

Adaptive Trajectory Tracking Control for WMR based on Cascade-design Method

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Abstract

This paper addresses the problem of adaptive trajectory tracking control for the wheeled mobile robot (WMR) based on cascade-design method. The condition is considered that there is a distance between the mass center of the robot and the geometrical center. The tracking error system is divided into two subsystems: the position error system and the orientation error system. In order to deal with the circumstance with the unknown parameters, an adaptive tracking control law is proposed. Simulation results illustrate the effectiveness of the proposed control methods.

Keywords: *wheeled mobile robot (WMR); trajectory tracking control; adaptive control; cascaded system*

1. Introduction

Mobile robots have attracted much attention of many researchers because of its usefulness in many applications. Mobile robots, however, suffer nonholonomic constraints. That is, mobile robots can move only in the direction normal to the axis of the driving wheels [1]. Kinematic model of parallel wheeled mobile robot (WMR) fails to meet Brockett's necessary condition for feedback stabilization. This means that no smooth or even continuous time invariant static state feedback law exists which makes the closed loop system locally asymptotically stable. This has attracted interest of researchers to the complicate problem of WMR control.

Trajectory tracking control is an important topic in the field of robot control. It refers to the case where a robot is required to track a time parameterized reference. Some researchers have studied the problem in recent years by using different techniques. In [2], a compensation adaptation law plus a nonlinear feedback term coupled to a dynamic nonlinear filter has been designed to produce tracking control. The input-output linearization has been applied in the tracking control of wheeled robots [3]. In [4-7], a fuzzy controller has been designed to realize tracking control for robots. In [8], the problem of PID tracking control of robotics has been considered. A sliding mode control scheme to solve the tracking problem based on the discrete time model of the robot [9] and a sliding-mode control method for wheeled-mobile robots in polar coordinates have been proposed [10]. In [11], a switching fuzzy logic controller for mobile robots with a bounded curvature constraint has been presented. A tracking control method for a robot has been presented, which combines adaptive approach and neural dynamics [12].

In this paper, we will propose an adaptive trajectory tracking controller based on cascade-design method. The main differences between the above methods and the cascade-design method: controllers based on the cascade-design method are much simpler, because the coupling terms are neglected when some conditions are satisfied without affecting performance of the entire systems. And the condition that there is a distance between the mass center and the geometrical center will be considered. Parameter

uncertainties will be dealt with in an adaptive framework by augmenting Lyapunov functions with terms that are quadratic in the parameter errors.

This paper is organized as follows. Section 2 presents the preliminaries and the problem formulation. An adaptive control law for the robot to deal with parameter uncertainty is derived in Section 3. Section 4 presents the simulation results. Finally, we make a brief conclusion on the paper in Section 5.

2. Preliminaries and Problem Formulation

2.1. Preliminaries

We start with a basic lemma in this section. Consider the cascaded system

$$\begin{cases} \dot{z}_1 = f_1(t, z_1) + g(t, z_1, z_2)z_2 \\ \dot{z}_2 = f_2(t, z_2) \end{cases} \quad (1)$$

where $z_1 \in R^n$, $z_2 \in R^p$, $f_1(t, z_1)$ is continuously differentiable in (t, z_1) and $f_2(t, z_2)$, $g(t, z_1, z_2)$ are continuous in their arguments, and locally Lipschitz in z_2 and (z_1, z_2) respectively.

Lemma1[13]. Cascaded system (1) is globally uniformly asymptotically stable (GUAS) if the following three assumptions hold:

(i) Assumption on Σ_1 : the system $\dot{z}_1 = f_1(t, z_1)$ is GUAS and there exists a continuously differentiable function $V(t, z_1): \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}_+$ that satisfies

$$\begin{aligned} W_1(z_1) &\leq V(t, z_1) \leq W_2(z_1), \quad \forall t \geq t_0, \quad z_1 \in \mathbb{R}^n \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial z_1} f_1(t, z_1) &\leq 0, \quad \forall \|z_1\| \geq \eta \\ \left\| \frac{\partial V}{\partial z_1} \right\| \|z_1\| &\leq c V(t, z_1), \quad \forall \|z_1\| \geq \eta \end{aligned}$$

where $W_1(z_1)$ and $W_2(z_1)$ are positive definite proper functions and $c, \eta > 0$ are constants;

(ii) Assumption on the interconnections: the function $g(t, z_1, z_2)$ satisfies for all $t \geq t_0$

$$\|g(t, z_1, z_2)\| \leq \theta_1(\|z_2\|) + \theta_2(\|z_2\|)\|z_1\|$$

where $\theta_1(\cdot)$, $\theta_2(\cdot): \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are continuous functions;

(iii) Assumption on Σ_2 : the system $\dot{z}_2 = f_2(t, z_2)$ is GUAS and for all $t_0 \geq 0$:

$$\int_{t_0}^{\infty} \|z_2(t, t_0, z_2(t_0))\| dt \leq \kappa(\|z_2(t_0)\|)$$

where $\kappa(\cdot)$ is a K-class function.

2.2. Problem Formulation

We consider the mobile robot of a two-wheeled and its mass center does not coincide with its geometrical center, shown in Figure 1. It is located in the two-dimensional $X - Y$ coordinate.

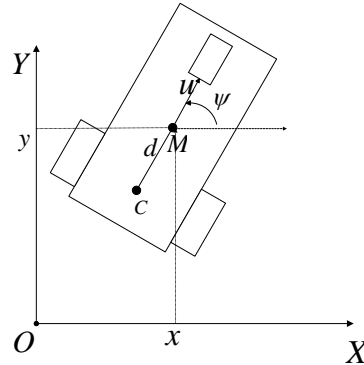


Figure 1. The Model of Two-wheeled Robot

In Figure 1, C is the geometrical center of two driving rear wheels, M is the mass center of the robot and d is the distance between C and M . (x, y) denotes the position of the robot in the $X - Y$ coordinate. ψ is the heading angle of the robot.

The mobile robot has the nonholonomic constraints that the driving wheels purely roll and do not slip. As the mass center of the robot does not coincide with its geometrical center, this nonholonomic constraint is written as the following equation

$$\dot{x} \sin \psi - \dot{y} \cos \psi - d \dot{\psi} = 0$$

The kinematic model of a wheeled mobile robot with two degrees of freedom is given by the following equations

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \cos \psi & -d \sin \psi \\ \sin \psi & d \cos \psi \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ \omega \end{pmatrix} \quad (2)$$

where u and ω represent linear velocity and angular velocity, respectively. The equations of reference robot can be described by

$$\begin{pmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\psi}_r \end{pmatrix} = \begin{pmatrix} \cos \psi & -d \sin \psi \\ \sin \psi & d \cos \psi \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_r \\ \omega_r \end{pmatrix}$$

The error state equations of the wheeled robot become

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\psi}_e \end{pmatrix} = \begin{pmatrix} -u + \omega y_e + u_r \cos \psi_e - d \omega_r \sin \psi_e \\ -x_e \omega - d \omega + u_r \sin \psi_e + d \omega_r \cos \psi_e \\ \omega_r - \omega \end{pmatrix} \quad (3)$$

The dynamic equations of the wheeled robot can be written as

$$\begin{pmatrix} F \\ N \end{pmatrix} = M \begin{pmatrix} \dot{u} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} m & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \dot{u} \\ \dot{\omega} \end{pmatrix}$$

where F denotes the driving force, N denotes the rotation torque, and m and I are the mass and the mass moment of inertia of the robot, respectively. Each rear wheel is powered by a motor which generates a control torque $\tau_i, i = 1, 2$ and

$$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \frac{R}{2} \begin{pmatrix} 1 & \frac{1}{L} \\ 1 & -\frac{1}{L} \end{pmatrix} \begin{pmatrix} F \\ N \end{pmatrix}$$

where R is the radius of the rear wheels, L is half of the length of the axis between two rear wheels. At this point, it is convenient to redefine the control inputs u_1 and u_2

as $u_1 \square \tau_1 + \tau_2$ and $u_2 \square \tau_1 - \tau_2$. This yields from $F = \frac{1}{R}(\tau_1 + \tau_2)$ and $N = \frac{L}{R}(\tau_1 - \tau_2)$

$$\begin{cases} u_1 = RF \\ u_2 = \frac{R}{L}N \end{cases} \quad (4)$$

As mentioned above, the control objective in this paper now is to stabilize error system (3). The problem considered can be formulated as follows:

Derive a feedback control for u_1 and u_2 such that error system (3) is globally asymptotically stable when m, I, R and L are unknown.

3. Adaptive Trajectory Tracking Controller Design

3.1. Kinematic Controller

Theorem1. Consider error system (3) with the control law

$$\begin{cases} \omega = \omega_r + k_1\psi_e \\ u = u_r + k_2x_e \end{cases} \quad (5)$$

where $k_1, k_2 > 0$, then the closed-loop error system

$$\begin{cases} \dot{x}_e = -u_r - k_2x_e + (\omega_r + k_1\psi_e)y_e + u_r \cos \psi_e - d\omega_r \sin \psi_e \\ \dot{y}_e = -(x_e + d)(\omega_r + k_1\psi_e) + u_r \sin \psi_e + d\omega_r \cos \psi_e \\ \dot{\psi}_e = -k_1\psi_e \end{cases} \quad (6)$$

is globally asymptotically stable.

Proof. Substituting (5) into (3), closed-loop error system (6) can be written in the cascaded form as following

$$\begin{cases} \begin{pmatrix} \dot{x}_e \\ \dot{y}_e \end{pmatrix} = \begin{pmatrix} -k_2 & \omega_r \\ -\omega_r & 0 \end{pmatrix} \begin{pmatrix} x_e \\ y_e \end{pmatrix} + \Pi \psi_e \\ \dot{\psi}_e = -k_1\psi_e \end{cases} \quad (7)$$

where

$$\Pi = \begin{pmatrix} k_1 y_e - u_r \int_0^1 \sin(s\psi_e) ds - d\omega_r \int_0^1 \cos(s\psi_e) ds \\ k_1 x_e + dk_1 - u_r \int_0^1 \cos(s\psi_e) ds + d\omega_r \int_0^1 \sin(s\psi_e) ds \end{pmatrix}$$

(7) satisfies the conditions of Lemma 1, so closed-loop system (6) is globally asymptotically stable.

3.2. Dynamic Controller

The above control law (5) is derived in the kinematic model of the robot. This control law can be extended to the dynamic model using the backstepping technique. If the dynamic factors are considered, the kinematic error model (3) can be extended to the following model

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\psi}_e \\ \dot{u} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} -u + \omega y_e + u_r \cos \psi_e - d\omega_r \sin \psi_e \\ -x_e \omega - d\omega + u_r \sin \psi_e + d\omega_r \cos \psi_e \\ -k_1 \psi_e \\ \frac{u_1}{c_1} \\ \frac{u_2}{c_2} \end{pmatrix} \quad (8)$$

where $c_1 = mR$, $c_2 = RI/L$. The objective in this section is to design control input u_1 and u_2 such that the system of (8) is globally asymptotically stable.

Introduce error variable

$$z = \begin{pmatrix} z_u \\ z_\omega \end{pmatrix} = \begin{pmatrix} u - \alpha_u \\ \omega - \alpha_\omega \end{pmatrix}$$

where

$$\begin{cases} \alpha_u = u_r + k_2 x_e \\ \alpha_\omega = \omega_r + k_1 \psi_e \end{cases}$$

For the subsystem

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \end{pmatrix} = \begin{pmatrix} -k_2 & \omega_r \\ -\omega_r & 0 \end{pmatrix} \begin{pmatrix} x_e \\ y_e \end{pmatrix}$$

choose the Lyapunov function candidate

$$V_1 = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2$$

augment the Lyapunov function with the term $z^T M z$ to obtain

$$V_2 = V_1 + \frac{1}{2} z^T M z$$

with derivative

$$\dot{V}_2 = \dot{V}_1 + m z_u \dot{z}_u + I z_\omega \dot{z}_\omega = -k_2 x_e^2 + z_u \left(\frac{u_1}{R} - m \dot{\alpha}_u - x_e \right) + z_\omega \left(\frac{L}{R} u_2 - I \dot{\alpha}_\omega \right)$$

Select the control law as

$$\begin{cases} u_1 = c_1 \dot{\alpha}_u + c_3 x_e - \bar{k}_1 z_u \\ u_2 = c_2 \dot{\alpha}_\omega - \bar{k}_2 z_\omega \end{cases} \quad (9)$$

where $c_3 = R$, $\bar{k}_1, \bar{k}_2 > 0$, then

$$\dot{V}_2 = -k_2 x_e^2 - \bar{k}_1 z_u^2 - \bar{k}_2 z_\omega^2 \leq 0$$

it can conclude that the system (8) is globally asymptotically stable.

3.3. Adaptive Controller

In this section, the control law developed is tackled to ensure robustness against parameters uncertainties.

Theorem 2. Consider the model of the robot described by (8), together with the control law

$$\begin{cases} u_1 = \hat{c}_1 \dot{\alpha}_u + \hat{c}_3 x_e - \bar{k}_1 z_u \\ u_2 = \hat{c}_2 \dot{\alpha}_\omega - \bar{k}_2 z_\omega \end{cases} \quad (10)$$

and the parameter adaptation law

$$\begin{cases} \dot{\hat{c}}_1 = -\frac{\dot{\alpha}_u z_u}{\gamma_1} \\ \dot{\hat{c}}_2 = -\frac{\dot{\alpha}_\omega z_\omega}{\gamma_2} \\ \dot{\hat{c}}_3 = -\frac{x_e z_u}{\gamma_1} \end{cases} \quad (11)$$

where $\gamma_1, \gamma_2 > 0$ are adaptation gains, the closed-loop system described by (10), (11), and (8) is globally asymptotically stable in presence of unknown parameters.

Proof. Control law (9) can be rewritten as

$$\begin{cases} u_1 = \hat{c}_1 \dot{\alpha}_u + \hat{c}_3 x_e - \bar{k}_1 z_u \\ u_2 = \hat{c}_2 \dot{\alpha}_\omega - \bar{k}_2 z_\omega \end{cases}$$

where \hat{c}_i and $\Delta c_i = c_i - \hat{c}_i$ ($i = 1, 2, 3$) are nominal value of the parameters c_i and parameter estimation errors respectively, and $\Delta \dot{c}_i = -\dot{\hat{c}}_i$.

Consider the augmented candidate Lyapunov function

$$V_3 = V_2 + \frac{\gamma_1 (\Delta c_1^2 + \Delta c_3^2)}{2c_3} + \frac{\gamma_2 \Delta c_2^2}{2c_4}$$

where $c_4 = R/L$, $\gamma_1, \gamma_2 > 0$. Then

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + \gamma_1 \left(-\frac{\dot{\hat{c}}_1 \Delta c_1}{c_3} - \frac{\dot{\hat{c}}_3 \Delta c_3}{c_3} \right) - \frac{\gamma_2 \dot{\hat{c}}_2 \Delta c_2}{c_4} \\ &= -k_2 x_e^2 - \frac{\bar{k}_1}{c_3} z_u^2 - \frac{\bar{k}_2}{c_4} z_\omega^2 - \frac{\Delta c_1}{c_3} (\dot{\alpha}_u z_u + \gamma_1 \dot{\hat{c}}_1) - \frac{\Delta c_3}{c_3} (x_e z_u + \gamma_1 \dot{\hat{c}}_3) - \frac{\Delta c_2}{c_4} (\dot{\alpha}_\omega z_\omega + \gamma_2 \dot{\hat{c}}_2) \end{aligned}$$

Choose the parameter adaptation law as

$$\begin{cases} \dot{\hat{c}}_1 = -\frac{\dot{\alpha}_u z_u}{\gamma_1} \\ \dot{\hat{c}}_2 = -\frac{\dot{\alpha}_\omega z_\omega}{\gamma_2} \\ \dot{\hat{c}}_3 = -\frac{x_e z_u}{\gamma_1} \end{cases}$$

to yield

$$\dot{V}_3 = -k_2 x_e^2 - \frac{\bar{k}_1}{c_3} z_u^2 - \frac{\bar{k}_2}{c_4} z_\omega^2 \leq 0$$

Then the closed-loop system described by (10), (11) and (7) is globally asymptotically stable.

4. Simulation Results

In order to illustrate the performance of the proposed control scheme, the adaptive controller is applied to the trajectory tracking of the wheeled robot. The objective is to regulate the position and attitude of the robot to zero. Two kinds of circumstances have been considered. One is the situation that the target point is behind the reference point, another is that the target point is before the reference point.

4.1. A Simulation for the Target Point behind the Reference Point

The following initial conditions for the reference robot are adopted in the simulations: $x_r(0) = 0.5m$, $y_r(0) = 1m$, $\psi_r(0) = \pi/4 rad$. The reference velocities are set to $u_r = 1m/s$ and $\omega_r = 0.5rad/s$. The initial states of the controller robot are $x(0) = 0.2m$, $y(0) = 1.5m$, $\psi(0) = \pi/3 rad$, $u(0) = 2m/s$, $\omega(0) = 3rad/s$. The parameters of the robot $m = 1kg$, $R = 2m$, $L = 3m$ and $I = 2kg \cdot m^2$ are unknown, and $d = 3m$ is known. The initial estimated values for the robot parameters are $\hat{c}_1(0) = 2$, $\hat{c}_2(0) = 2$, $\hat{c}_3(0) = 2$. The control parameters are selected as $k_1 = 1$, $k_2 = 2$, $\bar{k}_1 = 10$, $\bar{k}_2 = 5$, $\gamma_1 = 5$, $\gamma_2 = 3$. Figures 2-6 show the results of the simulations.

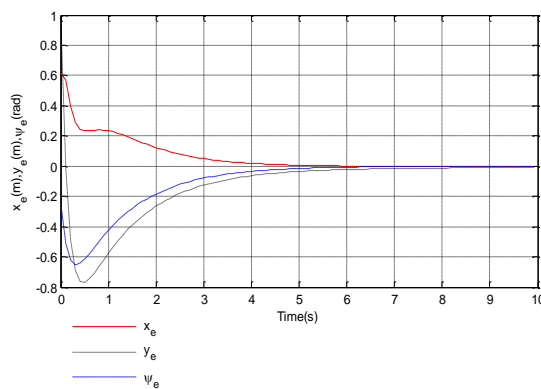


Figure 2. Positions and Orientation Errors Curve

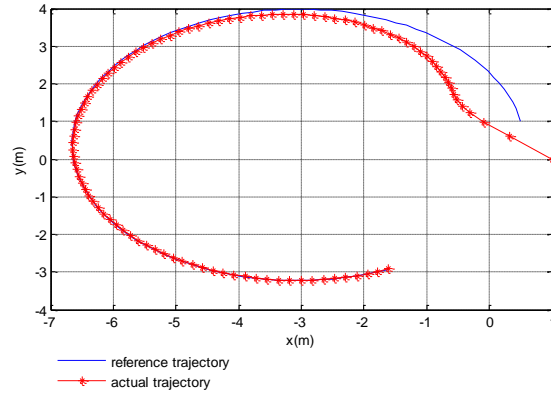


Figure 3. Robot Reference and Actual Trajectory

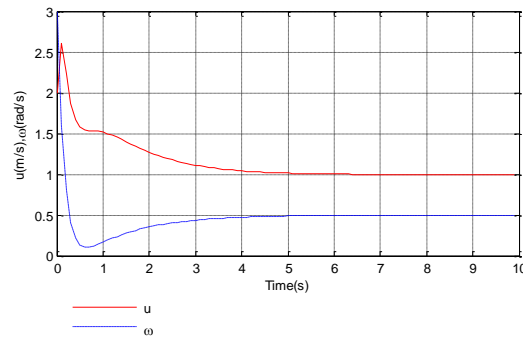


Figure 4. Velocity Curve

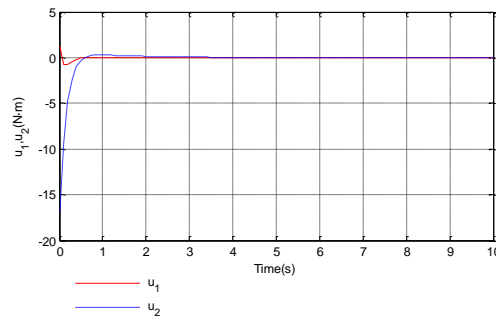


Figure 5. Control Input Curve

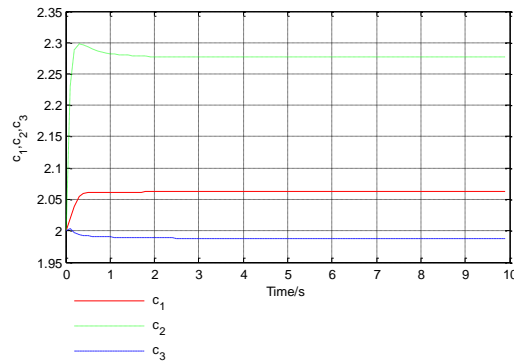


Figure 6. The Estimation Parameter Curve

4.2. A Simulation for the Target Point before the Reference Point

The following initial conditions for the reference robot are adopted in the simulations: $x_r(0) = 0.5m$, $y_r(0) = 0.5m$, $\psi_r(0) = \pi/4 rad$. The reference velocities are set to $u_r = 0.3m/s$ and $\omega_r = 0.15rad/s$. The initial states of the controller robot are $x(0) = 1m$, $y(0) = 0m$, $\psi(0) = \pi/3 rad$, $u(0) = 1m/s$, $\omega(0) = 2rad/s$. The parameters of robot $m = 1kg$, $R = 2m$, $L = 3m$ and $I = 2kg \cdot m^2$ are unknown, and $d = 3m$ is known. The initial estimated values for the robot parameters are $\hat{c}_1(0) = 2$, $\hat{c}_2(0) = 2$, $\hat{c}_3(0) = 2$. The control parameters are selected as $k_1 = 2$, $k_2 = 2$, $\bar{k}_1 = 10$, $\bar{k}_2 = 5$, $\gamma_1 = 5$, $\gamma_2 = 1$. Figures 7-11 show the results of the simulations.

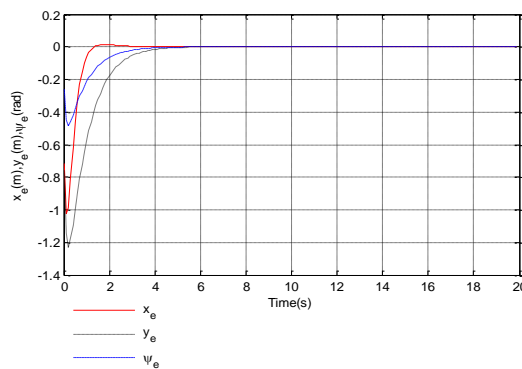


Figure 7. Positions and Orientation Errors Curve

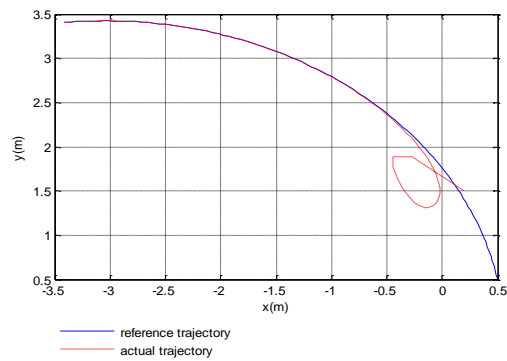


Figure 8. Robot Reference and Actual Trajectory

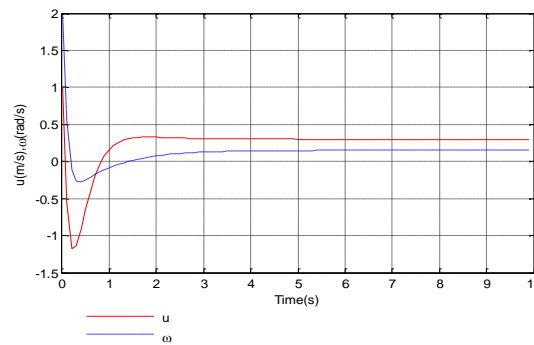


Figure 9. Velocity Curve

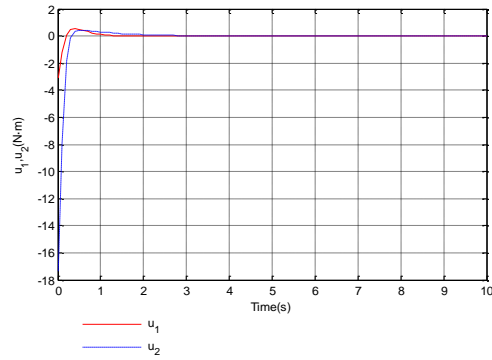


Figure 10. Control Input Curve

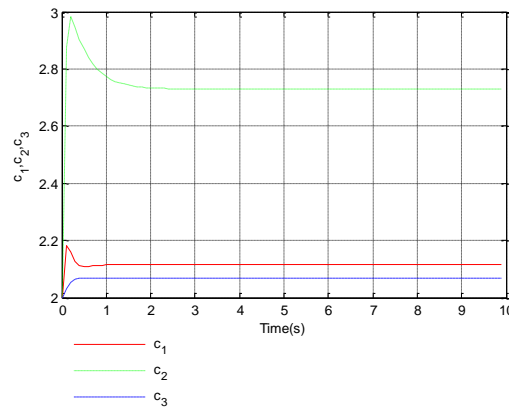


Figure 11. The Estimation Parameter Curve

In Figure 2 and Figure 7, notice how, in spite of parameter uncertainty, the errors of positions and orientation converge to zero. The robot converges to the desired trajectory in Figure 3 and Figure 8. As Figure 4 and Figure 9 show, the velocities converge to the reference velocities. Figure 5 and Figure 10 illustrate the control input u_1 and u_2 needed for tracking. From Figure 6 and Figure 11, it can be seen that the estimation parameters are convergence. It can be shown, however that $\Delta c_i, i = 1, 2, 3$ do not converge to zero. Nevertheless, the proposed controller for the robot guarantees the stable response and convergence to the reference trajectory smoothly. Therefore, the simulation results demonstrate that the adaptive controller is effective to solve the trajectory tracking problem.

5. Conclusions

In the presence of parameter uncertainties, the laws of adaptive trajectory tracking control for the WMR have been derived. And the mass center of the robot does not coincide with its geometrical center. The key idea is that the cascade-design method has been applied to divide the tracking control problem into two problems: position and orientation tracking problems. Controller design has been obtained by backstepping techniques and Lyapunov theory. The developed adaptive control laws have been ensured robustness against the unknown model parameters. Simulations have demonstrated the validity of the designed trajectory tracking control scheme.

Acknowledgments

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