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High Thrust Station Keeping Maneuvers for Geostationary Satellites

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Abstract

This paper explain how the station keeping maneuvers are habitually planned in ground based station keeping control systems for GEO satellites equipped with high thrust propulsion systems consisting of chemical thrusters able to achieve radial, tangent and normal thrust acceleration components in unrelated ways. In fact, by using North-South high thrust maneuvers and Est-West high thrust maneuver. A numerical simulation is performed to illustrate this approach.

Keywords: *Geostationary satellite, station keeping, Correction cycle duration, mean longitude drift, VOP*

1. Introduction

The operation of geostationary satellites requires that their latitude and longitude remain confined during the whole spacecraft life. To this purpose, a suitable station keeping strategy is implemented, whose objectives are the set of maneuvers that have to be executed in order to thwart the effects of natural perturbing forces affecting the spacecraft position [1]. The strategy is decided by predicting the changes of the orbital parameters on the basis of simplified models for the spacecraft dynamics which take into account only the main natural perturbing forces: the luni-solar attraction force, the solar radiation pressure and the earth gravitational force.

More recently, however, the use of electric propulsion systems is being considered as a viable alternative to the classical chemical actuators and is rapidly becoming the baseline on new telecom satellite platforms [2-3]. The use of low thrust propulsion systems is compulsory when the dimension of the station keeping box is very small. In this case the accelerations produced by the thrusters have to be of the same order of magnitude as the accelerations induced by the environmental perturbing forces.

In addition, the non circular shape of the Earth's equator, the non-homogeneous Earth's mass distribution and the solar radiation pressure cause a GEO satellite to be slowly drawn to one of two equilibrium points along the equator [3-4], resulting in East-West librations about these points.

To counteract the undesirable effects of the environmental perturbing forces, sufficient fuel is loaded into the geostationary satellites to periodically activate the on board propulsion system and to correct the satellite trajectory over all the planned mission duration [4-5]. The making of these periodic GEO satellite trajectory corrections are also called station keeping maneuvers. North-South (NS) station keeping (SK) maneuvers counteract the changes of latitude and keep the satellite in the assigned position range around zero latitude value; they consist in thrust accelerations along the *N* axis of the *RTN* reference frame [5-7].

East-West (EW) station keeping maneuvers counteract the variations in longitude and keep the satellite in the assigned position range around the station longitude on the geostationary belt; they consist in thrust accelerations along the *R* axis and/or the *T* axis of the

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RTN reference frame. Once a GEO satellite has exhausted its fuel, it becomes unusable and it is no more operational, because no control prevents its inclination to grow and the satellite to drift in longitude [8-9]. A number of guidelines and recommendations for end-of-mission disposal were issued by national and international institutions to protect the geostationary orbit environment.

2. GEO Satellite Station Keeping (SK) Problem Statement

The GEO satellite station keeping problem is a maneuver planning problem. The goal is to find the thrust time histories of each thrusters of the propulsion system over all the spacecraft life time. Such thrust time histories must generate thrust acceleration components such that the GEO satellite orbital requirements are met minimizing the propellant mass consumption. The advantage of this minimization is twofold: one can choose to increase the spacecraft life time for the same propellant mass or to reduce the necessary amount of propellant mass allowing the spacecraft to take extra payload mass on board.

During the whole mission life time T_M, spacecraft drift intervals without maneuvers alternate with spacecraft drift intervals with maneuvers [6-7]. The first ones are under the effect of the environmental perturbing accelerations only. The second ones are under the effect of the environmental perturbing accelerations and the thrust accelerations. This is true for both maneuvers performed by acceleration impulses and maneuvers performed by acceleration pulses. For the first ones the spacecraft drift intervals with maneuvers narrow to points where instantaneous changes of the velocity vector take place.

The station keeping problem can be seen, instead of a problem of maneuver planning, as a problem of trajectory optimization. In this second formulation the problem consists in finding the optimal GEO trajectory meeting the orbital requirements and minimizing the duration and/or the number of the spacecraft drift intervals with maneuvers.

The following formulation of the SK maneuver planning problem is based on orbit propagation model and does not take into account the measures of the spacecraft state vector which are actually provided by a measured orbit model (also called orbit determination model). In other words, at this time, we make the hypothesis that the orbit found by orbit propagation (which is obtained integrating the equations of motion) coincides during the whole mission life time $T_M = t_{fM} - t_{iM}$ with the orbit found by orbit determination (which provides the best estimate of the spacecraft state vector).

Let be [7]:

$$\frac{d\boldsymbol{s}}{dt} = f(\boldsymbol{s}, \boldsymbol{a}_t), \qquad \boldsymbol{s}(t_{iM}) = \boldsymbol{s}_i \tag{1}$$

a system of nonlinear differential equations describing the translational dynamics of a GEO satellite subject to the effect of the environmental perturbing accelerations and to the effect of the acceleration vector at induced by a propulsion system composed by N_t thrusters. The vector $\boldsymbol{s}$ of the system (1) is:

$$\boldsymbol{s} = \left[\boldsymbol{x} \;\; \boldsymbol{\Theta} \;\; \mathrm{T} \;\; m_{p1} \; \dots \; m_{p2} \; \dots \; m_{pN_t}\right] \tag{2}$$

It is made up of the following quantities.

The spacecraft state vector $\boldsymbol{x}$, whose components can be given by the three position components plus the three velocity components or by the six equinoctial orbital elements. Its dynamics is described by one of the GEO satellite models in perturbed Keplerian conditions [7-8]:

$$\boldsymbol{x} = [a \;\; P_1 \;\; P_2 \;\; Q_1 \;\; Q_2 \;\; l_\theta]^T \tag{3}$$

Where a is the semi-major axis, (P_1, P_2) is the eccentricity, (Q_1, Q_2) is the Inclination and l_θ is the mean longitude in terms equinoctial orbital elements EOEs [10].

The Greenwich mean sidereal time Θ, which has a dynamics given by

$$\frac{d\Theta}{dt} = \omega_\oplus \tag{4}$$

Where $\omega_\oplus$ is the angular velocity of the Earth.

The number of Julian centuries T since 1.5 January 2000, which has a dynamics given by

$$\frac{dT}{dt} = 1 \tag{5}$$

The propellant masses m_{p1} , . . . ,m_{pi} , . . . ,m_{pNt} of the N_t thrusters, which have a dynamics given by

$$\frac{dm_{p1}}{dt} = -\frac{F_1}{gI_{sp1}}, \dots, \frac{dm_{pi}}{dt} = -\frac{F_1}{gI_{spi}}, \dots, \frac{dm_{pN_t}}{dt} = -\frac{F_1}{gI_{spN_t}} \tag{6}$$

Where m the Propellant mass, F values of the thrust level, I_{sp} the specific impulse and g Sea-level acceleration of gravity on the earth (constant).

The vector $\boldsymbol{a_t}$ of the system (1) is the acceleration vector induced by the propulsion system [7]:

$$a_t(t) = \boldsymbol{\Gamma}_1(\gamma_1, \sigma_1)\frac{F_1(t)}{m(t)} + \dots + \boldsymbol{\Gamma}_i(\gamma_i, \sigma_i)\frac{F_i(t)}{m(t)} + \dots + \boldsymbol{\Gamma}_{N_t}(\gamma_{N_t}, \sigma_{N_t})\frac{F_{N_t}(t)}{m(t)} \tag{7}$$

It depends on the following quantities.

The loaded spacecraft mass m, which is the sum of the constant spacecraft dry mass m_d and of the time varying propellant masses of each thruster m_p

$$m(t) = m_d + \sum_{i=1}^{N_t} m_{pi}(t) \tag{8}$$

The loaded spacecraft mass m is maximum at the beginning of the spacecraft life and equal to the so called Beginning of Life (BOL) mass. The dry mass m_d is constant and composed by the payload mass (*i.e.*, mass of load useful for the mission) and by the spacecraft bus dry mass (*e.g.*, vehicle dry mass, propulsion system mass) [10].

The configuration vector $\boldsymbol{\Gamma}_i$ of the ith thruster, which is function of the cant γ and slew σ angles of the thruster (see Figure 1).

$$\boldsymbol{\Gamma}_i(\gamma_i, \sigma_i) = [sin\gamma_i cos\sigma_i \; sin\gamma_i sin\sigma_i \; cos\gamma_i] \tag{9}$$

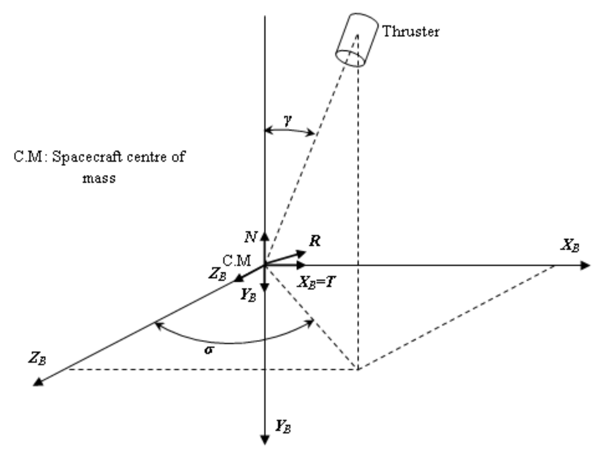

Figure 1. Thruster Cant and Slew Angles

The scalar quantity $F_i(t)/m(t)$, which is the time history of magnitude a_i of the acceleration vector induced by the ith thrust F_i

$$a_i(t) = \frac{F_i(t)}{m(t)} \quad \text{with } i=1,2,\ldots,N \tag{10}$$

In the following we will call a_i the thrust acceleration of the ith thruster.

The problem of station keeping maneuver planning consists in determining the N_p time histories of thrusts $F_i(t)$ over a time horizon equal to the mission duration $T_M = t_{fM} - t_{iM}$ such that over T_M the inequality constraints on the state $\boldsymbol{s}$ variables.

The main requirement of a geostationary satellite consists in having, during its whole life, longitudinal and latitudinal position confined in a rectangular box of (λ, φ) plane centered in (λ_s, 0) and with dimensions equal to $2\lambda_{max}$ and $2\varphi_{max}$ (see Fig 2).

$$-\lambda_{max} \leq \lambda - \lambda_s \leq +\lambda_{max} \tag{11}$$

$$-\varphi_{max} \leq \varphi \leq +\varphi_{max} \tag{12}$$

Where λ_s is a Sun's ecliptic geographical longitude.

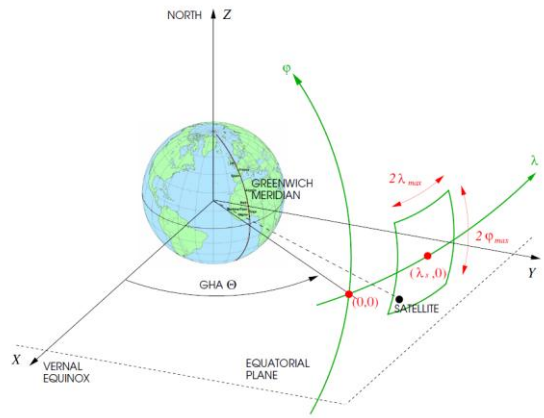

Figure 2. Deadband Rectangular Box in (λ, φ) Plane

This box is called deadband box or station keeping window and its sides of magnitude $2\lambda_{max}$ and $2\varphi_{max}$ (usually specified in degrees) are respectively called the longitude and latitude deadbands. A circular confinement area may also be prescribed for the latitude and longitude, but this is usually handled like the previous case by using the square box inscribed in the circle.

The orbital requirement (Eq. 12) on the latitude φ can be replaced with a requirement of spacecraft inclination $0 \leq i \leq i_{max}$. This last requirement is translated in a constraint on the magnitude of the inclination vector which is $\tan\left({}^{i}/_{2}\right) = \sqrt{Q_1^2 + Q_1^2}$ see (Fig 3).

3. High Thrust SK Maneuver Planning

The ground based SK maneuver planning is executed to reach the two main ground based SK control system goals: to minimize the overall fuel consumption and to minimize the operational effort at the control centre in order to achieve the GEO satellite orbital requirements. The way to execute the ground based SK maneuver planning consists in solving a series of parameter optimization problems in different phases of the planning: strategic and tactical phases.

a. Strategic Phases

During the strategic phases the planning is independently performed for the East-West SK maneuvers and for the North-South SK maneuvers. In both cases, the strategic defines the objectives for reaching the main ground based SK goals and develops action plans for reaching those objectives over SK correction cycles with duration T_C. We will denote T_{Ck} the duration of the kth correction cycle.

The action plans for reaching the above objectives are expressed in terms of maneuver execution dates (days d and times of the day's t_m) and in terms of sizes of the maneuver executed with high thrust accelerations. The hypothesis of high thrust is translated to the hypothesis of impulsiveness of thrust acceleration components which consequently are assumed to have the following time histories.

$$a_{tR}(t) = \Delta v_{tR}\delta(t - t_{mR}) \tag{13}$$

$$a_{tT}(t) = \Delta v_{tT}\delta(t - t_{mT}) \tag{14}$$

$$a_{tN}(t) = \Delta v_{tT}\delta(t - t_{mN}) \tag{15}$$

Where a_{tR} ,a_{tT} *and* a_{tN} is the thrust acceleration components in RTN frame; v_{tR} ,v_{tT} *and* v_{tN} is the velocity magnitude of the spacecraft subject to the only thruster forces in RTN frame; t_m is the impulsive maneuver time.

When radial, tangent and normal maneuvers are respectively executed at the instant of the day t_{mR}, t_{mT} and t_{mN}. Therefore the sizes of the maneuvers are nothing else than the amount of thrust velocity increments Δv_{tR}, Δv_{tT} , Δv_{tN} along the main directions of the RTN reference frame at each times t_{mR}, t_{mT} , t_{mN} of the prefixed correction days $d_{R,}$ d_T , d_N. In case of high thrust corrections, such days are those corresponding to the time t_{iCk} of the beginning of the SK correction interval T_{Ck}. The determination of all the other quantities is done basing on a simplified model of thrust acceleration effect on orbital parameters. The equations of such a model comes from the linearization of Gauss' variation of parameters equations (VOP)

$$\frac{d\boldsymbol{x}_t}{dt} = \boldsymbol{K}(\boldsymbol{x}_t) - \boldsymbol{\omega}_\oplus + \boldsymbol{G}(\boldsymbol{x}_t,t)a_t(t) \tag{16}$$

When

$$\boldsymbol{\omega}_\oplus = [0\;0\;0\;0\;0\;\omega_\oplus]^T$$

$$\boldsymbol{K} = [0\;0\;0\;0\;0\;n]^T \tag{17}$$

Where n is the mean motion; $\boldsymbol{G}$ is the vector disturbing contribution to the VOP equations [……].

Around the Keplerian nominal station keeping state vector

$$\boldsymbol{x}_{\boldsymbol{Ksk}} = [a_k\;0\;0\;0\;0\;\lambda_s]^T \tag{18}$$

Where a_k is a Keplerian semi major axis (constant).

And the zeros thrust acceleration vector $\boldsymbol{a_t}$=$\mathbf{0}_{3x0}$:

$$\frac{da_t}{dt} = 2a_k\frac{a_{tT}(t)}{v_{sk}} \tag{19}$$

$$\frac{dP_{1t}}{dt} = -cosK_{sk}(t)\frac{a_{tR}(t)}{v_{sk}} + 2\sin K_{sk}(t)\frac{a_{tT}(t)}{v_{sk}} \tag{20}$$

$$\frac{dP_{2t}}{dt} = +sinK_{sk}(t)\frac{a_{tR}(t)}{v_{sk}} + 2\cos K_{sk}(t)\frac{a_{tT}(t)}{v_{sk}} \tag{21}$$

$$\frac{dQ_{1t}}{dt} = \frac{1}{2}sinK_{sk}(t)\frac{a_{tN}(t)}{v_{sk}} \tag{22}$$

$$\frac{dQ_{2t}}{dt} = \frac{1}{2}cosK_{sk}(t)\frac{a_{tN}(t)}{v_{sk}} \tag{23}$$

$$\frac{dl_{\theta t}}{dt} = -\frac{2}{3}\frac{n_k}{a_t}(a_t - a_k) - 2\frac{a_{tR}(t)}{v_{sk}} \tag{24}$$

With $K_{sk}(t)=\lambda_s+\Theta(t)$ (Θ here is the right ascension of the Greenwich meridian), $v_{sk}=\omega_{\oplus} a_k$ and the Keplerian mean motion is $n_k = \sqrt{GM_{\oplus}/a_k^3}$.

The integration of equations (19)–(24) with impulsive accelerations equations (13)–(15), leads to express the equinoctial parameter variation induced by thrust accelerations (orbital element impulsive corrections) in function of the velocity change budgets Δv_t and of the maneuver instants t_m

$$\Delta P_{1t} = -cosK_{sk}(t_{mR})\frac{\Delta v_{tR}}{v_{sk}} + 2\sin K_{sk}(t_{mR})\frac{\Delta v_{tT}}{v_{sk}} \tag{24}$$

$$\Delta a_t = 2a_k\frac{\Delta v_{tT}}{v_{sk}} \tag{25}$$

$$\Delta P_{2t} = sinsK_{sk}(t_{mR})\frac{\Delta v_{tR}}{v_{sk}} + 2\cos K_{sk}(t_{mR})\frac{\Delta v_{tT}}{v_{sk}} \tag{26}$$

$$\Delta Q_{1t} = \frac{1}{2}sinK_{sk}(t_{mN})\frac{\Delta v_{tN}}{v_{sk}} \tag{27}$$

$$\Delta Q_{2t} = \frac{1}{2}cosK_{sk}(t_{mN})\frac{\Delta v_{tN}}{v_{sk}} \tag{28}$$

$$\Delta l_{\theta} = -2\frac{\Delta v_{tR}}{v_{sk}} \tag{29}$$

Moreover, the first term at the right-hand side of Eq. (24) is the mean motion deviation induced by thrust effect. With impulsive accelerations, its integration entails a variation not of the mean longitude l_{Θ} but of the mean longitude drift $\dot{l}_{\Theta}$ given by

$$\Delta\dot{l}_{\theta} = -2n_k\frac{\Delta v_{tT}}{v_{sk}} \tag{30}$$

This formula is deduced from the variation of the semi-major axis and it is very useful in the high thrust EW maneuver planning.

The structure of the above equations and the hypothesis of non-correlation between the generations of the RTN thrust acceleration components justify the practice of doing the strategic planning of East-West SK maneuvers independently from the strategic planning of North-South SK maneuvers. Moreover the structure of the previous equations shows that the eccentricity and the inclination corrections are sensitive to the correction instants.

b. Tactical Phases

During the tactical phases the attention is focused on the immediate execution of the individual detailed activities. The operator at the control centre determines which thrusters of the propulsion system have to be switched on and how long. In this phase a more realistic model of the on-board propulsion system is used. In this model the duration and amplitude of the accelerations are non-infinitesimal.

$$a_t(t) = \begin{cases} \frac{\Delta v_t}{T_M} & t_m^{on} \le t \le t_m^{off} \\ 0 & otherwise \end{cases} \tag{31}$$

With $T_m = t_m^{off} - t_m^{on}$ ‘see Fig. 3).Hence, When

$$\begin{bmatrix} a_{tR}(t) \\ a_{tT}(t) \\ a_{tN}(t) \end{bmatrix} = \frac{1}{m}\begin{bmatrix} +1 & 0 & 0 & -1 & 0 & 0 \\ 0 & +1 & 0 & 0 & -1 & 0 \\ 0 & 0 & +1 & 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} F_{+Z_B} \\ F_{-X_B} \\ F_{+Y_B} \\ F_{-Z_B} \\ F_{+X_B} \\ F_{-Y_B} \end{bmatrix} \tag{32}$$

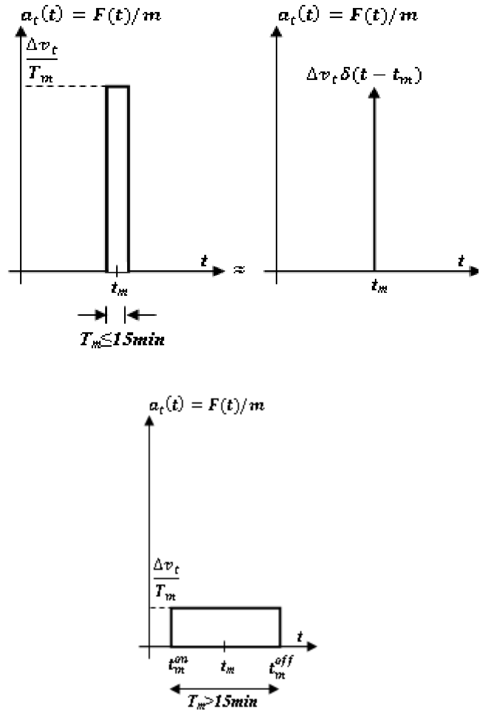

Figure 3. Accelerations Induced by a Thrust Impulse and by a Thrust Pulse

And when a single maneuver (in terms of Δv_t) is executed by switching on a single thruster, the choice of the thruster to switch on and the duration of the on interval is trivial and depends on thrust levels of each thruster and on the spacecraft mass only. Presumably the thrust durations T_m will be short given that in the strategic planning phase the maneuvers in terms of Δv_t budgets have been computed under the impulsiveness hypothesis. The strategic planning is moreover coherent with the hypothesis of the independence between EW and NS maneuvers.

4. Strategic Planning of North-South High Thrust SK Maneuvers

North-South high thrust maneuvers (also called inclination or latitude maneuvers) require fuel 20 times as much as required for the East-West maneuvers. Inclination high thrust maneuvers can be planned by maneuver optimization programs of various degrees of sophistication.

Most of the inclination maneuvers are planned to compensate only the secular drift of the inclination vector [11], who determine and tabulate the impulse requirements obtained with various methods of compensation for inclination drift). Our model to describe the evolution of the orbital pole (three-dimensional inclination vector perpendicular to the orbital plane) not only under the effect of the Sun's and Moon's attraction but also of the Earth's precession. We propose a strategic planning at high degree of optimization where only the direction of the inclination vector variation (i.e., the time of the correction day) is calculated.

From the simulation results, we have plotted (dashed line) the trace described by the tip of the inclination vector $\boldsymbol{i} = \boldsymbol{tan(i/2)n}$ in the (Q_2,Q_1) plane over eight weeks. The trace of $\boldsymbol{i}$ reaches the inclination tolerance circle after 36 day. Day 36 after the start of the simulation will be the days of the NS maneuver see Figure 4. The size of maneuver will be such that the instantaneous change $\Delta \boldsymbol{i}$ of the inclination vector $\boldsymbol{i}$ is equal to the maximum amplitude of the inclination dead band box ***(2 tan i_{max}/2***) and such that the inclination vector after the maneuver points at the opposite side of the tolerance circle in the direction of the natural drift. Then, from

$$|\Delta v_i|^2 = \Delta Q_{1t}{}^2 + \Delta Q_{2t}{}^2 = \frac{v_{tN}^2}{4v_{sk}^2} = 4tan^2\frac{imax}{2} \tag{33}$$

We deduce

$$\Delta v_{tN} = 4v_{sk}tan\frac{i_{max}}{2} \tag{34}$$

The time of the day for the impulsive maneuver is obtained from the argument of the vector change $\Delta \boldsymbol{i}$, *i.e.*, from the slope of the natural inclination drift whose time history can be approximated as a straight line. Then, from

$$\arg(\Delta i) = \frac{Q_{1t}}{Q_{2t}} = tan[\lambda_s + \theta(t_{mN})] \tag{35}$$

And consequently

$$\theta(t_{mN}) = \arctan[\arg(\Delta i) - \lambda_s] \tag{36}$$

Figure 5 and Figure 6 show the results over eight weeks of such a NS maneuver with $\boldsymbol{i}_{max}$ = 0.1 deg in the (Q_2,Q_1) and (t,φ) planes respectively. The initial value of inclination is zero. The maneuver found in terms of Δv_{tN} gives Δv_{tN} = 10.73 *m/s* to perform at $t_{mN} = -16.2$ hours of the 36th day, *i.e.*, at t_{mN} = 7.8 hours of the 33th day (after the beginning of the simulation for a spacecraft with mass m = 4500 kg). We have assumed to execute this velocity variation with a chemical thruster at high thrust F_{+YB} = 80 Newton which gives a maneuver duration T_{mN} = 10 minutes

$$T_{mN} = \frac{m\Delta v_{tN}}{F_{+Y_B}} \tag{37}$$

The simulation results plotted in Figures 5.7 and 5.8 have been obtained integrating over eight weeks the Gauss' VOP equations with a normal acceleration component given by

$$a_{tN}(t) = \begin{cases} \frac{F_{+Y_B}}{m} & if\ t_{mN} - \frac{T_{mN}}{2} \le y \le t_{mN} + \frac{T_{mN}}{2} \\ 0 & otherwise \end{cases} \tag{38}$$

By a simulink variable-step solver based on an explicit Runge-Kutta (4, 5) formula: ode45. Relative and absolute tolerances have been chosen equal to 10^{-12}. Figure 7 is a zoom of the latitude time history over the maneuver day. We can observe the instantaneous change in the latitude time history derivative (*i.e.*, in the spacecraft latitudinal velocity) at the maneuver instant.

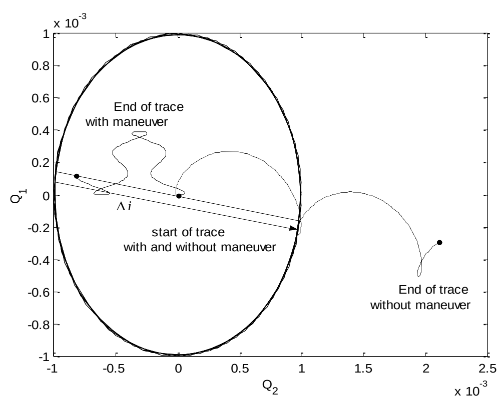

Figure 4. Trace described by the tip of the inclination vector without maneuver (dashed line) and with impulsive maneuver (solid line).the circle is the inclination tolerance circle of radius *tan(i_{max}/2)*with i_{max}*=0.1 deg.*

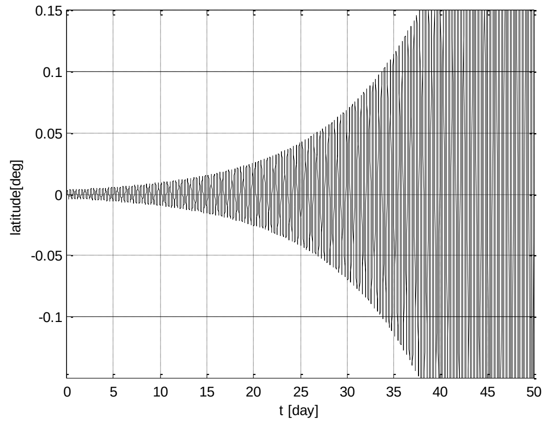

Figure 5. Latitude Described without Maneuver; the Horizontal Line are the Limits of the Latitude Tolerance Ranger *2arcsin [2tan (*$_{imax}$*/2)] ≈2i*$_{max}$ in Width

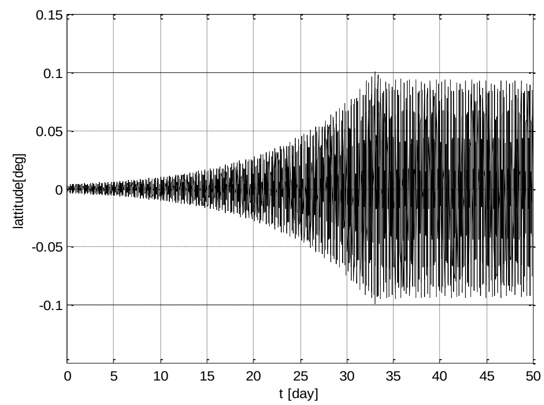

Figure 6. Latitude DManeuver; the Horizontal Line are the Limits of the Latitude Tolerance Ranger *2arcsin [2tan (*$_{imax}$*/2)] ≈2i*$_{max}$ in Width

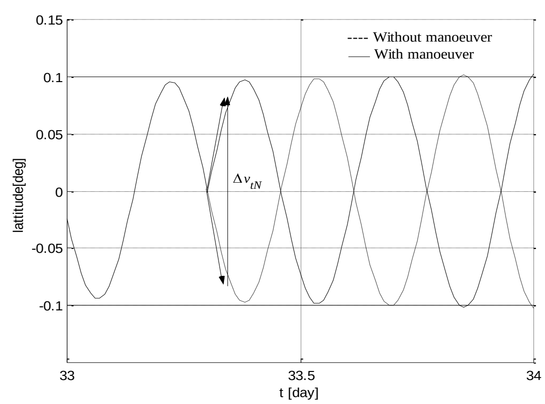

Figure 7. Zoom of Figure 5 and Figure 6 over the Maneuver Day

5. Strategic Planning of East-West High Thrust SK Maneuvers

East-West high thrust maneuvers use generally less fuel than inclination maneuver but they are more complex to plan for several reasons:

1. A tangent thrust acceleration does not change directly the mean longitude l_Θ but only the mean motion drift

$$n - n_k = n_k D = -\frac{3}{2}\frac{n_k}{a_k}(a - a_k) \tag{39}$$

Where D is a mean motion deviation rate.

2. A tangent thrust acceleration changes also the eccentricity components and consequently the geographical longitude (see eq.20-eq.21).

3. The longitude drift is sensitive to disturbances of attitude and inclination.

An EW maneuver strategic planning, we recognize the need of regularizing the number of corrections to be performed at the beginning of each correction cycle. Moreover, the correction cycle duration T_C is maximized following a waiting strategy, which consists in deciding to execute a tangent maneuver when the longitude reaches the dead band box border and in using all the allowed range during the spacecraft longitudinal drift without maneuvers. We illustrate this strategy with a simple example based on a simplified version of our Gauss' VOP equations of motion.

In Figure 7 we have plotted the time histories of the mean longitude l_Θ and its derivative $\dot{l}_\Theta$obtained integrating between $t_0 = 0$ and $t_f = 28\ day$ the nonlinear Gauss' VOP equations with eccentricity components equal to zero and with only the Earth's gravity acceleration acting on semi-major axis a and mean longitude l_Θ. The mean longitude time history can be approximated with a parabola

$$l_\Theta(t) = \alpha + \beta t + \frac{\gamma}{2}t^2 \tag{40}$$

The time history of its derivative (also known as the mean longitude drift) can be approximated with the straight line

$$\dot{l}_\Theta(t) = \beta + \gamma t \tag{41}$$

The coefficients α, β and γ can be extrapolated by the simulation time histories to give

$$\alpha = l_\Theta(t_0), \beta = \dot{l}_\Theta(t_0), \gamma = \frac{\dot{l}_\Theta(t_f) - \beta}{t_f - t_0} \tag{42}$$

For $t_0 = 0$. The second derivative $\ddot{l}_\Theta(t)$ (the mean longitude acceleration) is equal to γ and it is nearly constant because it depends on the nearly constant tangent acceleration component induced by the non uniform Earth's gravity attraction. To strategically plan the high thrust maneuvers under the impulsiveness hypothesis, we cannot modify the second derivative of the parabola (*i.e.*, its concavity) because the high thrust maneuvers induce only velocity variations (*i.e.*, instantaneously velocity changes) and not total velocity changes resulting in finite accelerations. We can act only on coefficients α and β. Coefficient β modifies the mean longitude drift $\dot{l}_\Theta(t)$ .Coefficients α fixes the position of the parabola in the (t, l_θ) because in this phase of the strategic planning we have assumed eccentricity always equal to zero, the mean longitude $l_\theta$coincides with the geographical longitude λ and the mean longitude drift $\dot{l}_\Theta$ coincides with the longitudinal angular velocity of the spacecraft.

In Figure 7 the limits of the allowed zone in the (t, λ) plane are also drawn (horizontal lines). Longitude λ reaches its maximum allowed value

$$\lambda_s + \lambda_{max} = 60.1° \ with\ \lambda_s = 60° \ and\ \lambda_{max} = 0.1° \tag{43}$$

At the instant

$$t_{iC1} = 3.5483\ day \tag{44}$$

(*i.e.*, at the 7th minute of the 13th hour in the 3rd day after the simulation beginning). At this instant an EW maneuver is needed such that the longitude slope changes brusquely and a new parabolic trend begins.

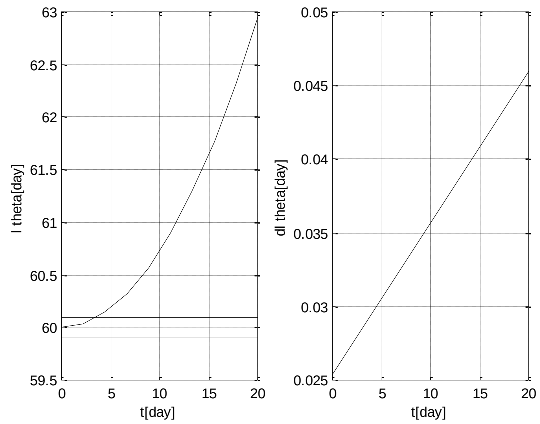

Figure 8. Time Histories of the Mean Longitude and of the Mean Longitude Drift Obtained Integrating over 3 Weeks Simplified Nonlinear Gauss' VOP Equations

Two arcs of trajectory juxtapose without interruption in the longitude component

$$\lambda(t) = \begin{cases} \alpha + \beta t + \frac{\gamma}{2}t^2 \ \ if\ t_0 \le t \le t_{iC1} \\ \alpha + \tilde{\beta} t + \frac{\tilde{\gamma}}{2}t^2 \ \ if\ t_{iC1} \le t \le t_{fC1} = t_{iC1} + t_C \end{cases} \tag{45}$$

Which entails the following continuity condition

$$\Delta\alpha + t_{iC1}\Delta\beta = 0 \tag{46}$$

Where $\Delta\alpha = (\alpha - \tilde{\alpha}), \Delta\beta = (\beta - \tilde{\beta})$ and t_{iC1} is known. The variation $\Delta\beta$ is the variation of the mean longitude drift $\dot{l}_{\Theta t}$ induced by an impulsive tangent thrust at the maneuver instant $t_{mT} = t_{iC1}$

$$\Delta\beta = \Delta\dot{l}_{\Theta t}(t_{mT}) = \dot{l}_{\Theta t}(t^{+}_{mT}) - \dot{l}_{\Theta t}(t^{-}_{mT}) \tag{47}$$

Once the amount of $\Delta\beta$ is known, the amount of tangent velocity variation Δv_{tT} induced by a thruster executing an impulsive EW maneuver is known too. In fact, from Eq. 47 we have

$$\Delta\dot{l}_{\Theta t}(t_{mT}) = -3n_k \frac{\Delta v_{tT}}{v_{sk}} \tag{48}$$

Which entails

$$\Delta v_{tT} = -\frac{a_k \Delta\beta}{3} \tag{49}$$

To find the variation $\Delta\beta$, the target condition

$$\lambda(t_{mT} + T_C) = \lambda(t_{mT}) = \lambda_s + \lambda_{max} \tag{50}$$

And the maximum range condition

$$\lambda(t_{mT} + T_C/2) = \lambda_s - \lambda_{max} \tag{51}$$

Have to be imposed. From the target condition and the continuity condition we obtain

$$\tilde{\beta} + \gamma t_{mT} + \frac{\gamma}{2}T_C = 0 \tag{52}$$

Subtracting term by term the maximum range condition

$$\tilde{\alpha} + \tilde{\beta} t_{mT} + \tilde{\beta}\frac{T_C}{2} + \frac{\gamma}{2}\left(t^2{}_{mT} + 2t_{mT}T_C + \frac{T_C{}^2}{4}\right) = \lambda_s - \lambda_{max} \tag{53}$$

From the target condition

$$\tilde{\alpha} + \tilde{\beta} t_{mT} + \tilde{\beta}\, T_C + \frac{\gamma}{2}\left(t^2{}_{mT} + t_{mT}T_C + T_C{}^2\right) = \lambda_s + \lambda_{max} \tag{54}$$

We obtain

$$\frac{T_C}{2}\left(\tilde{\beta} + \gamma t_{mT}\right) + \frac{\gamma}{2} 3 \frac{T_C{}^2}{4} = 2\lambda_{max} \tag{55}$$

Thanks to Eq. (5.72), we find from Eq. 55 the duration T_C of the correction cycle

$$T_C = 4\sqrt{\frac{\lambda_{max}}{\gamma}} \tag{56}$$

As the values of β and T_C are known, we find $\Delta\beta = (\beta - \tilde{\beta})$

$$\Delta\beta = \beta + \gamma t_{mT} + 2\sqrt{\gamma\lambda_{max}} \tag{57}$$

In this simulation, such a variation is induced by a high thrust maneuver with Δv_{tT} =-0.14 m/s. Figure 9 shows the effect of this tangent high thrust maneuver on the mean longitude drift time histories approximated straight line.

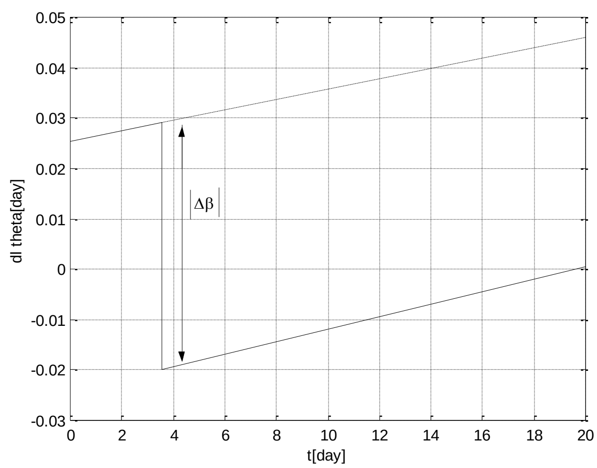

Figure 9. Effect of Tangent High Thrust Maneuver on the Mean Longitude (solid lines)

6. Conclusion

In these papers, we are interested in North-South and East-West station keeping strategy of geostationary satellites equipped with chemical propulsion systems: in order to compensate changes in the orbit parameters, chemical thrusters are typically fired once every two weeks during a time interval T_m of few tens of minutes, providing forces of some tens of Newton. Given the small ratio between T_m and the geostationary orbital period, chemical thrusts can be considered with good approximation as impulsive. It thus makes sense to define station keeping strategy by excluding from the dynamics equations the non conservative forces (*i.e.*, the thrusts).

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