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The Fuzzy $(PI+D)^2$ Sliding Mode Scheme to Motor Vibration Control

Saman Rahbar, Farzin Piltan, Ehsan Pouladi, Hossein Davarpanah and Somayeh Jowkar

Intelligent System and Robotic Lab, Iranian Institute of Advance Science and Technology (IRAN SSP), Shiraz/Iran
Email: piltan_f@iranssp.com, WWW.IRANSSP.COM/english

Abstract

This paper proposes new fuzzy type sliding mode controller for multi-DOF joints to reduce/eliminate motor vibration. The fuzzy $(PI+D)^2$ single-input single-output (SISO) fuzzy system is applied to modify each element of the control in a sliding mode controller. The proposed method is designed based on the Lyapunov method. Mathematical proof for the stability and the convergence of the system is presented. Various operation situations such as the set point control and the trajectory control are simulated. The simulation results demonstrate that the chattering and the steady state errors, which usually occur in the classical sliding mode control, are eliminated and satisfactory trajectory tracking is achieved.

Keywords: *Trajectory tracking, PI fuzzy logic controller, D controller, sliding mode control, fuzzy control, Lyapunov method, stability, robustness, multi degrees of freedom joints*

1. Introduction

Classical sliding mode control (CSMC) is a powerful scheme for nonlinear systems with uncertainty [1]. This control scheme suffers from some problems, however. In order to guarantee the stability of the sliding mode system, the boundary of the uncertainty has to be estimated. A large value has to be applied to the control gain when the boundary is unknown. Unfortunately, this large control gain may cause chattering on the sliding surface and therefore deteriorate the system performance. Several approaches for reducing the chattering have been proposed, among which the well-known one is to apply a saturation function [1] to the control gain when the sliding surface is within a boundary of the sliding hyper-plane. This approach, however, does not guarantee the convergence of the output. In other words, there exists a nonzero steady-state error in the output. An alternative way to solve the chattering problem is the application of the fuzzy logic [2] in the construction of the control input. Fuzzy logic has proven to be a potent tool in the sliding mode control of time-invariant linear systems [3] as well as time-varying nonlinear systems [4]. It provides methods for formulating linguist rules from expert knowledge and is able to approximate any real continuous system to arbitrary accuracy. Thus, it offers a simple solution dealing with the wide range of the system parameters. In recent years, artificial intelligence theory has been used in nonlinear control systems. Neural network, fuzzy logic and neuro-fuzzy are synergically combined with nonlinear classical controller and used in nonlinear, time variant and uncertain system. Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory. This controller can be used to control nonlinear, uncertain, and noisy systems. This method is free of some model techniques as in model-based controllers. As

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mentioned that fuzzy logic application is not only limited to the modelling of nonlinear systems but also this method can help engineers to design a model-free controller. Control of nonlinear system using model-based controllers is based on system dynamic model. These controllers often have many problems for modelling. Conventional controllers require accurate information of dynamic model of system, but most of time these models are MIMO, nonlinear and partly uncertain therefore calculate accurate dynamic model is complicated. The main reasons to use fuzzy logic methodology are able to give approximate recommended solution for uncertain and also certain complicated systems to easy understanding and flexible. Fuzzy logic provides a method to design a model-free controller for nonlinear plant with a set of IF-THEN rules. Mohan and Bhanot [13] have addressed comparative study between some adaptive fuzzy and a new hybrid fuzzy control algorithm for manipulator control. They found that self-organizing fuzzy logic controller and proposed hybrid integrator fuzzy give the best performance as well as simple structure.

Multi-degree-of-freedom (DOF) actuators are finding wide use in a number of Industries. Currently, a significant number of the existing robotic actuators that can realize multi-DOF motion are constructed using gear and linkages to connect several single-DOF motors in series and/or parallel. Not only do such actuators tend to be large in size and mass, but they also have a decreased positioning accuracy due to mechanical deformation, friction and backlash of the gears and linkages. A number of these systems also exhibit singularities in their workspaces, which makes it virtually impossible to obtain uniform, high-speed, and high-precision motion. For high precession trajectory planning and control, it is necessary to replace the actuator system made up of several single-DOF motors connected in series and/or parallel with a single multi-DOF actuator. The need for such systems has motivated years of research in the development of unusual, yet high performance actuators that have the potential to realize multi-DOF motion in a single joint. One such actuator is the spherical motor. Compared to conventional robotic manipulators that offer the same motion capabilities, the spherical motor possesses several advantages. Not only can the motor combine 3-DOF motion in a single joint, it has a large range of motion with no singularities in its workspace. The spherical motor is much simpler and more compact in design than most multiple single-axis robotic manipulators. The motor is also relatively easy to manufacture. Figure 1 shows the multi-DOF actuators [4].

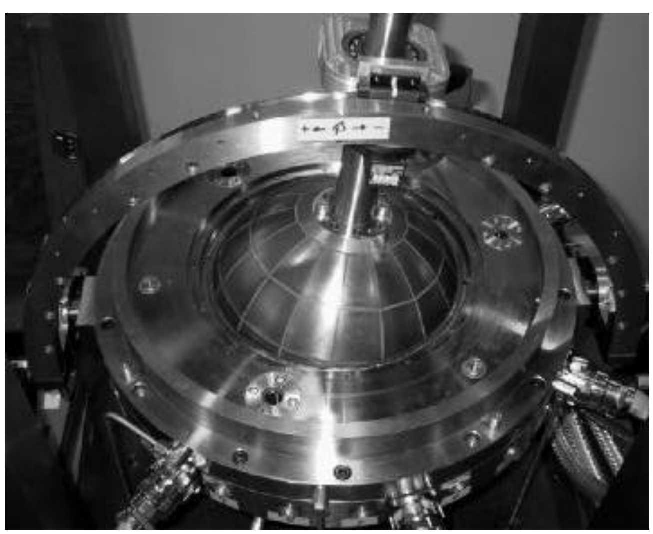


Figure 1. Multi DOF Actuator

In this research the new technique of fuzzy $(PI+D)^2$ sliding mode controller is used to reduce/eliminate the effect of motor vibration. This paper is organized as follows;

- Section 2, is served as a modeling and formulation of spherical motor.
- Part 3, introduces and describes the control design for reduce the motor vibration.
- Section 4 presents the simulation results and discussion of this algorithm applied to a spherical motor and the final section describe the conclusion.

2. Theory: Modeling and Formulation

Since its inception, the field of multi-DOF actuator dynamics has presented many issues in refining both theory and operations; one of the most challenging areas of study has been the problem of computational efficiency in the dynamics of mechanisms. Many efficient algorithms in dynamics have been developed to address this problem.

Dynamic equation is the study of motion with regard to forces. Dynamic modeling is vital for control, mechanical design, and simulation. It is used to describe dynamic parameters and also to describe the relationship between displacement, velocity and acceleration to force acting on multi-DOF actuators. To calculate the dynamic parameters which introduced in the following lines, four algorithms are very important.

i. **Inverse dynamics**, in this algorithm, joint actuators are computed (e.g., force/torque or voltage/current) from end-effector position, velocity, and acceleration. It is used in feed forward control.
ii. **Forward dynamics** used to compute the joint acceleration from joint actuators. This algorithm is required for simulations.
iii. **The joint-space inertia matrix**, necessary for maps the joint acceleration to the joint actuators. It is used in analysis, feedback control and in some integral part of forward dynamics formulation.
iv. **The operational-space inertia matrix**, this algorithm maps the task accelerations to task actuator in Cartesian space. It is required for control of end-effector.

Several different methods are available to compute multi-DOF actuators dynamic equations. These methods include the Newton-Euler (N-E) methodology, the Lagrange-Euler (L-E) method, and Kane's methodology [5].

The Newton-Euler methodology is based on Newton's second law and several different researchers are signifying to develop this method [5-8]. This equation can be described the behavior of a multi-DOF actuators joint-by-joint from base to endeffector, called forward recursion and transfer the essential information from end-effector to base frame, called backward recursive.

The literature on Euler-Lagrange's is vast but a good starting point to learn about it is in[9-10]. Calculate the dynamic equation multi-DOF actuators using E-L method is easier because this equation is derivation of nonlinear coupled and quadratic differential equations.

The Kane's method was introduced in 1961 by Professor Thomas Kane[9]. This method used to calculate the dynamic equation of motion without any differentiation between kinetic and potential energy functions.

The dynamics of multi-DOF actuators illustrate the relationship between force and motion. The generalized force for a multi-DOF actuator can be described as a second-order nonlinear differential equation. The dynamic equation of multi-DOF actuators is derived using the Lagrangian. The Lagrangian is derived by subtracting potential energy from kinetic energy.

The dynamic equation of multi-DOF actuator governed by the following equation [17-19]:

$$\boldsymbol{H}(\boldsymbol{q})\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\\ \ddot{\gamma}\end{bmatrix}+\mathbf{B}(\boldsymbol{q})\begin{bmatrix}\dot{\alpha}\dot{\beta}\\ \dot{\alpha}\dot{\gamma}\\ \dot{\beta}\dot{\gamma}\end{bmatrix}+\boldsymbol{C}(\boldsymbol{q})\begin{bmatrix}\dot{\alpha}^2\\ \dot{\beta}^2\\ \dot{\gamma}^2\end{bmatrix}=\begin{bmatrix}\tau_x\\ \tau_y\\ \tau_z\end{bmatrix} \tag{1}$$

- τ is actuation torque
- H (q) is a symmetric and positive define inertia matrix
- B(q) is the matrix of coriolios torques
- C(q) is the matrix of centrifugal torques.

The angular acceleration is found as to be:

$$\begin{bmatrix}\ddot{\alpha}\\ \ddot{\beta}\\ \ddot{\gamma}\end{bmatrix}=\boldsymbol{H}^{-1}(\boldsymbol{q}).\left\{\boldsymbol{\tau}-\mathbf{B}(\boldsymbol{q})\begin{bmatrix}\dot{\alpha}\dot{\beta}\\ \dot{\alpha}\dot{\gamma}\\ \dot{\beta}\dot{\gamma}\end{bmatrix}-\boldsymbol{C}(\boldsymbol{q})\begin{bmatrix}\dot{\alpha}^2\\ \dot{\beta}^2\\ \dot{\gamma}^2\end{bmatrix}\right\} \tag{2}$$

3. Control Design for Trajectory Tracking

Controller is a device which can sense information from linear or nonlinear system to improve the systems performance. The main targets in designing control systems are [11]:

- Stability
- good disturbance rejection
- small tracking error

Several industrial nonlinear plant are controlled by linear methodologies (*e.g.*, Proportional-Derivative (PD) controller, Proportional- Integral (PI) controller or Proportional-Integral-Derivative (PID) controller), but when system works with various payloads and have uncertainty in dynamic models this technique has limitations.

From the control point of view, uncertainty is divided into two main groups [12]:

- Uncertainty in unstructured inputs (*e.g.*, noise, disturbance)
- uncertainty in structure dynamics (*e.g.*, payload, parameter variations)

In some applications multi-DOF actuators are used in an unknown and unstructured environment, therefore strong mathematical tools used in new control methodologies to design nonlinear robust controller with an acceptable performance (*e.g.*, minimum error, good trajectory, disturbance rejection). Sliding mode controller is an influential nonlinear controller to certain and uncertain systems which it is based on system's dynamic model [14].

Sliding mode controller (SMC) is a powerful nonlinear controller which has been analyzed by many researchers especially in recent years. This theory was first proposed in the early 1950 by Emelyanov and several co-workers and has been extensively developed since then with the invention of high speed control devices [15]. The main reason to opt for this controller is its acceptable control performance in wide range and solves two most important challenging topics in control which names, stability and robustness. Sliding mode control theory for control joint of robot manipulator was first proposed in 1978 by Young to solve the set point problem ($\dot{\boldsymbol{q}}_{\boldsymbol{d}}=\mathbf{0}$) by discontinuous method in the following form [16-19];

$$\boldsymbol{\tau}_{(q,t)}=\begin{cases}\boldsymbol{\tau}_i^+(\boldsymbol{q},\boldsymbol{t}) & \boldsymbol{if}\ \ \boldsymbol{S}_i>0\\ \boldsymbol{\tau}_i^-(\boldsymbol{q},\boldsymbol{t}) & \boldsymbol{if}\ \ \boldsymbol{S}_i<0\end{cases} \tag{3}$$

where $\boldsymbol{S}_i$ is sliding surface (switching surface), $\boldsymbol{i}=\mathbf{1},\mathbf{2},\ldots\ldots,\boldsymbol{n}$ for *n*-DOF joint, $\boldsymbol{\tau}_i(\boldsymbol{q},\boldsymbol{t})$ is the $\boldsymbol{i}^{\boldsymbol{th}}$ torque of joint. Sliding mode controller is divided into two main sub controllers:

- Corrective control($\boldsymbol{U_c}$)
- Equivalent controller($\boldsymbol{U_{eq}}$).

Discontinues controller causes an acceptable tracking performance at the expense of very fast switching. Conversely in this theory good trajectory following is based on fast switching, fast switching is caused to have system instability and chattering phenomenon. Fine tuning the sliding surface slope is based on nonlinear equivalent part [20]. However, this controller is used in many applications but, pure sliding mode controller has two most important challenges: chattering phenomenon and nonlinear equivalent dynamic formulation in uncertain parameters[21].

Design a robust controller for multi-DOF-joints is essential because these joints have highly nonlinear dynamic parameters. Consider a nonlinear single input dynamic system is defined by:

$$\boldsymbol{x^{(n)} = f(\vec{x}) + b(\vec{x})u} \tag{4}$$

Where u is the vector of control input, $\boldsymbol{x^{(n)}}$ is the $\boldsymbol{n^{th}}$ derivation of $\boldsymbol{x}$, $\boldsymbol{x = [x, \dot{x}, \ddot{x}, \dots, x^{(n-1)}]^T}$ is the state vector, $\boldsymbol{f(x)}$ is unknown or uncertainty, and $\boldsymbol{b(x)}$ is of known *sign* function. The main goal to design this controller is train to the desired state; $\boldsymbol{x_d = [x_d, \dot{x}_d, \ddot{x}_d, \dots, x_d{}^{(n-1)}]^T}$, and trucking error vector is defined by:

$$\boldsymbol{\widetilde{x} = x - x_d = [\widetilde{x}, \dots, \widetilde{x}^{(n-1)}]^T} \tag{5}$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$\boldsymbol{U_C = \widehat{U} - K(\vec{x}, t) \cdot \mathrm{sgn}(s)} \tag{6}$$

where the switching function $\mathbf{sgn}(\boldsymbol{S})$ is defined as

$$\boldsymbol{sgn(s)} = \begin{cases} \mathbf{1} & \boldsymbol{s} > 0 \\ \mathbf{-1} & \boldsymbol{s} < 0 \\ \mathbf{0} & \boldsymbol{s = 0} \end{cases} \tag{7}$$

and the $\boldsymbol{K(\vec{x}, t)}$ is the positive constant.

Based on above discussion, the sliding mode control law for multi-DOF-joints is written as:

$$\boldsymbol{U = U_{eq} + U_c} \tag{8}$$

where, the model-based component $\boldsymbol{U_{eq}}$ is the nominal dynamics of systems and calculated as follows:

$$\boldsymbol{U_{eq}} = \left[\boldsymbol{H^{-1}(q)} \left(\mathbf{B}\boldsymbol{(q)} \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} + \boldsymbol{C(q)} \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} \right) + \dot{\boldsymbol{S}} \right] \boldsymbol{H(q)} \tag{9}$$

and $\boldsymbol{U_c}$ is computed as;

$$\boldsymbol{U_c} = \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{S}) \tag{10}$$

The sliding mode control of multi-DOF-joint is calculated as;

$$\begin{bmatrix} \widehat{\tau_\alpha} \\ \widehat{\tau_\beta} \\ \widehat{\tau_\gamma} \end{bmatrix} = \left[\boldsymbol{H^{-1}}(\boldsymbol{q}) \left(\mathrm{B}(\boldsymbol{q}) \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} + \boldsymbol{C}(\boldsymbol{q}) \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} \right) + \dot{S} \right] \boldsymbol{H}(\boldsymbol{q}) + \boldsymbol{K} \cdot \mathbf{sgn}(\boldsymbol{S}) \tag{11}$$

The lyapunov formulation can be written as follows [22],

$$\boldsymbol{V} = \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{S^T}.\boldsymbol{H}.\boldsymbol{S} \tag{12}$$

the derivation of V can be determined as,

$$\dot{\boldsymbol{V}} = \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{S^T}.\dot{\boldsymbol{H}}.\boldsymbol{S} + \boldsymbol{S^T}\ \boldsymbol{H}\dot{\boldsymbol{S}} \tag{13}$$

the dynamic equation of multi-DOF actuator can be written based on the sliding surface as

$$\boldsymbol{H}\dot{\boldsymbol{S}} = -\boldsymbol{V}\boldsymbol{S} + \boldsymbol{H}\dot{\boldsymbol{S}} + \boldsymbol{B} + \boldsymbol{C} \tag{14}$$

it is assumed that

$$\boldsymbol{S^T}\left(\dot{\boldsymbol{H}} - \mathbf{2}\boldsymbol{B} + \boldsymbol{C}\right)\boldsymbol{S} = \mathbf{0} \tag{15}$$

by substituting (14) in (15)

$$\dot{\boldsymbol{V}} = \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{S^T}\dot{\boldsymbol{H}}\boldsymbol{S} - \boldsymbol{S^T}\boldsymbol{B} + \boldsymbol{C}\boldsymbol{S} + \boldsymbol{S^T}\left(\boldsymbol{H}\dot{\boldsymbol{S}} + \boldsymbol{B} + \boldsymbol{C}\boldsymbol{S}\right) = \boldsymbol{S^T}\left(\boldsymbol{H}\dot{\boldsymbol{S}} + \boldsymbol{B} + \boldsymbol{C}\boldsymbol{S}\right) \tag{16}$$

suppose the control input is written as follows

$$\widehat{\boldsymbol{U}} = \widehat{\boldsymbol{U_{Nonlinear}}} + \widehat{\boldsymbol{U_c}} = \left[\widehat{\boldsymbol{H^{-1}}}(\boldsymbol{B} + \boldsymbol{C}) + \dot{\boldsymbol{S}}\right]\widehat{\boldsymbol{H}} + \boldsymbol{K}.\boldsymbol{sgn}(\boldsymbol{S}) + \boldsymbol{B} + \boldsymbol{C}\boldsymbol{S} \tag{17}$$

by replacing the equation (17) in (10)

$$\dot{\boldsymbol{V}} = \boldsymbol{S^T}(\boldsymbol{H}\dot{\boldsymbol{S}} + \boldsymbol{B} + \boldsymbol{C} - \widehat{\boldsymbol{H}}\dot{\boldsymbol{S}} - \widehat{\boldsymbol{B} + \boldsymbol{C}\boldsymbol{S}} - \boldsymbol{K}\boldsymbol{sgn}(\boldsymbol{S}) = \boldsymbol{S^T}\left(\widetilde{\boldsymbol{H}}\dot{\boldsymbol{S}} + \widetilde{\boldsymbol{B} + \boldsymbol{C}\boldsymbol{S}} - \boldsymbol{K}\boldsymbol{sgn}(\boldsymbol{S})\right) \tag{18}$$

and

$$\left|\widetilde{\boldsymbol{H}}\dot{\boldsymbol{S}} + \widetilde{\boldsymbol{B} + \boldsymbol{C}\boldsymbol{S}}\right| \leq \left|\widetilde{\boldsymbol{H}}\dot{\boldsymbol{S}}\right| + \left|\widetilde{\boldsymbol{B} + \boldsymbol{C}\boldsymbol{S}}\right| \tag{19}$$

the Lemma equation in multi-DOF actuator can be written as follows

$$\boldsymbol{K_u} = \left[\left|\widetilde{\boldsymbol{H}}\dot{\boldsymbol{S}}\right| + |\boldsymbol{B} + \boldsymbol{C}\boldsymbol{S}| + \boldsymbol{\eta}\right]_{\boldsymbol{i}}, \boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \ldots \tag{20}$$

and finally;

$$\dot{V} \leq -\sum_{i=1}^{n} \eta_i \, |S_i| \tag{21}$$

To reduce or eliminate the chattering, various papers have been reported by many researchers. In this research intelligent method is used to reduce/eliminate the chattering in presence of uncertainty with respect to improve the robustness. To reduce the chattering fuzzy logic controller is used and applied to conventional SMC. This type of controller has double inputs and an output.

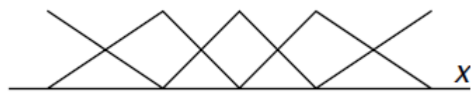
Fuzzy control emerged on the foundations of Zadeh's fuzzy set theory. It is a methodology of intelligent control that mimics human thinking and reacting by using a multivalent fuzzy logic and elements of artificial intelligence (simplified deduction principles). The word "fuzzy" is used here to describe terms that are either not well-known or not clear enough, or their closer specification depends on subjectivity, estimation, and even the intuition of the person who is describing these terms. In everyday life there are a lot of situations characterized by a certain degree of ambiguity whose description includes terms and expressions such as *majority*, *many*, *several*, *not exactly*, or *quite possible*, all of which can be qualified as "fuzzy terms." On the other hand, terms like *false*, *true*, *possible*, *necessary*, *none*, or *all* reflect crisp meanings, and in such a context, represent "exact terms." The fuzzy logic concept had very strong opponents in the beginning. They believed that any form of vagueness or imprecision could be equally well described with the theory of probability. Furthermore, opponents claimed that fuzzy logic theory was only a theory without real potential for practical applications. In the field of automatic control, the strongest opponents assumed that traditional control techniques were superior to fuzzy logic or at least equal in effect. The kind of a structure a fuzzy controller will have will primarily depend on the controlled process and the demanded quality of control. Since the application area for fuzzy control is really wide, there are many possible controller structures, some differing significantly from each other by the number of inputs and outputs, or less significantly by the number of input and output fuzzy sets and their membership functions forms, or by the form of control rules, the type of inference engine, and the method of defuzzification. All that variety is at the designer's disposal, and it is up to the designer to decide which controller structure would be optimal for a particular control problem. For example, if the controlled process exhibits integral behavior, then a so-called *non-integral* or *PD-type* fuzzy controller whose crisp output value represents absolute control input value could provide the required quality of control. On the other hand, a so-called *integral* or *PI-type* fuzzy controller whose crisp output value represents an increment of control input value could be a satisfactory solution for the control of static systems. The usage of fuzzy algorithms is not limited to fuzzy logic controllers. Fuzzy algorithms can be used equally well as nonlinear adaptation mechanisms, universal approximators or as auxiliary units added to some conventional control solutions. We should not fail to mention that fuzzy controllers are very convenient as supervisory controllers. Sometimes, fuzzy logic algorithms are also used as modal or fuzzy state controllers. Despite the variety of possible fuzzy controller structures, the basic form of all common types of controllers consists of:

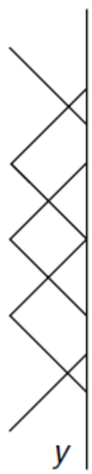
- Input fuzzification (binary-to-fuzzy[B/F]conversion)
- Fuzzy rule base (knowledge base)
- Inference engine

- Output defuzzification (fuzzy-to-binary [F/B] conversion).

The most frequently used structure of a fuzzy controller is the double input–single output (DISO) structure. In case of designing such a controller, a very convenient form of displaying the complete fuzzy rule base is a *fuzzy rule table*. Every rule in the fuzzy rule table is represented by an output fuzzy set engaged in the THEN part of the rule. The rule position within the fuzzy rule table is determined by coordinates of inputs fuzzy sets engaged in the IF part of the rule. Thus the fuzzy rule table provides straight insight into the essence of the fuzzy rule base and automatically eliminates the creation of contradictory fuzzy rules. The tabular format also makes an elegant entry of new fuzzy rules possible.

Table 1 shows the fuzzy rule-table of a DISO fuzzy controller with l = 5 triangular fuzzy sets defined for both inputs *x (error)* and *y (integral of error)*, and output *u* as following: negative large (NL); negative small (NS); around zero (Z); positive small (PS); and positive large (PL). For the studied $l \times l = 5 \times 5$ table a number of fuzzy rules may increase up to $l^2 = 25$ rules.





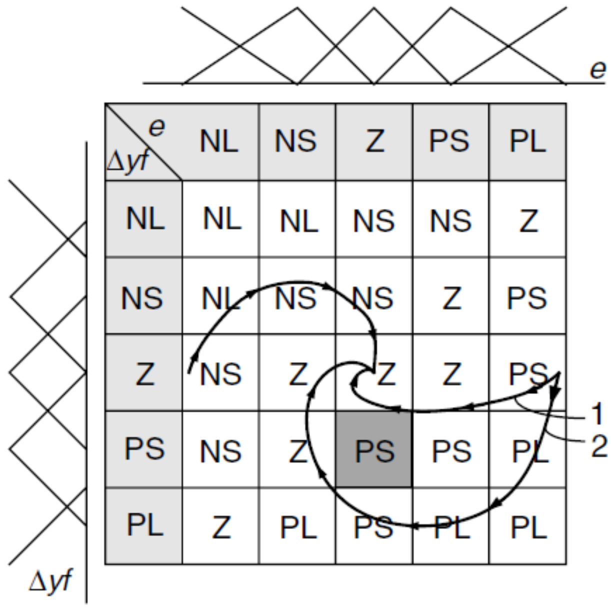
y \ x	NL	NS	Z	PS	PL
NL	NL	NL	NS	NS	Z
NS	NL	NS	NS	Z	PS
Z	NS	Z	Z	Z	PS
PS	NS	Z	PS	PS	PL
PL	Z	PS	PS	PL	PL

Table 1. A Fuzzy Rule Base Displayed as a Fuzzy Rule Table

The shaded rule in Table 1 can be read as follows:

IF *x* is Z AND *y* is PS THEN *u* is PS

A short glance at the table confirms the completeness (all 25 rules are there) and the continuity of the displayed fuzzy rule-base (consistency is automatically provided). A fuzzy rule table can also be viewed as the state space of two process variables *x* and *y* (e.g., let *x* = *e(k)* — control error, $\Delta y_f = integral\,of\ x$ — integral of error($\sum e$), where k is the substitute for kT_d, and T_d is a sampling interval). By using a fuzzy rule table, we get the chance to see the corresponding phase trajectories resulting from consecutive switching of fuzzy rules (Table 2).



Δyf \ e	NL	NS	Z	PS	PL
NL	NL	NL	NS	NS	Z
NS	NL	NS	NS	Z	PS
Z	NS	Z	Z	Z	PS
PS	NS	Z	PS	PS	PL
PL	Z	PL	PS	PL	PL

Table 2. Phase Trajectories Drawn in a Fuzzy Rule Table

We have already mentioned that the design of a fuzzy controller is actually a heuristic search for the best fitted static nonlinear mapping function between controller inputs and outputs. As a result of mapping, every discrete trajectory $[e(k), y_{f\,(k)}]$ has a matching controller output series $u_{FC(k)}$, $k = 0, 1, \ldots, \infty$ space composed of a phase plane and a corresponding fuzzy control surface lying above the plane. Every controller output sequence $u_{FC(k)}$ belongs to this fuzzy control surface. Any changes made in the fuzzy rule table during the design process will change the path of phase trajectories. Therefore, these trajectories are very useful for getting a better insight into the progress of an ongoing controller design. By following the trajectory during a transient response one can easily find which fuzzy control rules are activated and how they contribute to crisp output value. A fuzzy rule table viewed as a phase plane is frequently used for heuristic assessment of closed-loop system stability, as it offers an elegant way to investigate the influence of individual control rules (their THEN parts) on the shape of phase trajectories. In order to bring more generality into the process of controller design, we would advise normalization of controller input and output domains. The universe of discourse of fuzzy controller inputs and outputs varies from one application to another. To avoid having to make adjustments for each application, inputs and outputs can be scaled to fit the normalized universes of discourse. When we use the term normalized fuzzy controller, we have in mind a controller whose fuzzification, fuzzy rule base and defuzzification parts operate with normalized values usually lying in the interval $[-1, 1]$.

The normalization of inputs should be performed before proceeding with fuzzification:

$$\boldsymbol{x_N}(\boldsymbol{k}) = \boldsymbol{K_x x}(\boldsymbol{k}) \tag{22}$$

where x is controller input, $\boldsymbol{x_N}$ is normalized controller input, and $\boldsymbol{K_x}$ is the scaling factor. Thus, $e(k)$ and $\sum e(k)$ after normalization become:

$$\boldsymbol{e_N(k) = K_e e(k)} \quad \textbf{(23)}$$

$$\sum \boldsymbol{e_N\,(k) = K_{\sum e} \sum e\,(k)} \quad \textbf{(24)}$$

Figure 2 shows the phase trajectory with matching fuzzy controller.

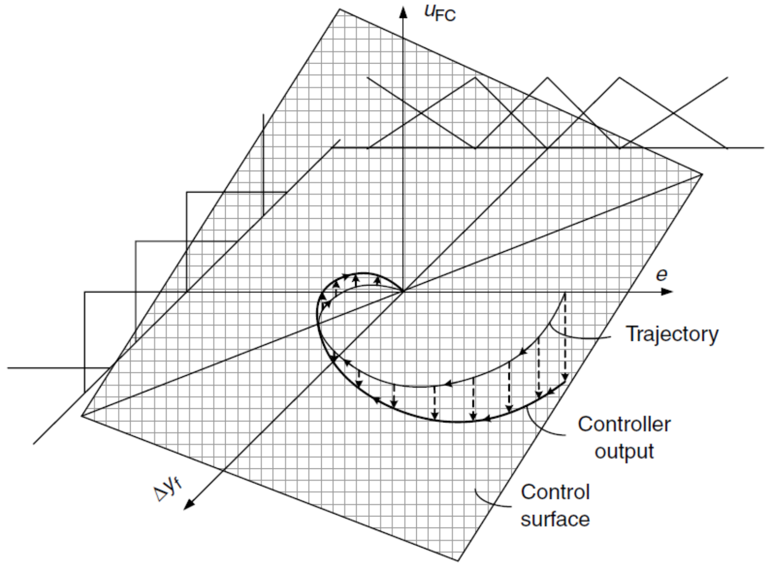


Figure 2. Phase Trajectory with Matching Fuzzy Controller Output

The normalized variable$\boldsymbol{e_N(k)}$ is thereafter converted into its fuzzy equivalent. The impact of input scaling factors on phase trajectories can be quite significant. We show in Table 2 two trajectories, the more shortened trajectory 1, obtained without scaling, and the more stretched trajectory 2, obtained by the scaling of inputs. Normalized trajectory 2 activates nine fuzzy rules, while trajectory 1 only three. Apparently, different input values trigger different fuzzy control rules, which eventually result in completely different controller output values. In general, it is much easier to achieve the desired control quality with a larger number of activated fuzzy rules. That is why the scaling of inputs should be done carefully so that we can use the full fuzzy rule base. Because of inadequate scaling, the fuzzy rule table may be imperfectly partitioned, which may cause many of the rules to remain inactive even though the rule base is complete. Figure 3 shows the DISO fuzzy controller.

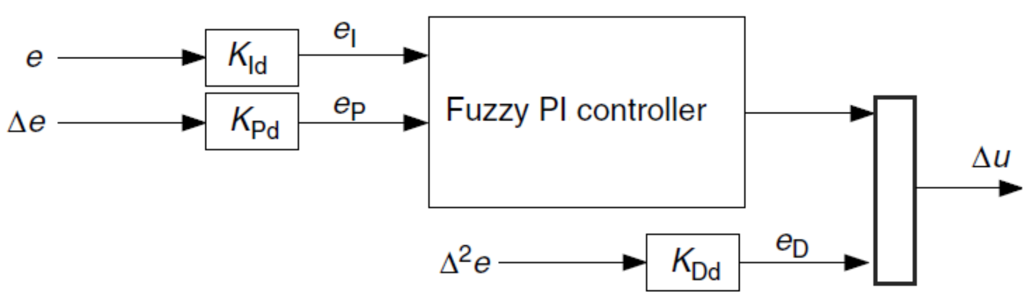


Figure 3. A DISO Fuzzy PI+D control

After reduce the chattering based on DISO fuzzy controller, the second SISO fuzzy controller is used to data tracking in uncertain condition. No matter how complicated the control of a plant may seem, the majority of control loops in industrial control systems utilize standard P, PI, PD, or PID control algorithms (here denoted as P-I-D) with fixed parameter values set during the commissioning. The synthesis of P-I-D controller parameters based on well-known design methods normally requires a mathematical model, which can precisely describe the dynamical behavior of a control object. Values of P-I-D controller parameters obtained in such a way describe a linear control law adequate for a selected operating point. If such a controller is applied to a nonlinear control system, the performance of the system will vary depending on the variations of control object parameters. Also, the usage of a linear control law will cause different responses of a nonlinear system for the same magnitude of positive and negative reference input changes.

Different design strategies have been developed with the purpose to overcome the disadvantages of linear P-I-D controllers. Such strategies transform a linear P-I-D controller into P-I-D-like structures of fuzzy controllers PI+D. When designing a fuzzy controller by emulating of a linear PI+D controller, we assume that the fuzzy controller should inherit the linear character of its model. In order to evaluate the quality of such a transformation, different measures of achieved linearity have been introduced. In terms of the influence that different fuzzy reasoning methods (fuzzy implications) have on the achieved linearity of PI+D-like fuzzy controllers, theoretical results show that the vast majority of fuzzy PI+D controller is actually nonlinear PI+D controller. In this research it has been proven that PI+D controllers are nonlinear PI+D controllers with P-gain, I-gain, and D-gain changing with the output of the controlled system, providing that they have five triangular input fuzzy sets for each input variable, fuzzy rules with a triangular in the consequent part, Zadeh's AND operator and the centroid defuzzifier. Although the making a perfect fuzzy copy of a linear PI+D controller could be an interesting design goal, it is more important to use a linear PI+D controller as a starting point for the initial setting of a fuzzy controller, because its prime role is not to mimic the original, but to use all of the original's intrinsic nonlinear control potential. This can be achieved through various adaptive and self-organizing (self-learning) design concepts. When a more general solution is wanted, then phase space and phase plane are utilized.

A PI+D controller has the following form in continuous time domain:

$$\boldsymbol{u}(\boldsymbol{t}) = \left(\boldsymbol{K_p e} + \boldsymbol{K_i} \sum \boldsymbol{e}\right) + \boldsymbol{K_D \dot{e}} \qquad \textbf{(25)}$$

Figure 4 shows a fuzzy PI+D controller composed of a linear PI+D controller and a SISO fuzzy controller with $e(k)$ and $e_F(k)$ as its input and output. If our goal is to minimize the number of rules, then we may use a very simple configuration of a fuzzy PI+D controller. This is a structure which contains a SISO fuzzy controller and a standard linear PI+D controller. The fuzzy controller has $e(k)$ as its input and $e_F(k)$ as its output. The number of input fuzzy sets l defines the total number of fuzzy rules. Providing that we are using triangular fuzzy sets, where only two adjacent sets are overlapping at output $e_F(k)$. The output $e_F(k)$ can be generated according to the COG principle. Then the output of the SISO fuzzy controller gets the form:

$$e_F(k) = \sum_{\boldsymbol{l=1}}^{\boldsymbol{M}} \boldsymbol{\theta^T \zeta(x)} \qquad \textbf{(26)}$$

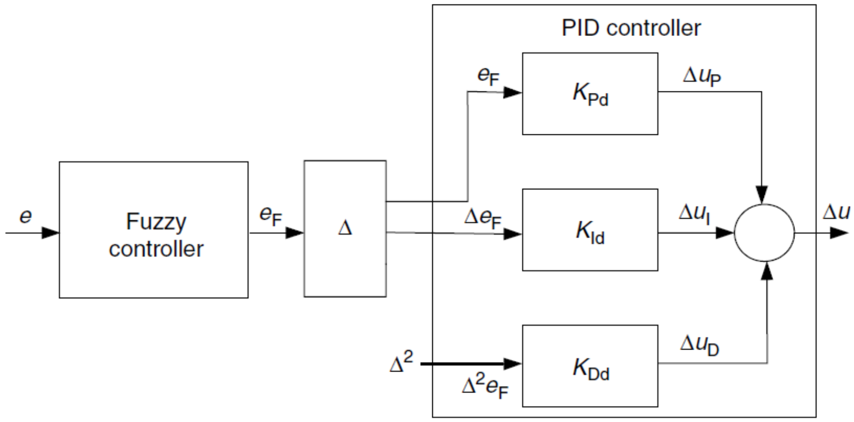


Figure 4. A Fuzzy PI+D Control

$\boldsymbol{\theta^T}$ is adjustable parameter and $\boldsymbol{\zeta}(\boldsymbol{x})$ is;

$$\boldsymbol{\zeta}(\boldsymbol{x}) = \frac{\sum_i \boldsymbol{\mu}(\boldsymbol{x_i})\boldsymbol{x_i}}{\sum_i \boldsymbol{\mu}(\boldsymbol{x_i})} \quad \textbf{(27)}$$

$\boldsymbol{\mu}(\boldsymbol{x_i})$ is membership function.

Figure 5 shows the membership function in SISO fuzzy control.

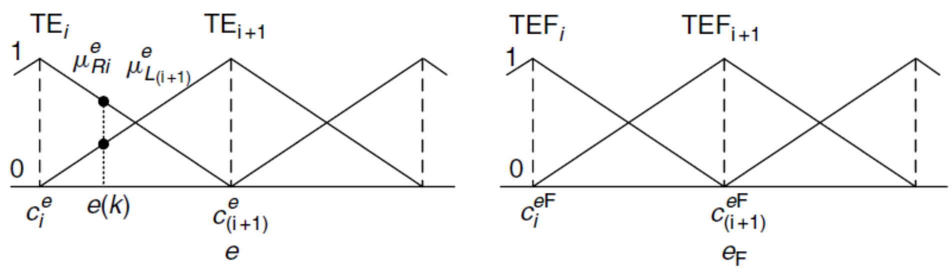


Figure 5. Membership Functions of a SISO Fuzzy Controller

Then the fuzzy rule table has only these three rules:

$$\boldsymbol{FR1}: IFe\ is\ N\ THEN\ e_F\ is\ N$$
$$\boldsymbol{FR2}: IFe\ is\ Z\ THEN\ e_F\ is\ Z$$
$$\boldsymbol{FR3}: IFe\ is\ P\ THEN\ e_F\ is\ P$$

We have achieved full compatibility of a fuzzy PI+D controller, but it would be much more effective to use a nonlinear potential of the SISO fuzzy controller.

4. Results and Discussion

The proposed $(PI + D)^2FSMC$ has been designed using MATLAB provided by *Mathworks* Company. This **M**atlab **S**oftware-**B**ased **C**ontroller (MSBC) is used to make a comparison with the proposed design.

Comparison between the Proposed $(PI + D)^2 FSMC$ and SMC

Data Tracking: $(PI + D)^2$ fuzzy logic sliding mode controller has been design using MSBC and sliding mode controller was designed using (MSBC), these controllers have been designed in the same project. The $(PI + D)^2$ fuzzy logic sliding mode controller has been designed by estimate and reduce the output fluctuations of the sliding mode controller. Hence, the proposed method has designed by using two blocks of $(PI + D)$ fuzzy logic sliding mode controller. Therefore, it is desirable to evaluate the behaviour of the proposed design with different controller. $(PI + D)^2$ fuzzy logic sliding mode controller and sliding mode controller subjected with step input have been used in unity feedback control system with second order plant to make a comparison with these two techniques, simulation environments have used in this comparison. The Mean differences between these two controllers are in chattering phenomenon in certain condition. Figure (6) shows the step response of the multi DOF joints controlled by $(PI + D)^2$ fuzzy logic sliding mode controller and sliding mode controller, it seem that the sliding mode has a steady state error (0.048) while the $(PI + D)^2$ fuzzy logic sliding mode controller is much close to zero, that mean the proposed method effects on both part of the plant output response (transient + steady state). While sliding mode controller effects on the transient response (reduce overshoot), without removing the error.

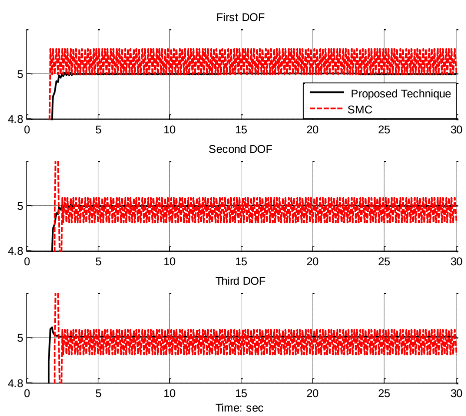


Fig 6: Data tracking SMC and Proposed control

In this nonlinear system, sliding mode controller has chattering in certain/uncertain situation. This challenge caused to motor vibration and oscillation.

Disturbance Rejection: the comparison with the sliding mode controlled system responses (Figure 7) shows significant increase of robustness due to usage of the proposed controller. If we look at amplitude and chattering characteristics of such a controller, the ability to keep an almost constant (near to zero) chattering in presence of uncertainty, can be interpreted as an achievement of a wide region of uncertainty. This suggests that the change of sliding surface slope gain could be sufficient to compensate for noticed changes of system dynamics. As can be seen, proposed method is more robust than conventional sliding mode controller; however this type of controller is a robust and stable.

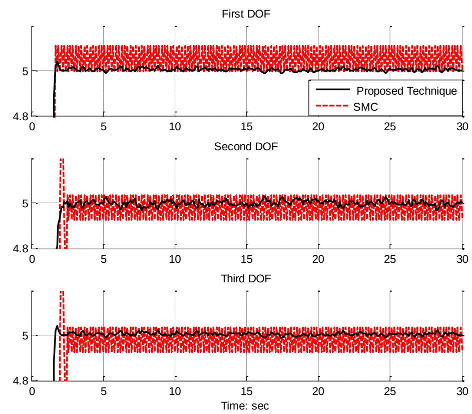


Figure 7. Disturbance Rejection: Proposed Method and SMC

Regarding to Figure 7; proposed method is more robust than conventional sliding mode controller. From a theoretical point of view, determination of any linear reference model is not a problem. From a practical point of view, conditions in the field may not always be in favor of easy assessment of process dynamics, so determination of the recursive equation coefficients of a higher-order reference model could be a problem. Therefore, the definition of reduced-order reference models is advised based on intelligent theory.

5. Conclusion

From a theoretical point of view, determination of any linear reference model is not a problem. From a practical point of view, conditions in the field may not always be in favor of easy assessment of process dynamics, so determination of the recursive equation coefficients of a higher-order reference model could be a problem. Therefore, the definition of reduced-order reference models is advised based on intelligent theory. In this research to reduce the chattering and improve the date tracking in certain and uncertain condition, $(PI + D)^2$ fuzzy logic sliding mode controller is introduced. This design divided into two main parts: reduce the chattering with respect to design DISO fuzzy sliding mode controller and improve the data tracking with respect to design SISO fuzzy sliding mode controller. Regarding to this method stability, robust and data error are improve compare to conventional sliding mode controller.

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Iranian center of Advance Science and Technology (IRAN SSP) is one of the independent research centers specializing in research and training across of Control and Automation, Electrical and Electronic Engineering, and Mechatronics & Robotics in Iran. At IRAN SSP research center, we are united and energized by one mission to discover and develop innovative engineering methodology that solve the most important challenges in field of advance science and technology. The IRAN SSP Center is instead to fill a long standing void in applied engineering by linking the training a development function one side and policy research on the other. This center divided into two main units:

- Education unit
- Research and Development unit

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Authors

Saman Rahbar, he has been working at "Design High Precision and Fast Dynamic Controller for Multi-Degrees of Freedom Actuator for Experimental Research and Education" project at Iranian Institute of Advance Science and Technology, Sanaat Kade Sabz Passargad Research Center (IRAN SSP) as "**research assistant**" of a research team composed of 21 researchers since Aug. 2013 to date.

Farzin Piltan, he was born on 1975, Shiraz, Iran. In 2004 he is jointed Institute of Advance Science and Technology, Research and Development Center, IRAN SSP. Now he is a dean of Intelligent Control and Robotics Lab. He is led of team (47 researchers) to design and build of nonlinear control of industrial robot manipulator for experimental research and education and published about 54 Papers in this field since 2010 to 2012, team supervisor and leader (9 researchers) to design and implement intelligent tuning the rate of fuel ratio in internal combustion engine for experimental research and education and published about 17 Journal papers since 2011 to 2013, team leader and advisor (34 researchers) of filtering the hand tremors in flexible surgical robot for experimental research and education and published about 31 journal papers in this field since 2012 to date, led of team (21 researchers) to design high precision and fast dynamic controller for multi-degrees of freedom actuator for experimental research and education and published about 7 journal papers in this field since 2013 to date, led of team (22 researchers) to research of full digital control for nonlinear systems (e.g., Industrial Robot Manipulator, IC Engine, Continuum Robot, and Spherical Motor) for experimental research and education and published about 4 journal papers in this field since 2010 to date and finally led of team (more than 130 researchers) to implementation of Project Based-Learning project at IRAN SSP research center for experimental research and education, and published more than 110 journal papers since 2010 to date. In addition to 7 textbooks, Farzin Piltan is the main author of more than 115 scientific papers in refereed journals. He is editorial review board member for 'international journal of control and automation (IJCA), Australia, ISSN: 2005-4297; 'International Journal of Intelligent System and Applications (IJISA)', Hong Kong, ISSN:2074-9058; 'IAES international journal of robotics and automation, Malaysia, ISSN:2089-4856; 'International Journal of Reconfigurable and Embedded Systems', Malaysia, ISSN:2089-4864. His current research interests are nonlinear control, artificial control system and applied to FPGA, robotics and artificial nonlinear control and IC engine modeling and control.

Ehsan Pouladi, he has been working at “Design High Precision and Fast Dynamic Controller for Multi-Degrees of Freedom Actuator for Experimental Research and Education” project at Iranian Institute of Advance Science and Technology, Sanaat Kade Sabz Passargad Research Center (IRAN SSP) as “**research assistant**“ of a research team composed of 21 researchers since Aug. 2013 to date.

Hossein Davarpanah, he has been working at “Design High Precision and Fast Dynamic Controller for Multi-Degrees of Freedom Actuator for Experimental Research and Education” project at Iranian Institute of Advance Science and Technology, Sanaat Kade Sabz Passargad Research Center (IRAN SSP) as “**research assistant**“ of a research team composed of 21 researchers since Aug. 2014 to date.

Somayeh Jowkar, she has been working at “DesignHigh Precision and Fast Dynamic Controller for Multi-Degrees of Freedom Actuator for Experimental Research and Education” project at Iranian Institute of Advance Science and Technology, Sanaat Kade Sabz Passargad Research Center (IRAN SSP) as“**research assistant**“of a research team composed of 21 researchers since Aug. 2014 to date.