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Fractal Analysis of the Agricultural Products Prices Time Series

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Abstract

It is very necessary to research the fluctuation of agricultural products prices as well as the data character. In this paper, the wholesale price of agricultural product is considered. The monofractal analysis and multifractal analysis methods are introduced to study wholesale price. We select the appropriate algorithm to calculate the fractal characteristics of the celery wholesale price. Results show that the price is monofractal and multifractal. The methods of prediction and risk assessment based on fractal analysis are available for agricultural product price.

Keywords: *wholesale price, agricultural products, monofractal, multifractal*

1. Introduction

The prices of China's agricultural products have been unstable for about ten years. The abnormal volatilities are frequent, which have a bad effect on farmers' production decision making and people's consumption. For the fluctuation of agricultural products prices, some researchers [1-4] analyzed fluctuation causes and effects, the other researchers focused on agricultural prices prediction [5-7]. But some typically analytical methods, such as the linear analysis methods, have encountered challenges. So researchers developed various methods to predict agricultural prices besides the traditional regression analysis method, including the neural networks, the grey method, the combinational model and Gray's theory, *etc.* [8-12]. Hence, it is very necessary to select the optimal method to predict prices according to the data character. In this paper, we aim to discuss the monofractal and multifractal characteristics of agricultural product price.

In the 1960s, the fractal theory was introduced to analyze irregular or non-smooth [13-14]. Fractal theory can explain the characteristics and behavior of complex systems. Presently, fractal theory is applied extensively [15-18]. For the fluctuation of agricultural products prices, some researches also involved the method of fractal analysis [19-23]. Most of the literatures studied future prices, while few articles considered wholesale prices. We point out that some agriculture products have no future prices and the wholesale price is different from the future price. Clearly, the wholesale market prices have more effect on the daily lives of the residents. So in this paper, we study the wholesale price and give the empirical analysis of celery price.

If the time series is monofractal or multifractal, then both the fractal prediction method and fractal risk assessment method can be applied to agricultural wholesale price. Our method of detecting fractality can also be used as a tool to analyze the wholesale price characteristics.

The difficulty of this research mainly lies in two sides. One is the capacity of wholesale price data is always small. At present, it is difficult to obtain long term and high frequent data of wholesale prices. The other is the big noise in agricultural wholesale prices. So, we must select the appropriate method very cautiously. For instance, since the variances

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of price data in each window are too small, the popular multifractal algorithm-MFDFA approach cannot be used in our empirical analysis.

The paper is organized as follows: in Section 2, we present the methods to detect fractality, including the monofractal analysis and multifractal analysis. In Section 3, we analyze the celery price time series. The results show it is a monofractal series as well as a multifractal series. In the last section, we make some conclusions.

2. Methodology

2.1. Monofractal Analysis

The rescaled range analysis (R/S) is a widely used non-parametric statistical method applied in monofractal analysis or unifractal analysis. The modern R/S analysis mainly contains two parts. One is computing the Hurst exponent. The other is the test of significance.

Computing the Hurst exponent begins with splitting of the origin time series $X=\{X_t, t=1,2,\cdots,N\}$ into non-overlapping segments $D_a(a=1,2,\cdots,A)$ of size n, yielding A segments altogether. Denote the element in every segment by $X_{i,a}, i=1,2,\cdots,n$ and denote their mean by $\bar{X}_a$. In the second step, the cumulative deviation is calculated in each segment $D_a(a=1,2,\cdots,A)$,

$$\hat{X}_{k,a}=\sum_{i=1}^{k}(X_{i,a}-\bar{X}_a), k=1,2,\cdots,n.$$

In the third step, the ranges and the standard deviations S_a in each segment are calculated,

$$R_a=\max_{1\le k\le n}(\hat{X}_{k,a})-\min_{1\le k\le n}(\hat{X}_{k,a}),\ S_a=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_{i,a}-\bar{X}_a)^2}.$$

Finally, the rescaled range is averaged over all segments $(R/S)_n=\frac{1}{A}\sum_{a=1}^{A}R_a/S_a$. In general, the relationship $(R/S)_n=cn^H$ holds for different n, where H is the Hurst exponent and c is a constant. Therefore, the Hurst exponent can be estimated by linear regression

$$\log(R/S)_n=\log c+H\log n.$$

The autocorrelation function can be obtained by $C=2^{2H-1}-1$, which is used to describe the influence of the present on the future. The fractal dimension can also be calculated from Hurst exponent by using of the simple relation: $D=2-H$, which is a statistical quantity that gives an indication of how completely a fractal appears to fill space, as one zooms down to finer and finer scales. The values of H obtained by the rescaled range analysis are limited to $0<H<2.$ These indices $H=0.5,$ $D=1.5,$ $C=0$ indicate a random walk. While $0.5<H<1, D<1.5, 0<C\le 0$ indicate long-term positively correlated behavior, which means that the series is persistent and has a long-term memory. Values $0\le H<0.5, D>1.5, -0.5\le C<0$ indicate long-term anti-correlated behavior of the data, which means that, after a period of decreasing, the data tend to increase in the next period, and vice versa.

Since the R/S values are random variables, normally distributed, we can expect that the values of H would also be normally distributed. The expected variance of the Hurst exponent would be $1/N$, where N is the total number of observations in the sample. Thus we can build a test in order to be able to accept or reject the following null hypothesis: the underlying process is Gaussian. To achieve this goal we can construct the following test statistic: $t = \frac{H - E(H)}{\sqrt{1/N}}$, where $E(H)$ is the mean of H if independent. Here, we use the following estimate value:

$$E(H) = \left(\frac{n-0.5}{n}\right) \times \left(\frac{\pi n}{2}\right)^{-0.5} \times \sum_{r=1}^{n-1} \sqrt{\frac{n-r}{r}} .$$

The so called V statistic is defined as follows:

$$V_n = \frac{(R/S)_n}{\sqrt{n}},$$

which can be used to test for the stability of the Hurst exponent. For an independent random process, the plot of V_n versus $\log(n)$ should be flat. If the process is persistent, the graph should be upwardly sloping. While if the process is anti-persistent, the graph should be downward sloping. In addition, one can find out the "breaks" which occur when the V chart flattens out. At those points, the long memory process has dissipated.

We can also test the persistence by using of the disrupted data. If the original data has a long term memory or the process is persistent, the disrupted data should have a closer to 0.5 or lower Hurst exponent. But if the process is an independent random process, the disrupted data should have the same Hurst exponent with the original data.

2.2. Multifractal Analysis

Sometimes a monofractal structure can only reflect one characteristic of the fluctuation. But, the multifractal process can be applied to a more complete description of the fluctuation behavior.

A stochastic process $\{X(t)\}$ is called multifractal if it has stationary increments and satisfies

$$E(|X(t+\Delta t) - X(t)|^q) = c(q)(\Delta t)^{\tau(q)+1} \quad (1)$$

for all $t \in \mathrm{T}, q \in \mathrm{Q}$,where T and Q are intervals on the real line, $\tau(q)$ and $c(q)$ are functions with domain Q, T and Q have positive lengths, $0 \in \mathrm{T}, [0,1] \subseteq \mathrm{Q}$, and that Δt is the time interval.

The principle of the multifractal analysis is to describe the local characteristics by calculating some parameters, for example, the scaling function, the generalized (multifractal) Hurst exponent, the local Hölder exponent, and the multifractal spectrum, *etc*.

A multifractal process is globally scaling, in the sense that its moments satisfy the scaling relationship (1). The function $\tau(q)$ is called the scaling function of the multifractal process ($\tau(q)$ is also called the Renyi scaling exponent). Multifractal processes with linear $\tau(q)$ are called monoscaling or monofractal. Self-affine processes are monofractal. More generally, linear scaling functions $\tau(q)$ are determined by their slope. Multifractal processes with non-linear functions $\tau(q)$ are multiscaling. In a multifractal process, $\tau(0)$=-1 and $\tau(q)$ is concave.

According to (1), the generalized Hurst exponent is defined as follows:

$$\{E(|X(t+\Delta t) - X(t)|^q)\}^{1/q} = c(q)(\Delta t)^{H(q)}$$

where $q > 0$. The function $H(q)$ contains information about averaged generalized volatilities at the time increment Δt (only $q = 1, 2$ are used to define the volatility). In particular, $H(1)$ indicates the Hurst exponent defined in Section 2.1. Obviously,

$$H(q) = \frac{1+\tau(q)}{q}.$$

For a continuous stochastic process, a function defined in a neighborhood of a point t has a Hölder exponent $\alpha(t)$ if

$$|X(t+\Delta t)-X(t)| \sim C(t)(\Delta t)^{\alpha(t)},$$

where $C(t)$ is the prefactor at t. The Hölder exponent is sometimes called the "local strength of singularity". It reflects the level of irregularity at t. The distribution of Hölder exponents can be represented by the multifractal spectrum and is denoted by the function $f(\alpha)$. By the Legendre transformation of the scaling function, one can obtain the following equalities

$$\alpha = \frac{d\tau(q)}{dq},\ f(\alpha) = q\alpha - \tau(q). \tag{2}$$

The function $f(\alpha)$ can be considered as the fractal dimension of the set of points having local Hölder exponent α. It indicates a multifractal characteristic that the support of $f(\alpha)$ has more than one point. Thus we can judge a multifractal process intuitively from the multifractal spectrum.

Presently, there are several approaches to calculate a multifractal spectrum. The common approach of multifractal analysis is the structure function approach, which has a simple principle and can be used in stationary time series as well as non-stationary time series. The multifractal DFA (MF-DFA) algorithm and wavelet transform modulus maxima (WTMM) algorithm are also important approaches. The WTHH algorithm is large in computing capacity and is especially suitable for large-scale data. The MF-DFA algorithm is used in the analysis of non-stationary fractal time series.

In general, it is difficult to collect high frequency price data of agricultural products. The daily data, monthly data, or yearly data of agricultural products can be collected easily. Thus, the data capacity of price data of agricultural products obtained is small, in general. For instance, the total number of daily data for fifteen years is about $15\times 365=5475$. Furthermore, some price time series are stationary, some price time series are non-stationary. Therefore, in this paper, we give the structure function approach for price time series of agricultural products, which can be used to stationary and non-stationary time series and can be suitable of small data set.

The steps of the structure function approach are as follows. Firstly, normalize the original price time series $\{p_t, t=1,2,\cdots,N\}$ and obtain the time series $\{P_t, t=1,2,\cdots,N\}$, where $P_t = \frac{p_t}{\sum p_t}$ .The index variation with time can be divided into many time intervals (boxes) of size ε.

Secondly, calculate the average probability $P_i(\varepsilon)$, which is the sum of all P_t inside the box i of size ε. Then one should choose the suitable moment order q, and compute the partition function $M_q(\varepsilon)=\sum_{i=1}^{n} P_i^q(\varepsilon)$, where n is the number of all the boxes of size ε and $q\in(-\infty,+\infty)$.

Since M_q can be expressed as

$$M_q \propto \varepsilon^{\tau(q)}, \tag{3}$$

the third step is to obtain $\tau(q)$ from the slope of the linear part of $\ln M_q(\varepsilon) \sim \ln\varepsilon$ curve.

Finally, compute $f(\alpha)$ by performing the relationship (2).

The width of the multifractal spectrum is $\Delta\alpha=\alpha_{\max}-\alpha_{\min}$ and the difference of the fractal dimensions of the maximum probability subset $\alpha=\alpha_{\min}$ and the minimum one $\alpha=\alpha_{\max}$ is $\Delta f = f(\alpha_{\max}) - f(\alpha_{\min})$.

3. Empirical Analysis

3.1. Data

The data used in this paper are the complete historical records, available, of the weekly price taken from agricultural wholesale markets in Shandong Province of China (data source: China Price Information Network), that is, the arithmetic average prices from February 2003 to May 2014 which aggregate 594 weekly data. We select the celery data to study and compare. The price graph of the vegetable is drawn in Figure 1.

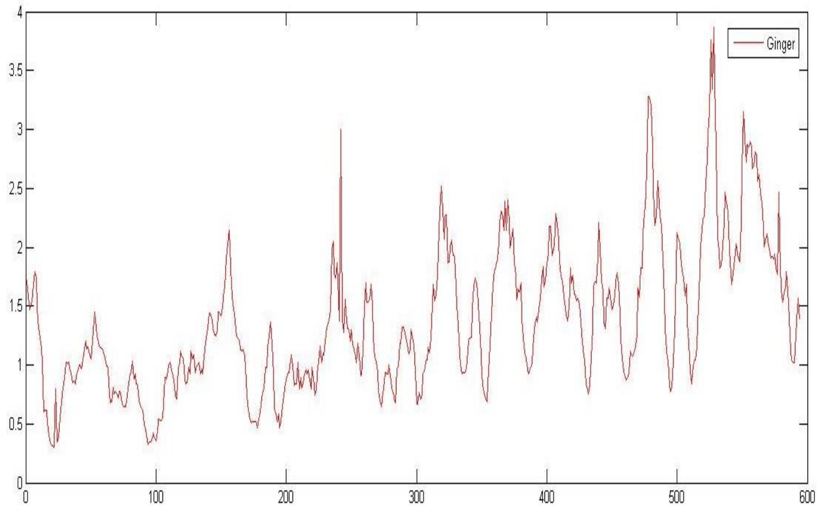

Figure 1. Price

We choose the data based on the following reasons:

(i) Shandong Province has been the largest vegetables production province in China, therefore Shandong Province's agricultural wholesale markets are very influential regionally; (ii) celery has been popular and necessary in China; (iii) on one hand, the monthly or yearly data are limited and cannot provide the correctness and rigor required by the fractal analysis, on the other hand it is very difficult to collect the complete historical daily records, therefore we choose the weekly data.

We first employ Eviews 6.0 to analyze the statistical characteristics of the celery (abbreviated as C) weekly prices and list the results in Table 1. The results of J-B test reject the null hypothesis of normal distribution at a 0.001 level of significance. The distribution of the price series exhibits high peak and fat tail. In addition, according to the ADF and PP tests, C is a stationary time series.

Table 1. Statistical Description

	Mean	Std.Dev.	Kurtosis	Skewness	Jarque-Bera /prob.
C	1.3676	0.6215	3.7867	0.8986	95.2666/0.0000

3. 2. R/S Analysis

The results of the R/S analysis of our original data are listed in Table 2, which indicate the celery weekly price series is persistent and have strong long-term memories. For this series, we obtained three disrupted sequences, namely C1, C2, C3 for celery prices series, and their parameters are listed in Table 2. Since the original structures are destroyed, the Hurst exponents of the three disrupted data are lower to 0.5.

Table 2. R/S Analysis of Celery Price Series

	H	C	D	t
C	0.8992	0.7391	1.0018	46.3083
C1	0.3089	-0.2328	1.6911	31.9220
C2	0.3653	-0.1703	1.6347	33.2975
C3	0.4010	-0.1282	1.5990	34.2975

The celery price curves of $\log(R/S)_n \sim \log n$ and $\log V_n \sim \log n$ are plotted in Figure 3 which show that a breaking point occurs where $\log n \approx 1.386 (n \approx 4\ weeks)$. At this point, the long memory process has dissipated. So the celery price series have a memory of almost 4 weeks. So the vegetable price has the 4-weeks cycle length.

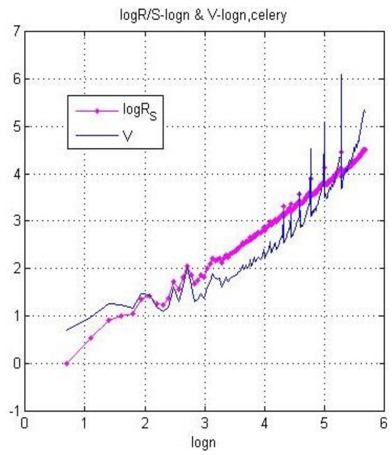

Figure 2. R/S Analysis and V Statistic, Original Data

From the above discussion, the price series exhibits the fractal characteristic. The sequence has strong long memory and its Hurst exponent is bigger than 0.5. Therefore, the vegetable price is not random and it can be predicted in short terms. Next, we explore whether the price series is multifractal. If the answer is yes, then the multifractal parameters can offer more comprehensive description.

3. 3. Multifractal Inspection

We start our multifractal inspection by drawing $\ln M_q(\varepsilon) \sim \ln \varepsilon$ curves. Ideal regular multifractal has strict linearity in $\ln M_q(\varepsilon) \sim \ln \varepsilon$ plot at all moments q. In this paper, q is taken when both $\frac{|d\alpha_{\max}|}{\Delta\alpha}$ and $\frac{|d\alpha_{\min}|}{\Delta\alpha}$ are less than 0.2%, where $d\alpha_{\max}$ is the increment of $\alpha_{\max}$ when the increment of q is 1, and $d\alpha_{\min}$ is the increment of $\alpha_{\min}$. Figure 3 shows the $\ln M_q(\varepsilon) \sim \ln \varepsilon$ curves of the time series of the ginger and celery prices with intervals of 4 of $|q|$. The left one is the curves of the ginger price. Here, the maximum moment order $|q|$ is 80. The box sizes ε are taken as 1/594, 1/297, 1/198, 1/99, 9/594, 1/54, 1/33, 22/594, 1/27, 1/18, 1/11, 1/9, 1/6, 1/3, and 1/2. It can be seen that the linearity of the $\ln M_q(\varepsilon) \sim \ln\varepsilon$ curves is good. We also can judge a multifractal process from a $\tau(q) \sim q$ graph.

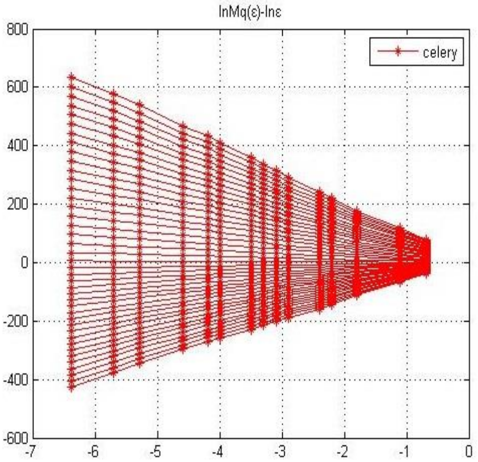

Figure 3. $\ln M_q(\varepsilon) \sim \ln\varepsilon$

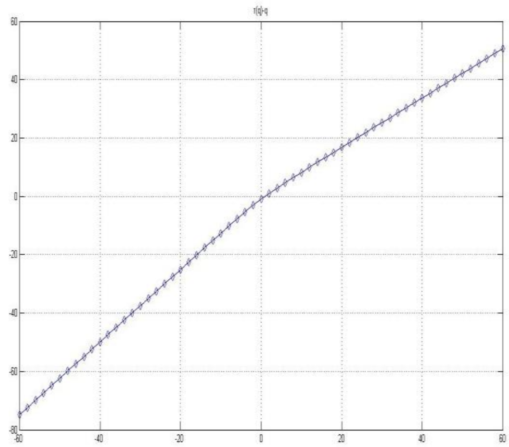

Figure 4. $\tau(q) \sim q$

Figure 4 shows the $\tau(q) \sim q$ curve of the celery price. For a monofractal time series, its $\tau(q) \sim q$ curve is linear while for a multifractal times series, its $\tau(q) \sim q$ curve is nonlinear and concave. So, the curve exhibits nonlinearity. The nonlinearity of the $\tau(q) \sim q$ curve of the ginger price is stronger than that of the celery price, which indicates the ginger time series is a stronger multifractal process. One can also find out this point from Figure 5 and Figure 6. Figure 5 displays $H(q) \sim q$ curve. The $H(q)$ range of the celery price is $[0.8562, 1.235]$. The $H(q)$ of the price time series is bigger than 0.5, which implies that the vegetable price have strong long memory.

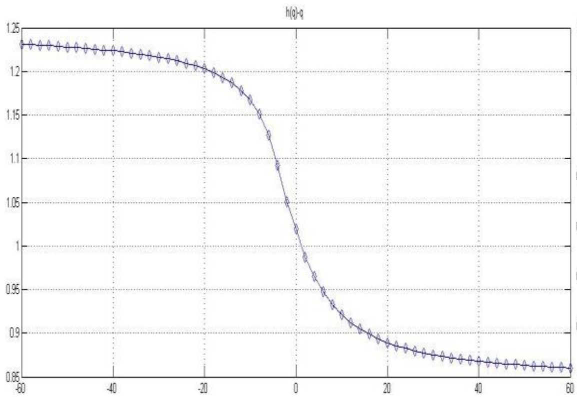

Figure 5. $H(q) \sim q$

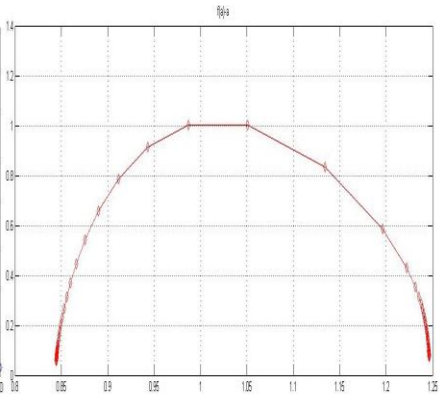

Figure 6. $f(\alpha) \sim \alpha$

Furthermore, evidence that the ginger price process has a stronger multifractal character can be obtained in view of its larger width of the multifractal spectrum. Figure 6 shows the curve of $f(\alpha) \sim \alpha$. Several indices about the multifractal spectrum of the price are list in Table 3. The width of the singular spectrum $\Delta\alpha$ reflects the difference between the maximum probability and minimum probability, which indicates the unevenness of probability distribution or the strength of multifractal. The larger the value of $\Delta\alpha$ is, the stronger the strength of multifractality is. The index Δf is the ratio of the number of maximum probability subset to the number of minimum probability subset. Its asymmetry to α reflects the uneven distribution of low- and high-intensity data. The large the $|\Delta f|$ is, the stronger the asymmetry is. If $\Delta f > 0$ the plot of $f(\alpha) \sim \alpha$ is left-hooked; if $\Delta f < 0$ the plot of $f(\alpha) \sim \alpha$ is right-hooked. So both $f(\alpha) \sim \alpha$ plots of the two prices are right-hooked, which means the prices have bigger opportunity locating in low price.

Overall, the vegetable price time series is multifractal; it has strong long-memory.

Table 3. Indices of the Multifractal Spectrum

	$\Delta\alpha$	Δf	$\alpha_{\max}$	$\alpha_{\min}$
C	0.4027	-0.0057	1.2469	0.8442

3. Conclusions

In this paper, we give the fractal analysis method of price of agriculture products and find out a series of evidences that there are both monofractal and multifractal characteristics in agricultural product price time series. We investigate the wholesale celery price data from February 2003 to May 2014. The Hurst exponent of the sequence is bigger than 0.5 which means that it has long memory and it is monofractal. Moreover, the $\ln M_q(\varepsilon) \sim \ln\varepsilon$ plot, the local Hölder exponent, the multifractal spectrum, and the scaling function indicate that the price time series are multifractal. The celery price has a cycle length (= 4 weeks).

Therefore, the wholesale vegetable price series is a monofractal series, as well as a multifractal series. The wholesale vegetables market is a fractal market and is not an efficient market. Our method of detecting fractality is available vegetable prices data. For a multifractal data, we can use the monofractal and multifractal methods to deal with the price prediction and risk assessment in the future.

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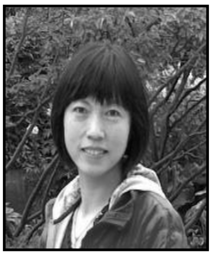

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