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Signatures of Chaotic Naturein Astronomical Data with Nonlinear Analysis Techniques

Shuang Zhou[1, 2] and Yong Feng[1]

[1]*Chongqing Key Laboratory of Automated Reasoning and Cognition, Chongqing Institute of Green and Intelligent Technology, Chinese Academy of Sciences, Chongqing 400714, P.R. China*
[2]*University of Chinese Academy of Sciences, Beijing 100049, P.R. China*
E-mail:zhoushuang@cigit.ac.cn

Abstract

The significance of treating astronomical phenomenon as a chaotic system instead of a stochastic system for a better understanding of the underlying has been grown considerably in recent years. Unfortunately, the most important disadvantage of these techniques is the dependence on a single approach for identifying the chaotic nature and the variables involved. In this paper, an attempt is proposed to detect chaotic nature of astronomical time series using various nonlinear analysis techniques and prediction is also done to quantify the uncertainty involved. Mutual information function and Cao algorithm are used to determine the time delay and embedding dimension for the phase space reconstruction. The probability time and secular persistence or memory are calculated using maximal Lyapunov exponent and rescaled range analysis. Our analysis results indicate that dynamical behavior of astronomical process should be controlled by a low-dimensional chaoticattractor, its chaotic motion can be considered as a long-term persistence on large scales. Moreover, the nonlinear prediction method employed support the short- and mid-term predictability nature of astronomical time series. We can arrive at a conclusion that the reasonably good predictions obtained using a chaos theory is helpful for modeling and understanding the underlying dynamics of the complex astronomical system.

Keywords: *information processing, rescaled range analysis, phase space portrait, maximal Lyapunov exponent*

1. Introduction

It is well known that both chaos and fractal are ubiquitous phenomenon in nature, and they establish that complex nonlinear irregular and unpredictable behavior could be the fruit of an ordinary deterministic system under several dominant interdependent variables [1-3]. Moreover, they are applied to deterministic dynamical systems which are a periodic and exhibit sensitive dependence on initial conditions. During the last three decades, a great deal of studies employed these theories to distinguish, model, and forecast the dynamical behavior of various natural phenomena, such as geophysics, economics, meteorology, material science, ethology, astronomy, and so on [4-6]. The outstanding achievements of these studies are very encouraging, because they not only demonstrated that the nonlinear dynamics of irregular systems can be described from a chaotic point of view but also made it develop in to a useful technique or method to solve physical problems in many other science fields, in particular for solar-activity variations [7-9]. Understanding the dynamics of astronomical phenomena is an

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important subject that has attracted scientific interest due to many technological applications as well as due to its impact in space weather, Earth's environment, and human life [10-12]. In the majority of cases we have access only to many measurable quantities which strongly depend on the unknown dynamics of such real world systems, which seem to be stochastic may present a nonlinear deterministic and potentially chaotic behavior. To better identify such a behavior, it is timely to apply appropriate techniques based on theories of chaotic and fractal conceptions.

There is no doubt that the Sun exhibits complex non linear and non-stationary behavior. During the last two hundred years, the most important problem of a possible link between long-term solar activity and the Earth's environment has attracted considerable attentions [13-14]. Periodicities with many solar flares and bright faculae are consistent with periods with higher irradiance in the solar visual and ultraviolet electronic spectrum, which acts on the ozone level. Moreover, it is recognized that the galactic cosmic rays can also take as cloud condensation nuclei, which should be connected variations of cloud coverage with long-term solar activity, because when the solar activity is not very high, more cosmic rays penetrate the Earth's magnetic field in this so-called 11-years Schwa be cycle. The frequently used indicator of long-term solar activity is the sunspot numbers, which has taken as the most important data set that is used to represent secular solar activity [15-16]. A variety of advanced analysis techniques have been applied to investigate the characteristic features of dynamical systems based on the empirical time series. An interesting review about the detection of chaos in solar system and space weather phenomena, such as sunspot numbers, flare activity, solar wind velocity, can be found in the review of Panchev & Tsekov [17]. However, so far it has not been clear whether solar irregularity is chaotic or stochastic. In recent years, more and more results have been revealed about deterministic chaos of solar activity [18]. On the other hand, several authors found that there is no evidence to prove that the solar activity is governed by a chaotic low-dimensional process [19].

The present paper attempts to use a variety of techniques for characterizing the dynamical behavior of monthly sunspot numbers. More specifically, we attempt to identify the possible presence of chaotic system in this astronomical observational data. The techniques employed range from classical statistical methods which provide standard indications regarding the nonlinear dynamics of natural phenomenon to solar activity that could provide comprehensive characterization of the dynamics. The statistical method used here is rescaled range analysis, where as mutual information approach, phase space portrait, and maximal Lyapunov exponent are employed for comprehensive characterization. The organization of the present work is as follows: brief description on the observational data considered for investigation are given in Section 2, further the methods of analysis employed in this study are presented in Section 3, the analysis results and preliminary discussions are presented in Section 4.Finally,conclusionsdrawnfromthepresent paper are shown.

2. Observational Data

Sunspots are temporary phenomena in the photosphere of the Sun that appear visibly as dark spots compared to surrounding regions. They are caused by intense magnetic activity, which inhibits convection by an effect comparable to the eddy current brake, forming areas of reduced surface temperature. They usually appear as pairs, with each sunspot having the opposite magnetic pole to the other [20]. Although they are at temperatures of roughly 3000–4500K(2700–4200 C),the contrast with the surrounding material at about5780 K(5500 C)leaves them clearly visible as dark spots, as the luminous intensity of a heated black body(closely approximated by the photosphere) is a function of temperature to the fourth power. If the sunspot were isolated from the surrounding photosphere it would be brighter than the Moon. Sunspots expand and contract as they move across the surface of the Sun and can be as small as 16 kilometers and as large as 160,000 kilometers in diameter, making the larger ones visible from Earth without the aid of a telescope. They may also travel at relative speeds of a few hundred meters per second when they first emerge onto the solar photosphere. Manifesting intense magnetic activity, sunspots host secondary phenomena such as coronal loops (prominences) and reconnection events. Most solar flares and coronal mass ejections originate in magnetically active regions around visible sunspot groupings. Similar phenomena indirectly observed on stars are commonly called star-spot sand both light and dark spots have been measured.

Although the details of sunspot generation are still a matter of research in solar physics, it appears that sunspots are the visible counterparts of magnetic flux tubes in the Sun's convective zone that get "wound up" by differential rotation. If the stress on the tubes reaches a certain limit, they curl up like a rubber band and puncture the Sun's surface. Convection is inhibited at the puncture points; the energy flux from the Sun's interior decreases; and with it surface temperature. Sunspot populations quickly rise and more slowly fall on an irregular cycle of eleven years, although significant variations in the number of sunspots attending the 11-year period are known over longer spans of time [21-24]. For example, from 1900 to the 1960s, the solar maxima trend of sunspot count has been upward; from the 1960s to the present, it has diminished somewhat [25]. Over the last decades the Sun has had a markedly high average level of sunspot activity, it was last similarly active over 8000 years ago. The number of sunspots correlates with the intensity of solar radiation over the period since 1979, when satellite measurements of absolute radiative flux became available. Since sunspots are darker than the surrounding photosphere it might be expected that more sunspots would lead to less solar radiation and a decreased solar constant. However, the surrounding margins of sunspots are brighter than the average, and so are hotter. To sum up, more sunspots increase solar constant or brightness. The variation caused by the sunspot cycle to solar output is relatively small, on the order of 0.1% of the solar constant. Sunspots were rarely observed during the Maunder Minimum in the second part of the 17th century (approximately from 1645 to 1715).

In 1610, shortly after viewing the sun with his new telescope, Galileo Galilei made the first European observations of sunspots. Continuous daily observations were started at the Zurich Observatory in 1849 and earlier observations have been used to extend the records back to 1610. The sunspot number is calculated by first counting the number of sunspot groups and then the number of individual sun spots. Sunspot numbers are then given by the sum of the number of individual sunspots and ten times the number of groups. Since most sunspot groups have, on average, about ten spots, this formula for counting sunspots gives reliable numbers even when the observing conditions are less than ideal and small spots are hard to see. Monthly averages of the sunspot numbers show that the number of sunspots visible on the sun rises and falls with an approximate 11-year cycle [26-29]. There are actually at least two "official" sunspot numbers reported. The international sunspot numbers are compiled by the Solar Influences Data Analysis Center (SIDC) in Belgium, and another sunspot numbers are compiled by the America National Oceanic and Atmospheric Administration (NOAA).

In the present paper, the continuous time series of monthly sunspot numbers, which has been widely and frequently used to characterize the long-term solar activity, can be publicly downloaded from SIDC's website(http://sidc.oma.be/sunspot-data/).Due to this data series is noisy, thus before the phase space could be reconstructed, this data set must be smoothed. To eliminate the impact that large and active regions exert to sunspot numbers and keep the essential dynamics characteristic from this data set, we adopt a 13-point smoothed technique to smooth this data series. Both the original and smoothed kind of this data series cover the time interval from 1874 May to 2013 November, and the results are shown in Figure 1.

3. Methods of Analysis

Rescaled Range Analysis: The Hurst exponent for analyzing stochastic behavior of a certain time series is proposed by Hurst [30]. For a discrete time series *x(t)*, the Hurst exponent is calculated from the Hurst rescaled range through following steps: divide the time series into N intervals of length T; calculate the possible range R_k and the standard deviation S_k of each sub-period k; and then calculate the rescaled range $(R/S)_k$ and average the $(R/S)_k$ obtained for all the N sub-periods; finally repeating the above procedure with different sub-period length T, the evolution of with T is obtained. The Hurst exponent H is thus estimated with following function:

$$H = {d[\ln(R/S)]_T}\Big/{d[\ln(T)]}$$

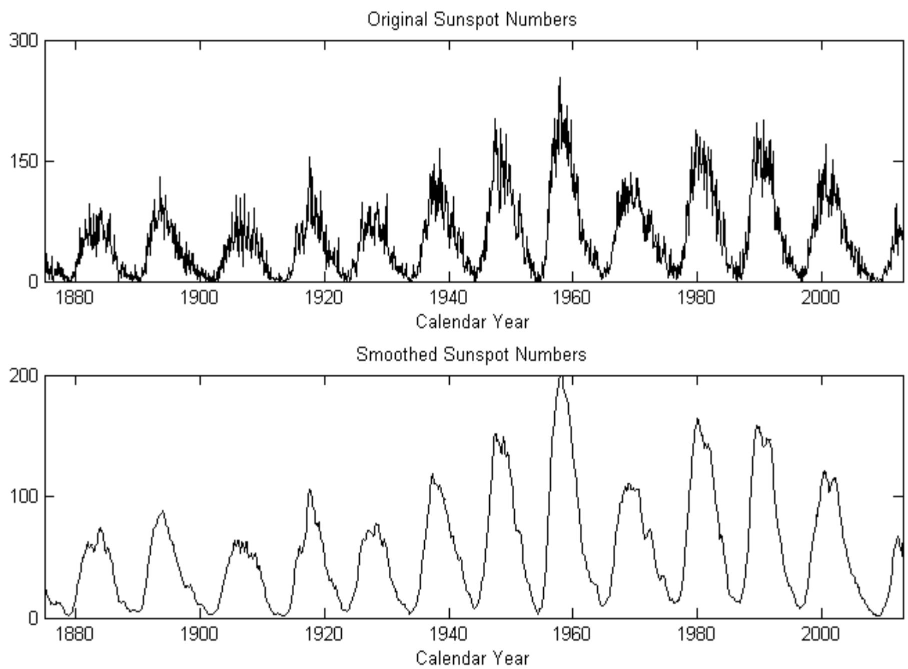

Figure 1. Plots of the Original (Upper Panel) and Smoothed (Lower Panel) Sunspot Numbers

The Hurst exponent is a measure of the secular memory of a time series. A Hurst exponent greater than 0.5 indicates that the time series exhibits a persistence behavior, namely, its trend whether increasing or decreasing will remain for a long period of time. A Hurst exponent of 0.5 implies that the behavior of the time series is random. A Hurst exponent less than 0.5

means that the time series exhibits an anti-persistence behavior, namely, its trend will likely reverse in the future.

Phase Space Portrait: If we want to investigate the complex dynamics of a time series, the first important step is to reconstruct an m-dimensional phase space [31]. For a certain time series *x (1), x(2), x(3),…, x(n)*, we can define the vector of the reconstructed phase space as follows:

$$y(t_i) = [\boldsymbol{x}(\boldsymbol{t_i}), \boldsymbol{x}(\boldsymbol{t_i} + \boldsymbol{\tau}), \ldots, \boldsymbol{x}(\boldsymbol{t_i} + (\boldsymbol{m} - \mathbf{1})\boldsymbol{\tau})]$$

Where *i=1, 2, 3, …, N, m* is the embedding dimension and τ is the time delay. Fraser & Swinney suggested a method named mutual information in 1986, it was confirmed that it is a tool for determining the reasonable value of τ [32]. Meanwhile, it differs from the traditional auto-correlation function, because this technique takes into account non-stationary correlations. Therefore, it is extensively applied to ascertain one of the most important variables in phase space from nonlinear and non-stationary time series. The mutual information between $S=\{s(t_1), s(t_2), \ldots, s(t_N)\}$ and $Q=\{q(t_1+\tau), q(t_2+\tau), \ldots, q(t_N+\tau)\}$ is the mean bits in this equation. We can predict the value of *S*by measurement from *Q*, and it can be expressed by:

$$I(S,Q) = H(Q) + H(S) - H(S,Q)$$

Where *H (Q)* is the entropy of *Q*, and *H(S)* is the entropy of *S*. *H(S, Q)* is the mutual-entropy between *S* and *Q*. In normally situation, the instantaneous value of the smallest mutual information is the optimal delay time of reconstructed phase space.

The embedding dimension of a time series can be calculated by the Cao algorithm [33-34]. For the time series *x(t)*, we should choose a proper embedding dimension m after selecting the suitable τ. The vector set $y_i(d)$ represents the *i*th vector acquired after we reconstruct the dimension *d*. If we define *a (i,d)*:

$$a(i,d) = \frac{\left\| y_i(d+1) - y_{n(i,d)}(d+1) \right\|}{\left\| y_i(d) - y_{n(i,d)}(d) \right\|}$$

Where $\| \ldots \|$ shows the certain value to Euclidean distance. After that, the mean value of *a(i,d)* can be defined as:

$$E(d) = \frac{1}{N - d\tau} \sum_{i=1}^{N-d\tau} a(i,d)$$

On the basis of this equation, the embedding dimension *E (d)* strongly depends on the dimension *d* and the time delay τ. Thus, to better investigate the dimension varies from *d* to *d*+1, we then define the following function:

$$E_1(d) = E(d+1)/E(d)$$

If a time series can be expressed by a certain attractor, and the *d* is larger than a given value of d_0, the $E_1(d)$ will not vary, and the obtained d should be the embedding dimension. In addition, Cao also denied another quantity $E_2(d)$ to distinguish fixed signal from random signal.

Maximal Lyapunov Exponent: The complex nonlinear system attracts to a stable periodic orbit or a fixed point, when the maximal Lyapunov exponent is not larger than zero.

Meanwhile, the modulus of the exponent shows the level of the stabilization [35]. At the same time, the dynamic system should be neutrally stable if the maximal Lyapunov exponent is equal to zero. Moreover, if the Lyapunov exponent is larger than zero, then we can define this system to be chaotic or unstable. That is to say, the maximal Lyapunov exponent can be considered as the most important Lyapunov exponent because it can be used to define the dynamic properties of a certain system [36]. Here, the Rosenstein method is very effective to calculate it. For each point of a certain phase spacey_j, we need to calculate its distance after i discrete time step of the adjacent. The nearest adjacent points in the phase space can be defined as:

$$d_j(i) = \min_i \left| y_{j+i} - y_{\hat{j}+i} \right|$$

For every i , calculate average value of ln $d_j(i)$:

$$y(i) = \frac{1}{q\Delta t} \sum_{j=1}^{q} \ln d_j(i)$$

Among which q is the nonzero number of $d_j(i)$. Define fitting curve by the famous least square technique, and then the biggest Lyapunov exponent is indicated as the slop of this fitting curve.

4. Results and Discussions

The rescaled range analysis is applied to the smoothed monthly-mean of SNs, and its Hurst exponent is shown in Figure 2. On the basis of such analysis, the Hurst exponent is estimated by the slope of the best fitting straight line of plot log$(R/S)_n$ against log n We can see that the Hurst exponent of SNs is 0.8045. As this value of sunspot time series considered here is between 0.5 and 1, namely, the solar activity is a long-term persistence, which exhibits strongly fractal Brownian motion [37].

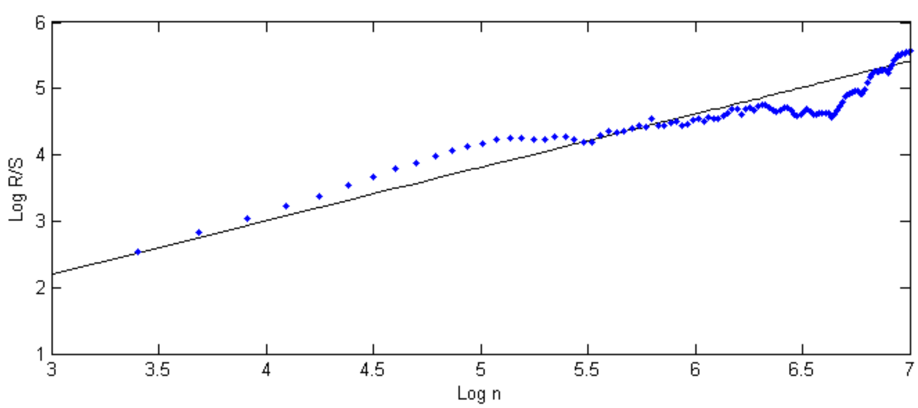

Figure 2. Rescaled Range Analysis for Smoothed Monthly Sunspot Numbers

Figure 3 shows the graph of mutual information of SNs as a function of time delay. As pointed out by Fraser & Swinney (1986), there a sonable value of the time delay is the value when the mutual information exhibits a marked first minimum. From this figure one can see that the reasonable value of time lag for SNs is 38 for SNs. We apply the mathematical algorithm written by Cao to estimate the embedding dimension of SNs, and the result is displaced in Figure 4. The idea is to increase the dimension d of the phase space up to the point. It is based on the fact that choosing too low embedding dimension resulting in points that are far part in the original phase space being moved closer together in the reconstruction phase space. But to avoid the choice of a threshold to decide whether a neighbor is false or not, Cao used relative change in the average distance between two neighboring points in R_d when the dimension is increased from d to d+1. When $E_1(d)$stopping changing, the embedding dimension of SNs is equal to three, what's more, the time series of the SNs is a fixed signal, because $E_2(d)$ doesn't always equal to 1. Moreover, we can obtain that the curves of $E_2(d)$ for SNs is not a straight line, implying that the secular solar activity exhibits the dynamical properties of a low-dimensional deterministic chaos. Figure 5 shows the reconstructed phase space by applying the time delay and embedding dimension from SNs. From this figure one can easily see that the phase space is totally unfolded, namely, it do not looks like a "ball of wool". In this figure, we can clearly see the infinite self-similarity which characterizes the stranger attractor, implying that the solar activity is a typical characteristic of a chaotic system [38]. We would like to point out that smoothing process can be helpful for recovering dynamic structure of a time series, and not injecting non-stationary behavior into the smoothed data.

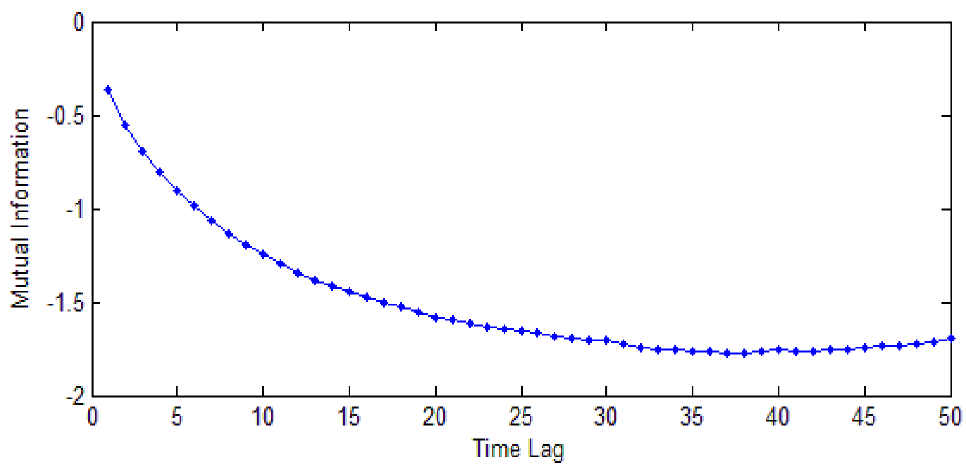

Figure 3. Mutual Information as a Function of Time Lag for Smoothed Sunspot Numbers

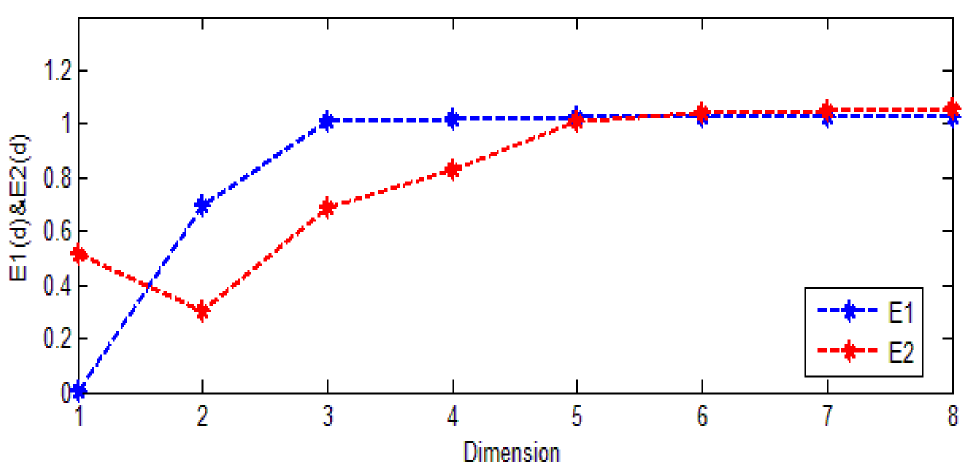

Figure 4. Variables of $E_1(d)$ and $E_2(d)$as a Function of Dimension for SNs

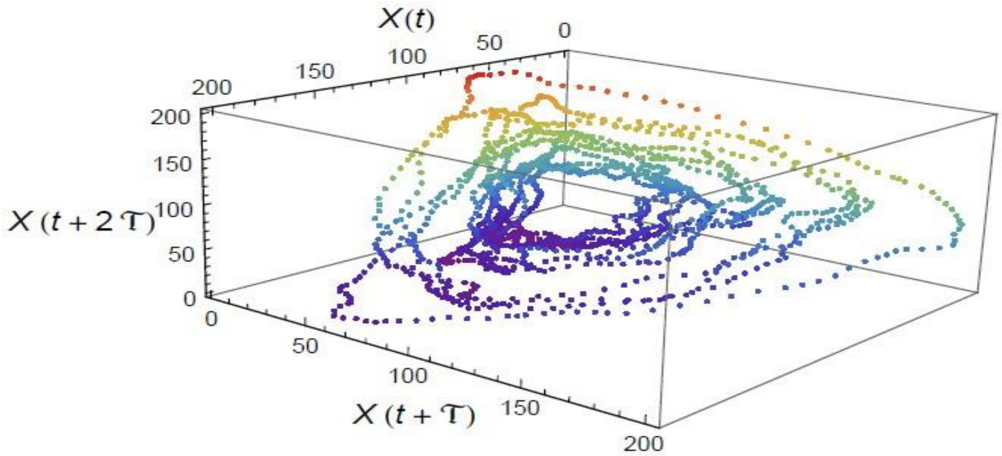

Figure 5. Reconstructed Phase Space with Estimated Parameters for SNs

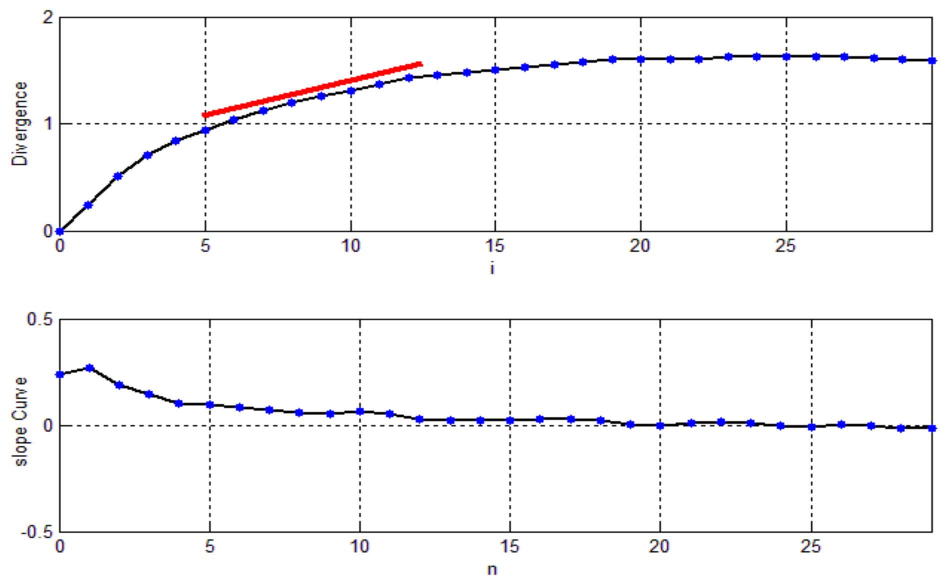

Figure 6. Divergence and Slope as a Function of Time Point for SNs

Figure 6 displays the divergence and the slope as a function of time point for SNs. As pointed out by Rosenstein, the slope of the red line showed in this Figure corresponding to theoretical value of the maximal Lyapunov exponent. Select the reasonable interval of nonzero value in slope curve graph to do straight line fitting, and the reliable value of the maximal Lyapunov exponent is 0.0248/month for SNs [39]. The analysis result also indicates that the secular solar activity should be a chaotic dynamic. We know that the strange attractor has a length which can be described as the Lyapunov time or predictability time [40]. If the divinable time exceeding the Lyapunov time, the dynamical state of the system ceases to be predictable [41]. We estimate that the Lyapunov time is equal to about 3.3602 years (1 month/0.0248).Therefore, we conclude that solar-activity forecast can be predicted for a short- to mid-term, due to its intrinsic complexity.

5. Conclusions

Chaos theory has opened new windows in scientific fields and is already considered by many researchers as one of the most important discovery in last century after relativity and quantum mechanics [42]. Deterministic chaos indicates the irregular or chaotic motion that is generated by nonlinear and non-stationary systems whose dynamical laws uniquely determine the temporal evolution of the system situation from previous history. It has confirmed that chaotic phenomena are abundant in nature and have far reaching consequences in many branches of sciences [43-44].

This paper describes a sequence of nonlinear analysis techniques, including rescaled range analysis, mutual information function, Cao method, phase space portrait, and maximal Lyapunov exponent, for discerning and investigating the chaotic behavior of long-term solar activity represented by time series of sunspot numbers. To decrease the noise signal and to preserve the secular dynamics of this complex non-stationary system, we first filtered the monthly sunspot numbers by a 13-point smoothed method. Subsequently, we applied the filtered data set to reconstruct the phase space and to calculate several important values, namely, the time delay, the embedding dimension and the maximal Lyapunov exponent. From the analysis results, we arrive at a conclusion that the dynamical behavior of SNs is a low-dimensional chaotic attractor and the solar activity is a chaotic phenomenon but not a stochastic behavior. The analysis results have shown that the chaotic characteristics obviously exist in solar activity due to the finite embedding dimension (3) and the maximal Lyapunov exponent (0.0248). Furthermore, the Hurst exponent of solar activity is larger than 0.5, implying that the chaotic motion of solar activity can be considered as a long-term persistence on large scales. The nonlinear prediction method employed support the short- and mid-term predictability nature of solar time series. We can arrive at a conclusion that the reasonably good predictions obtained using a chaos theory affirm the suitability of a chaotic approach for modeling and understanding the underlying dynamics of the complex astronomical process [45].

Acknowledgements

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Authors

Shuang Zhou, received his Bachelor of Science in Department of Mathematics fromTonghua Normal College, Tonghua City, in June of 2006 & Master of Science from Guangxi Normal University, Guilin City, in June of 2009. He is a Ph.D candidate in Chongqing Institute of Greenand Intelligent Technology, Chinese Academy of Sciences in Chongqing City. His research interests are in the areas of digital signal processing, and data mining technique.

YongFeng, he is a professor at the Chongqing Institute of Green and Intelligent Technology, Chinese Academy of Sciences. His research interests include symbolic-numeric computation and finite element method.