufacturers have three recycling options: 1) Both two retailers participate in recycling and they compete with each other; 2) Only one retailer recycles used products; 3) manufacturers collected directly (Figure1). The study aims to solve the influence of costs saving on recycling channel options. We also analyze the effect of consumer rate who return voluntarily used products in the market on chain members or consumers.

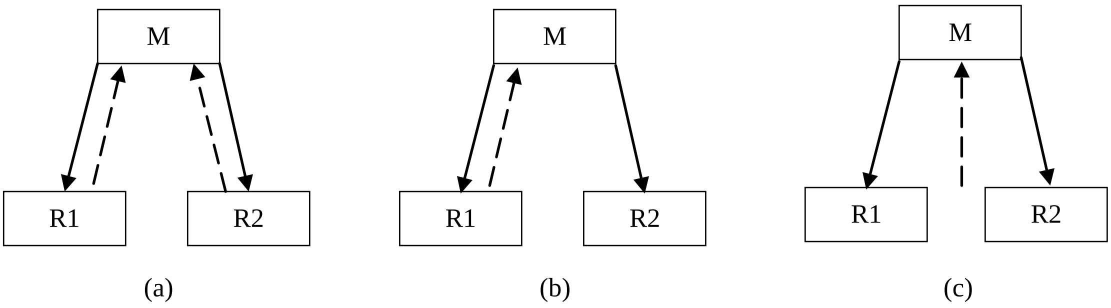

Figure 1. The Channel Chart

2.2. Model Assumptions and Variable Definitions

Under the condition of information symmetry in the supply chain, we assume that 100 percent of used products are put into remanufacturing and resale and we ignore depreciation of products. The unit cost production of new products is C_m, that of remanufacturing products is C_r. If$\Delta = C_m - C_r$, then we have $\Delta < C_m$. The unit buy-back rate is constant and usually it is zero. It should be pointed out that though the unit buy-back rate is non-zero, it cannot change the results. Manufacturers sell products to retailers as unit price w and the retailer i sells products to consumers as unit price P_i. In the market, the unit buy-back price of used products is b and manufacturers should spend m unit buy-back price to collect used products from retailers.

2.3. Product Demand Function and used Product Recovery Quantity Function

Assume that there exists Bertrand competition between the two retailers and the product demand function of the retailer i is

$$q_i = \alpha - \delta P_i + \gamma(P_j - P_i), \qquad 0 \le \alpha, 0 \le \delta, 0 \le \gamma \tag{1}$$

α means potential market size. δ stands for attract coefficient and means consumers who can be attracted by the low price. γ is switching coefficient and means consumers who wonder between the two retailers.

If both two retailers recycle products at the same time, they still have Bertrand competition. Based on above assumptions, the recovery quantity function of the retailer i is

$$g_i = \beta q_i + \delta b_i - \gamma(b_j - b_i), \qquad 0 \le \beta \le 1, 0 \le \delta, 0 \le \gamma \tag{2}$$

β means the consumer rate who voluntarily return used products. δ is attract coefficient and stands for consumers who are attracted by high buy-back price and return used products. γ is switching coefficient and means consumers who return used products but wonder between the two retailers.

If there are only one retailer or manufacturers collect directly, the recovery quantity function is

$$g = \beta(q_1 + q_2) + \delta b, \qquad 0 \le \beta \le 1, 0 \le b \tag{3}$$

3. The Closed-loop Supply Chain with Two Retailers

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Because manufacturers are Stackelberg leader, manufacturers and retailers have a two-stage complete information dynamic game. The game order is that the manufacturer first decides the wholesale price and buy-back price, the retailer then decides her own wholesale price and buy-back price according to the manufacturers' decision. We can obtain the optimal solution by using the reverse recursive method.

Based on above assumption, manufacturers' decision variable is wholesale price w and buy-back price m, its profits function is

$$\prod M = (w - C_m)(q_1 + q_2) + (\Delta - m)(g_1 + g_2) \tag{4}$$

And the profits function of retailer i is

$$\prod R_1 = (P_i - w)q_i + (m - b_i)g_i \tag{5}$$

The Hesse matrix of formula (5) is

$$\begin{pmatrix} \frac{\partial^2 \prod R_i}{\partial P_i^2} & \frac{\partial^2 \prod R_i}{\partial P_i \partial b_i} \\ \frac{\partial^2 \prod R_i}{\partial b_i \partial P_i} & \frac{\partial^2 \prod R_i}{\partial b_i^2} \end{pmatrix} = (\gamma + \delta)\begin{pmatrix} -2 & \beta \\ \beta & -2 \end{pmatrix} \tag{6}$$

From the formula (6), we know the Hesse matrix of the retailer i is negative. According to the first-order derivative, we can obtain the optimal wholesale price P_i and the optimal buy-back price b_i.

$$P_i = \frac{(\gamma + \delta)(\gamma + 2\delta)w - \beta\delta(\gamma + \delta)m + \alpha(\gamma - \beta^2\gamma + 2\delta - \beta^2\delta)}{\gamma^2 + (4 - \beta^2)\gamma\delta + (4 - \beta^2)\delta^2} \tag{7}$$

$$b_i = \frac{(\gamma + \delta)[(\gamma + 2\delta - \beta^2\delta)m + \beta\delta w - \alpha\beta]}{\gamma^2 + (4 - \beta^2)\gamma\delta + (4 - \beta^2)\delta^2} \tag{8}$$

We generate the formula (7) and (8) into formula (4), then we get the Hesse matrix of the formula (4).

$$\begin{pmatrix} \frac{\partial^2 \prod M}{\partial w^2} & \frac{\partial^2 \prod R_i}{\partial w \partial m} \\ \frac{\partial^2 \prod M}{\partial m \partial w} & \frac{\partial^2 \prod R_i}{\partial m^2} \end{pmatrix} = \frac{\gamma^2 + 3\gamma\delta + 2\delta^2}{\gamma^2 + 4\gamma\delta - \beta^2\gamma\delta + 4\delta^2 - \beta^2\delta^2}\begin{pmatrix} -4\delta & 2\beta\delta \\ 2\beta\delta & -4\delta \end{pmatrix} \tag{9}$$

From the formula (9), we know the Hesse matrix of the manufacturer is negative. According to the first-order derivative, we can obtain the optimal wholesale price m^* and the optimal buy-back price w^* of manufacturers.

$$m^* = \frac{\delta(2\gamma - \beta^2\gamma + 4\delta - \beta^2\delta)\Delta - \beta\gamma(\alpha - \delta C_m)}{\delta(\gamma + 2\delta)(2 - \beta)(2 + \beta)} \tag{10}$$

$$w^* = \frac{\delta C_m(2\gamma + 4\delta - \beta^2\delta) - \beta\gamma\delta\Delta + \alpha(2\gamma - \beta^2\gamma + 4\delta - \beta^2\delta)}{\delta(\gamma + 2\delta)(2 - \beta)(2 + \beta)} \tag{11}$$

We generate the formula (10) and (11) into formula (7) and (8), then we get the formula (12) and (13).

$$P_i^* = \frac{2\delta(\gamma + \delta)C_m - \beta\gamma(\gamma + \delta)\Delta + \alpha(2\gamma - \beta^2\gamma + 6\delta - 2\beta^2\delta)}{\delta(\gamma + 2\delta)(2 - \beta)(2 + \beta)} \tag{12}$$

$$b_i^* = \frac{(\gamma+\delta)[(2-\beta^2)\delta\Delta - \beta(\alpha-\delta C_m)]}{\delta(\gamma+2\delta)(2-\beta)(2+\beta)} \tag{13}$$

We generate the results P_i^* and b_i^* into formula (1) and (2), and combine with the formula (4), we can get the results of manufacturers' profits.

$$\prod M^* = \frac{2(\gamma+\delta)[\delta^2\Delta^2 + \beta\delta(\alpha-\delta C_m)\Delta + (\alpha-\delta C_m)^2]}{\delta(\gamma+2\delta)(2-\beta)(2+\beta)} \tag{14}$$

Proposition 1 when $\alpha - \delta C_m < \frac{(2-\beta^2)\delta\Delta}{\beta}$, we have $m^* > 0, b_i^* > 0$.

Theorem 1 when both two retailers participate in recycling, the manufacturers' profits increase with the increase of the consumer rate β. However sale price and buy-back price in the market, the whole price and buy-back price of manufacturers decrease with the increase of β.

Proof

$$\frac{\partial \prod M^*}{\partial\beta} = \frac{2(\gamma+\delta)}{\delta(\gamma+2\delta)(\beta-2)^2(\beta+2)^2}[2\beta\delta^2\Delta^2 + \delta\Delta(4+\beta^2)(\alpha-\delta C_m) + 2\beta(\alpha-\delta C_m)^2] > 0,$$

$$\frac{\partial P_i^*}{\partial\beta} = -\frac{\gamma+\delta}{\delta(\gamma+2\delta)(\beta-2)^2(\beta+2)^2}[\delta\Delta(4+\beta^2)+4\beta(\alpha-\delta C_m)] < 0,$$

$$\frac{\partial b_i^*}{\partial\beta} = -\frac{\gamma+\delta}{\delta(\gamma+2\delta)(\beta-2)^2(\beta+2)^2}[4\beta\delta\Delta+(4+\beta^2)(\alpha-\delta C_m)] < 0,$$

$$\frac{\partial m^*}{\partial\beta} = -\frac{\gamma}{\delta(\gamma+2\delta)(\beta-2)^2(\beta+2)^2}[4\beta\delta\Delta+(4+\beta^2)(\alpha-\delta C_m)] < 0,$$

$$\frac{\partial w^*}{\partial\beta} = -\frac{\gamma}{\delta(\gamma+2\delta)(\beta-2)^2(\beta+2)^2}[\delta\Delta(4+\beta^2)+4\beta(\alpha-\delta C_m)] < 0,$$

Under the condition of two retailers, retailers' enthusiasm of recycling decreases when consumers' environmental awareness enhances, that is when the consumer rate is large who voluntarily return used products. It leads to the decrease of the unit buy-back price. With the increase of the consumer rate, manufacturers' production costs decrease, which promotes reduce of the manufacturers' wholesale price and retailers' sale price. Then manufacturers' profits rise because of the recycling of waste products. And the reduce of sale price is beneficial to consumers in the market.

4. The Closed-loop Supply Chain with Only One Retailer

Game order is as follows, manufacturer sells products to the two retailers as the unit wholesale price $\overline{w}$. The retailer sells products as $\overline{P_i}$ in the market. The retailer 1 collects used products from consumers as a unit buy-back price of $\overline{b}$ and then manufacturer collects it from the retailer 1 as a unit buy-back price of $\overline{m}$. The decision variable of manufacturers is $\overline{w}$ and $\overline{m}$, The decision variable of retailer 1 is $\overline{P_1}$ and retailer 2 is $\overline{P_2}$.

Then we get the profits function of manufacturers,

$$\overline{\prod M} = (\overline{w} - C_m)(\overline{q_1} + \overline{q_2}) + (\Delta - \overline{m})\overline{g} \tag{15}$$

The profits function of retailer i is

$$\overline{\prod R_1} = (\overline{P}_1 - \overline{w})\overline{q_1} + (\overline{m} - \overline{b})\overline{g} \tag{16}$$

$$\overline{\prod R_2} = (\overline{P}_2 - \overline{w})\overline{q_2} \tag{17}$$

By verifying the Hesse matrix, we can judge the formula (16) and (17) are concave function. According to the first-order derivative of profits maximization, we can obtain $\overline{P}_1$, $\overline{P}_2$ and $\overline{b}$ respectively:

$$\overline{P}_1 = \frac{(\gamma+\delta)(6\gamma+4\delta+\beta^2\delta)w - 2\beta\delta(\gamma+\delta)m + \alpha(6\gamma-4\beta^2\gamma+4\delta-3\beta^2\delta)}{(3\gamma+2\delta)(2\gamma+4\delta-\beta^2\delta)} \tag{18}$$

$$\overline{P}_2 = \frac{(\gamma+\delta)(6\gamma+4\delta+\beta^2\delta)w - \beta\gamma\delta m + \alpha(6\gamma-2\beta^2\gamma+4\delta-\beta^2\delta)}{(3\gamma+2\delta)(2\gamma+4\delta-\beta^2\delta)} \tag{19}$$

$$\overline{b} = \frac{2\beta\delta(\gamma+\delta)w + \beta(\gamma+2\delta-\beta^2\delta)m - 2\alpha\beta(\gamma+\delta)}{\delta(2\gamma+4\delta-\beta^2\delta)} \tag{20}$$

We generate the formula (18) , (19) and (20) into formula (15) respectively. By verifying the Hesse matrix of manufacturers' profits function, we get it is negative. According to the first-order derivative of profits maximization of the manufacturers, we can obtain the optimal wholesale price and buy-back price respectively:

$$\overline{w^*} = \frac{\delta C_m(8\delta^2+24\gamma\delta-2\beta^2\gamma\delta+16\delta^2-3\beta^2\delta^2) - \beta\delta\Delta(\gamma+2\delta)(2\gamma+\delta)}{\alpha\delta} + \frac{2\alpha(\gamma+\delta)(4\gamma-2\beta^2\gamma+8\delta-3\beta^2\delta)}{\alpha\delta} \tag{21}$$

$$\overline{m^*} = \frac{2(\gamma+\delta)[\delta\Delta(4\gamma-2\beta^2\gamma+8\delta-3\beta^2\delta) - 2\beta(\alpha-\delta C_m)(2\gamma+\delta)]}{\alpha\delta} \tag{22}$$

Here, $\alpha = 16\gamma^2 - 4\beta^2\gamma^2 + 48\gamma\delta - 12\beta^2\gamma\delta + 32\beta^2 - 9\beta^2\delta^2 > 0$.

We generate the formula (21) , (22) into formula (18), (19) and (20) respectively. We can get

$$\overline{P_1^*} = \frac{C_m(\gamma+\delta)(24\gamma^2+42\gamma\delta+40\gamma\delta+16\delta^2+3\beta^2\delta^2) - \beta\Delta(\gamma+\delta)(6\gamma^2+15\gamma\delta+10\delta^2)}{\alpha(3\gamma+2\delta)} + \frac{\alpha(24\gamma^3-12\beta^2\gamma^3+112\gamma^2\delta-46\beta^2\gamma^2\delta+136\gamma\delta^2-56\beta^2\gamma\delta^2+48\delta^3-21\beta^2\delta^3)}{\alpha\delta(3\gamma+2\delta)} \tag{23}$$

$$\overline{P_2^*} = \frac{C_m(\gamma+\delta)(24\gamma^2+40\gamma\delta-2\beta^2\gamma\delta+16\delta^2-3\beta^2\delta^2) - \beta\Delta(\gamma+\delta)(6\gamma^2+11\gamma\delta+2\delta^2)}{\alpha(3\gamma+2\delta)} + \frac{\alpha(24\gamma^3-12\beta^2\gamma^3+112\gamma^2\delta-42\beta^2\gamma^2\delta+136\gamma\delta^2-46\beta^2\gamma\delta^2+48\delta^3-15\beta^2\delta^3)}{\alpha\delta(3\gamma+2\delta)} \tag{24}$$

$$\overline{b^*} = \frac{2(\gamma+\delta)[\delta\Delta(2\gamma-2\beta^2\gamma+4\delta-3\beta^2\delta) - \beta(\alpha-\delta C_m)(6\gamma+5\delta)]}{\alpha\delta} \tag{25}$$

According to the $\overline{P_1^*}$, $\overline{P_2^*}$, $\overline{b^*}$ and formula (1), (3), (15), we can get

$$\overline{\prod M^*} = \frac{2(\gamma+\delta)[\delta^2\Delta^2(\gamma+2\delta)+\beta\delta\Delta(2\gamma+3\delta)(\alpha-C_m)+4(\gamma+\delta)(\alpha-C_m)^2]}{\alpha\delta} \tag{26}$$

Proposition 2 when $\alpha-\delta C_m < \frac{(2\gamma-2\beta^2+4\delta-3\beta^2\delta)\delta\Delta}{\beta(6\gamma+5\delta)}$, we have $\overline{m^*}>0, \overline{b^*}>0$.

Theorem 2 The sale price of retailer 1 who participates recycling is lower than that of retailer 2 who doesn't recycle, that is $\overline{P_1}^* < \overline{P_2}^*$.

Proof

$$\overline{P_1}^* - \overline{P_2}^* = -\frac{2\beta(\gamma+\delta)[\beta(2\gamma+3\delta)(\alpha-\delta C_m)+2\delta(\gamma+2\delta)]}{\alpha(3\gamma+2\delta)} < 0$$

Under the condition of only one retailer, the retailer 1 get profits from collecting waste products. So he reduce his sale price to get more market size. It is bad for retailer 2. And this situation is not stable. Because the retailer 2 will get profits from the manufacturers.

Theorem 3 In the closed-loop supply chain with only one retailer, manufacturers' profits increase with the consumer rate increasing who voluntarily return used products. And the whole price and buy-back price of manufacturers decrease with the increase of β.

Proof

As shown in Theorem 1.

5. The Closed-loop Supply Chain in Which Manufacturers Collect Directly

Game order is as follows, manufacturer sells products to the two retailers as the unit wholesale price $\tilde{w}$. The retailer sells products as $\tilde{P}_i$ in the market. The manufacturers collect used products from consumers as a unit buy-back price of $\tilde{b}$. And The decision variable of manufacturers is $\tilde{w}, \tilde{b}$. The retailer is $\tilde{P}_i$.

Then we have

$$\overline{\prod M} = (\tilde{w}-C_m)(\tilde{q_1}+q_2)+(\Delta-\tilde{b})\tilde{g} \tag{27}$$

$$\overline{\prod R_i} = (\tilde{P}_i-\tilde{w})\,\tilde{q}_i \tag{28}$$

$$\tilde{P}_i = \frac{w\gamma+w\delta+\alpha}{\gamma+2\delta} \tag{29}$$

We generate the formula (29) into formula (27). By verifying the Hesse matrix of manufacturers' profits function, we obtain the optimal wholesale price and buy-back price respectively:

$$\widetilde{b^*} = \frac{\delta\Delta(\gamma-\beta^2\gamma+2\delta-\beta^2\delta)-\beta(\gamma+\delta)(\alpha-\delta C_m)}{c\delta} \tag{30}$$

$$\widetilde{w^*} = \frac{2\delta C_m(\gamma+2\delta)-\beta\delta\Delta(\gamma+2\delta)+2\alpha(\gamma-\beta^2\gamma+2\delta-\beta^2\delta)}{c\delta} \tag{31}$$

Here $c = 2\gamma-\beta^2\gamma+4\delta-\beta^2\delta > 0$.

We generate the formula (30) and (31) into formula (27) and (29) to get $\widetilde{p^*}$ and $\widetilde{\prod M^*}$.

$$\widetilde{p^*} = \frac{2\delta C_m(\gamma+\delta)-\beta\delta\Delta(\gamma+\delta)+2\alpha(\gamma-\beta^2\gamma+3\delta-\beta^2\delta)}{2c\delta} \tag{32}$$

$$\widetilde{\prod M^*} = \frac{2\delta C_m(\gamma+\delta) - \beta\delta\Delta(\gamma+\delta) + 2\alpha(\gamma - \beta^2\gamma + 3\delta - \beta^2\delta)}{2c\delta} \tag{33}$$

Proposition 3 when $\alpha - \delta C_m < \frac{(\gamma-\beta^2\gamma+2\delta-\beta^2\delta)\delta\Delta}{\beta}$, we have $\widetilde{b^*} > 0$.

Theorem 4 In the closed-loop supply chain in which manufacturers collect directly, manufacturers' profits increase with the consumer rate increasing who voluntarily return used products. And the whole price and buy-back price of manufacturers decrease with the increase of β.

Proof

As shown in Theorem 1.

6. A Comparative Analysis of the Closed-loop Supply Chain Models in Three Channel Options

This part analyzes the optimal decision of the manufacturer and competing retailers under the following three cases: two retailers participated in the recycle, one retailer participated in the recycle and the manufacturer collected directly. Manufacturers make decisions as measured by its profits.

When $\alpha - \delta C_m < min\left\{\frac{(2-\beta^2)\delta\Delta}{\beta}, \frac{(2\gamma-2\beta^2+4\delta-3\beta^2\delta)\delta\Delta}{\beta(6\gamma+5\delta)}, \frac{(\gamma-\beta^2\gamma+2\delta-\beta^2\delta)\delta\Delta}{\beta}\right\}$, the decision variables in the three cases are all positive. According to the formula (14), (26) and (33), the three manufacturers' profits ($\prod M^*, \overline{\prod M^*}, \widetilde{\prod M^*}$) subtract respectively. We can get: the saving costs between new and remanufacturing products Δ, the attract coefficient δ, the switching coefficient γ. And the potential market size α, which influence the manufacturers' decision.

1. $\prod M^* - \overline{\prod M^*} = \frac{2(\gamma+\delta)}{\alpha(\gamma+\delta)(4-\beta^2)}(A\Delta^2 + B\Delta + C)$

Here $A = \delta(12\gamma^2 - 3\beta^2\gamma^2 + 16\delta^2 - 5\beta^2\delta^2 + 32\gamma\delta - 8\beta^2\gamma\delta) > 0$,
$B = \beta(\alpha - \delta C_m)(8\gamma^2 - 2\beta^2\gamma^2 + 8\delta^2 - 3\beta^2\delta^2 + 20\gamma\delta - 5\beta^2\gamma\delta) > 0$,
$C = -\beta^2\delta(\alpha - \delta C_m)^2 < 0$.

Make $0 < \Delta_1 = \frac{-B+\sqrt{B^2-4AC}}{2A}$ and according to the nature of the quadratic function, we get that when $\Delta_1 < \Delta$, $\overline{\prod M^*} < \prod M^*$; when $0 < \Delta < \Delta_1$, $\prod M^* < \overline{\prod M^*}$.

2. $\overline{\prod M^*} - \widetilde{\prod M^*} = \frac{1}{2acb}(D\Delta^2 + E\Delta + F)$

Here $D = -\delta^2(\gamma + 2\delta)(8\gamma^2 + 16\delta^2 - 5\beta^2\delta^2 + 24\gamma\delta - 4\beta^2\gamma\delta) < 0$,
$E = -2\beta\delta(\gamma+\delta)(\alpha - \delta C_m)(8\gamma^2 + 8\delta^2 - 3\beta^2\delta^2 + 20\gamma\delta - 2\beta^2\gamma\delta)$,
$F = -2\beta^2(\gamma+\delta)(\alpha - \delta C_m)^2(4\gamma^2 + 4\gamma\delta - \delta^2)$.

Make $\Delta_2 = \frac{-E-\sqrt{E^2-4DF}}{2D} < \Delta_3 = \frac{-E+\sqrt{E^2-4DF}}{2D}$ and according to the nature of the quadratic function, we get that

1) if $4\gamma^2 + 4\gamma\delta - \delta^2 < 0$, when $\Delta_3 < \Delta$, $\overline{\prod M^*} < \widetilde{\prod M^*}$; when $0 < \Delta < \Delta_3$, $\widetilde{\prod M^*} < \overline{\prod M^*}$.

2) if $4\gamma^2 + 4\gamma\delta - \delta^2 > 0$ and $(\alpha - \delta C_m) > 0$, $\overline{\prod M^*} < \widetilde{\prod M^*}$.

3) if $4\gamma^2 + 4\gamma\delta - \delta^2 > 0$ and $(\alpha - \delta C_m) < 0$, when $0 < \Delta < \Delta_2$ or $\Delta_3 < \Delta$, $\overline{\prod M^*} < \widetilde{\prod M^*}$; when $\Delta_2 < \Delta < \Delta_3$, $\widetilde{\prod M^*} < \overline{\prod M^*}$.

3. $\prod M^* - \widetilde{\prod M^*} = \frac{\gamma}{2c\delta(\gamma+2\delta)(4-\beta^2)}(G\Delta^2 + H\Delta + I)$

Here $G = \delta^2(4\gamma - 3\beta^2\gamma + 8\delta - 4\beta^2\gamma) > 0$,

$\mathrm{H} = -2\beta^3\delta(\gamma+\delta)(\alpha-\delta C_m)$,
$\mathrm{I} = -\beta^2(\gamma+\delta)(\alpha-\delta C_m)^2 < 0$.

Make $0 < \Delta_4 = \frac{-H+\sqrt{H^2-4GI}}{2G}$ and according to the nature of the quadratic function, we get that when $\Delta_4 < \Delta$, $\prod M^* > \widetilde{\prod M^*}$; when $0 < \Delta < \Delta_4$, $\widetilde{\prod M^*} > \prod M^*$.

Combing the above conclusion, we can get that

1. when $4\gamma^2 + 4\gamma\delta - \delta^2 < 0$, that is $2\gamma + 2\sqrt{2}\gamma < \delta$

1) if $\max\{\Delta_1,\Delta_3,\Delta_4\} < \Delta$, we have $\overline{\prod M^*} < \widetilde{\prod M^*} < \prod M^*$.

2) if $0 < \Delta < \min\{\Delta_1,\Delta_3,\Delta_4\}$, $\overline{\prod M^*} > \widetilde{\prod M^*} > \prod M^*$.

3) if $\max\{\Delta_1,\Delta_4\} < \Delta < \Delta_3$, $\widetilde{\prod M^*} < \overline{\prod M^*} < \prod M^*$.

4) if $\Delta_3 < \Delta < \min\{\Delta_1,\Delta_4\}$, $\prod M^* < \overline{\prod M^*} < \widetilde{\prod M^*}$.

5) if $\max\{\Delta_1,\Delta_3\} < \Delta < \Delta_4$, $\overline{\prod M^*} < \prod M^* < \widetilde{\prod M^*}$.

6) if $\Delta_4 < \Delta < \min\{\Delta_1,\Delta_3\}$, $\widetilde{\prod M^*} < \prod M^* < \overline{\prod M^*}$.

2. when $0 < 4\gamma^2 + 4\gamma\delta - \delta^2$, that is $0 < \delta < 2\gamma + 2\sqrt{2}\gamma$

1) if $\max\{\Delta_1,\Delta_4\} < \Delta$, $\overline{\prod M^*} > \widetilde{\prod M^*} > \prod M^*$.

2) if $0 < \Delta < \min\{\Delta_1,\Delta_4\}$, $\prod M^* < \overline{\prod M^*} < \widetilde{\prod M^*}$.

3) if $\Delta_1 < \Delta < \Delta_4$, $\overline{\prod M^*} < \prod M^* < \widetilde{\prod M^*}$.

3. when $4\gamma^2 + 4\gamma\delta - \delta^2 > 0$, that is $0 < \delta < 2\gamma + 2\sqrt{2}\gamma$ and $(\alpha - \delta C_m) < 0$

1) if $\max\{\Delta_1,\Delta_3,\Delta_4\} < \Delta$, or $\max\{\Delta_1,\Delta_4\} < \Delta < \Delta_2$, $\overline{\prod M^*} < \prod M^* < \widetilde{\prod M^*}$.

2) if $\Delta_2 < \Delta < \min\{\Delta_1,\Delta_3,\Delta_4\}$, $\overline{\prod M^*} > \widetilde{\prod M^*} > \prod M^*$.

3) if $\max\{\Delta_1,\Delta_2,\Delta_4\} < \Delta < \Delta_3$, $\widetilde{\prod M^*} < \overline{\prod M^*} < \prod M^*$.

4) if $0 < \Delta < \min\{\Delta_1,\Delta_2,\Delta_4\}$, or $\Delta_3 < \Delta < \min\{\Delta_1,\Delta_4\}$, $\prod M^* < \overline{\prod M^*} < \widetilde{\prod M^*}$.

5) if $\max\{\Delta_1,\Delta_3\} < \Delta < \Delta_4$, or $\Delta_1 < \Delta < \min\{\Delta_2,\Delta_4\}$, $\overline{\prod M^*} < \prod M^* < \widetilde{\prod M^*}$.

6) if $\max\{\Delta_2,\Delta_4\} << \Delta < \min\{\Delta_1,\Delta_3\}$, $\widetilde{\prod M^*} < \prod M^* < \overline{\prod M^*}$.

From the above analysis, we conclude that the larger the saving costsΔ is, the higher the probability is in which manufacturers choose both the two retailers participate in recycling. Competition could promote the increase of recovery quantity and the decrease of manufacturers' costs and wholesale price. It increases the market size and further increase the profits. And it is also good for retailers. Because they can get more profits from recycling. However when the saving cost is small, the manufacturer focus more on potential market size. And it up to the competition whether to recycle herself or retailer collects directly.

Theorem 5 when the saving cost (cost between new products and reused products) is large enough, the manufacturer tends to collect used products in the consumer market through the competing retailers. And when the saving cost is small, the manufacturer collects herself or chooses one of the competing retailers to collect the used products, which depends on the market size, the attract coefficient and the switching coefficient.

7. Conclusions

Different from the existing literature, this paper constructs a new function, in which recovery quantity connected with buy-back price and sale quantity. We analyzes the optimal decision of the manufacturer and competing retailers under the following three cases respectively. It concludes that: 1) When the saving cost (cost between new products and reused products) is large enough, the manufacturer tends to collect used products in the consumer market through the competing retailers. 2) When the saving cost is small, the manufacturer collects herself or chooses one of the competing retailers to collect the used products, which depends on the market size, the attract coefficient and the switching coefficient. 3) Human's environmental awareness is very important, which could promote consumers to return used products voluntarily. It is win-win, on the one hand it increases the profits of the supply chain and decrease the sale price, on the other hand it reduces environmental pollutions.

These conclusions need further debate. In the future we can expand the number of retailer from two to several, which makes it more general and reliable.

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