Special Net Structure and its Application in Workflow Modeling

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Abstract

On the basis of the traditional net structure, a new concept of special net structure (SNS) is defined and its main characteristics are summarized. The SNS is a concolorous graph with colorless nodes and concolorous edges, which can be described by polychromatic sets theory formally and stored by Hash table in computer programming to reduce the storage space and diminish the search region. Furthermore, the SNS is applied to workflow modeling, and the basic model primitives, typical model constructs and typical conflict constructs are presented. Finally, the verification algorithm of workflow SNS model is advanced. Together with a case of service process, a prototype system of workflow modeling based on SNS is developed to illustrate the correctness and feasibility of the proposed approach.

Keywords: *special net structure; polychromatic sets; workflow modeling; verification algorithm*

1. Introduction

Net structure or graph structure comes from some difficult mathematical game, and is a subject with rapid development and wide application. In 1847, Kirchhoff used graph theory to analyze circuit network, which was a pioneer engineering application. With the development of science and technology, the graph theory has an outstanding advantage in solving the problems of operational research, network theory, information theory, control theory, game theory and computer science [1]. Net structure or graph structure has aroused the special attention of mathematics and engineering fields.

The modern enterprises are confronting many challenges from unprecedented and abrupt changes, including economic globalization, saturated market and rapid IT progress [2]. To keep their competitive advantages, more and more enterprises are adopting workflow technology to improve their production management and after service [3-6]. The workflow process model is the core of the workflow management systems [7-9]. Net structure or graph structure, such as Petri net, has been used to model workflow process. In nature, Petri net is a kind of graphical and formal modeling tool and is suitable for describing systems with concurrency and asynchrony [7, 10, 11]. However, it has some weaknesses for modeling workflow process [12]:

● The business process defined by Petri net is difficult to understand by a non-specialist.

● Petri net is inconvenient to represent data flow.

Polychromatic sets theory (PST) is a relatively new mathematics theory originally developed by the Russian scientist V.V. Pavlov [13]. PST has two important concepts: polychromatic sets (PS) and polychromatic graph (PG). The nodes and edges of PG all form polychromatic sets. The key idea of PST is to utilize a standardized mathematical model to simulate different objects [14, 15]. It has a significant advantage in problem formalization and has been applied to business process reengineering and workflow modeling [16, 17].

Based on the traditional net structure, a new concept of special net structure (SNS) was defined. The PST and Hash table are used to research the logic structure (formal description), physical structure (storage method) and verification algorithm of SNS. A prototype system of workflow modeling based on SNS is developed and a case is provided to illustrate the correctness and feasibility of the proposed approach.

2. Special Net Structure and its Features

The traditional net structure, also called graph structure, can be described as follows:

$$G = (A, C) \tag{1}$$

Where, A is called the set of nodes and C is called the set of edges. However, the traditional net structure does not describe the different characteristics of nodes and edges, but expresses their components. These shortcomings restrict the application range of traditional net structure in complex systems modeling.

In the engineering practice, a special net structure (SNS) is often used, which includes the same two types elements, nodes an edges, as the traditional net structure, and has some unique features as follows:

● SNS has and only has a start node and an end node. In the traditional net structure, the nodes are coequal, and their differences exist in their names while other different characteristics cannot be represented formally. However, SNS has and only has two special nodes: start node and end node. The former has next node(s) and has not prior node(s), while the latter has prior node(s) and has not next node(s).

● In SNS, any node must exist in a path from start node to end node. In the traditional net structure, the isolated node is allowable, which is not adjacent to any node. However, SNS does not include node without prior node(s), node without next node(s) or isolated node without prior node(s) and next node(s).

● In SNS, the relations of nodes are complex but their types are determined. In the traditional net structure, the relations of nodes are arbitrary, but their types are not distinguished. However, in SNS, the relations of nodes are arbitrary too, but their types are determined. That is to say, the SNS not only describes whether the interrelation exists in any two nodes, but also represents a connected kind of interrelation existing in any two connected nodes.

● In SNS, several nodes can constitute a basic model construct based on some rules, and several basic model constructs can nest each other into a more complicated net structure.

Because of these unique features, the special net structure has stronger simulation and operation abilities to verify the logical structure of described system, such as workflow process.

3. Formal Expression of Special Net Structure

3.1. Polychromatic Sets and Polychromatic Graph

Generally speaking, PG includes three constituents as follows:

$$PG = (F(G), PS_A, PS_C) \tag{2}$$

PS of nodes PS_A can be described as follows:

$$PS_A = (A, F(a), F(A), [A\times F(a)], [A\times F(A)], [A\times A\ (F)]) \tag{3}$$

Where, A is node set, $F(a)$ is node individual pigmentation, $F(A)$ is node unified pigmentation, $[A\times F(a)]$ represents the relation of nodes and individual pigmentation, $[A\times F(A)]$ represents the relation of nodes and unified pigmentation, and $[A\times A\ (F)]$ represents the relation of nodes and entity of unified pigmentation.

PS of edges PS_C can be described as follows:

$$PS_C = (C, F(c), F(C), [C\times F(c)], [C\times F(C)], [C\times C\ (F)]) \tag{4}$$

Where, C is edge set, and other constituents are same as PS of nodes PS_A.

$F(G)$ is unified pigmentation of PG. If the nodes of PG are colorless, $F(G) = F(C)$; If the edges of PG are colorless, $F(G) = F(A)$; If the nodes and edges of PG are all colorless, $F(G) = \phi$. This is to say, the general graph is a special polychromatic graph, which can be described by Equation (1).

If any node and/or edge just can be pigmented a color, this graph is called concolorous graph, which has concolorous node and/or edge and is a special polychromatic graph too.

In simulating the real system or object, the concept of contour in PS and PG displaces the mathematical terminology, color, to represent the abstracts of properties, attributes, parameters, characteristics and indexes etc. More detail on polychromatic sets theory can be found in the literature [18].

3.2. Formal Expression of SN*S*

By using prior node and next node, the nodes of SNS can be classified into start node, end node and activity node. From the compositions of SNS, every node itself need not be pigmented any color. The edges of SNS not only describe whether the interrelation exists in any two nodes, but also can be pigmented a certain color to represent a connected kind of interrelation existing in any two connected nodes. In fact, the SNS is a concolorous graph with colorless nodes and concolorous edges, and also is a special polychromatic graph, which can be described by polychromatic sets theory formally.

According to Equations (2)-(4), the unified pigmentation of SNS, PS of nodes and PS of edges can be described as follows:

$$F(G) = F(C) \tag{5}$$

$$PS_A = A \tag{6}$$

$$PS_C = (C, F(C), [C\times F(C)]) \tag{7}$$

Therefore, SNS can be described formally in polychromatic graph as follows:

$$PG_{SNS} = (A, C, F(C), [C\times F(C)]) \tag{8}$$

Where, A is node set of SNS, written as

$$A = \{a_i \mid 1 \le i \le n\} \tag{9}$$

C is edge set of SNS, written as

$$C = \{c_{i,j}, \mid 1 \le i, j \le n\} \tag{10}$$

In the practical operation, the edge set of SNS is computed by using Boolean matrix [A×A] as follows:

$$[A \times A] = \begin{array}{c} \begin{matrix} a_1 & \cdots & a_j & \cdots & a_n \end{matrix} \\ \begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{n1} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \end{array} \begin{matrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{matrix} \tag{11}$$

All elements c_{ij} = 1 constitute a set, which is the C, edge set of SNS.

$F(C)$ is edge unified pigmentation of SNS, which represents a set of interrelation types existing in any two connected nodes and can be written as:

$$F(C) = \{F_k \mid 1 \le k \le m\} \tag{12}$$

C×$F(C)$] represents the relation of edges and unified pigmentation in matrix, written as:

$$[C \times F(C)] = \begin{array}{c} \begin{matrix} F_1 & \cdots & F_k & \cdots & F_m \end{matrix} \\ \begin{bmatrix} r_{11} & \cdots & r_{1k} & \cdots & r_{1m} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ r_{i1} & \cdots & r_{ik} & \cdots & r_{im} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ r_{n1} & \cdots & r_{nk} & \cdots & r_{nm} \end{bmatrix} \end{array} \begin{matrix} c_{1,2} \\ \vdots \\ c_{i,j} \\ \vdots \\ c_{n-1,n} \end{matrix} \tag{13}$$

In the Boolean matrix [C×$F(C)$], if $c_{i,j}$, the edge of SNS, is pigmented a color F_k, a certain interrelation type, the Boolean element r_{ik} is 1.

According to Equations (9)-(13), SNS can be described further in polychromatic graph as follows:

$$PG_{SNS} = [(A \times A) \times F(C)] \tag{14}$$

Where, (A×A) is the Cartesian product of the nodes set A and itself. F_k is the abbreviation for $F_k(a_i, a_j)$. A logic sum of $F_k(a_i, a_j)$ is defined by a Boolean variable as

$$F(a_i, a_j) = \bigvee_{k=1}^{m} F_k(a_i, a_j) \tag{15}$$

The expression of SNS in PS is shown in Figure 1.

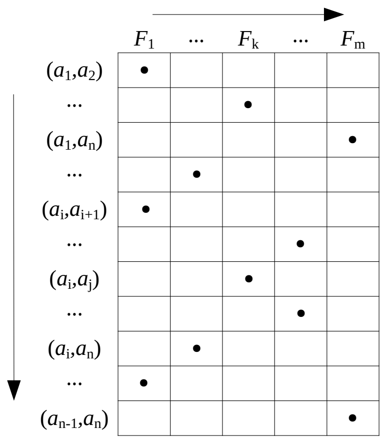


Figure 1. Expression of SNS in PS

4. Storage Method of Special Net Structure

In Figure 1, the SNS is an infinite 2-dimension table with the increase of node amount n and interrelation type m, and can be used to simulate complex system or object. However, the storage method of SNS in computer programming is an important problem. The simplest method is that 2-array is used to store this 2-dimension table. In the case of small scale problem, this method is feasible and acceptable. However, with the enhancement of problem scale, the node amount n and interrelation type m will increase sharply, and lead to two serious problems:

● The bigger storage space in computer programming;

● The bigger search region when structure is verified.

Actually, just only little information is useful while a lot of information is useless in the 2-dimension table shown in Figure 1. The useful information needs to store in computer programming. Based on this opinion, this paper proposes a storage method of *Hashtable* to store SNS. This is an economic storage form.

Hashtable is a set storing pairs of key/value. The pairs of key/value are organized in Hash code and every element is a pair of key/value stored in objects of *DictionaryEntry*. The key can not be an empty reference while the value can. In C# language, there are two common construct functions:

● Using default original capability, load gene, hash code providing program and comparer to initialize a new and empty instance. The syntax format is as follows:

public Hashtable ( );

● Using appointed original capability, default load gene, default hash code providing program and default comparer to initialize a new and empty instance. The syntax format is as follows:

public Hashtable (int capacity);

Where, capacity is an approximate element number of Hahtable object.

Using *Hashtable* to store SNS in computer programming, the useful information is reserved and the bigger storage space is solved. Further more, *Hashtable* possesses a lot of attributes, such as Count, Keys and Values, and a lot of methods, such as Add, Clear, Remove and Contains, which are fit for the reduction and verification of SNS. For example, if an interrelation type "F_k" is added between the node a_i and node a_j, a sentence is enough as follows:

hashtable.Add("a_i,a_j", "F_k");

More detail on programming methods of *Hashtable* can be found in the literature [19].

5. Application Special Net Structure into Workflow Modeling

The proposed special net structure has a lot of engineering applications. In this paper, the special net structure is applied into workflow modeling and a workflow SNS model is proposed.

5.1. Basic Model Primitives

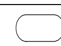
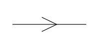
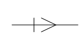
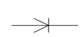
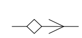
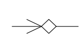
In the workflow SNS model, basic model primitives have two kinds:

● *Nodes*: start node, end node and activity node. All nodes constitute a set *A*.

● *Interrelations*: link-arc, and-split, and-join, or-split and or-join, represented by a set as $F(C) = (F_1, F_2, F_3, F_4, F_5)$.

Basic model primitives of workflow SNS model are shown in Table 1.

Table 1. Graphical Symbols of Basic Model Primitives

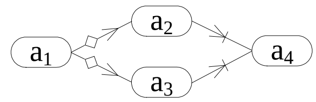
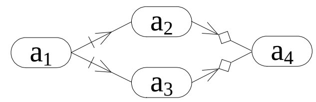
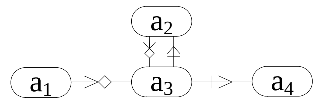
Primitive type	Basic primitive	Graphical symbol
Nodes	Start node	
	End node	
	Activity node	
Interrelations	Link-arc	
	And-split	
	And-join	
	Or-split	
	Or-join	

5.2. Typical Model Constructs and Conflict Constructs

Applying the above basic model primitives of workflow SNS model to describe the interrelations of nodes, four typical model constructs can be gained: sequential construct, parallel construct, choice construct and iterative construct [17]. These typical model constructs not only accord with the definitions proposed by WfMC, but also with standard process control structures proposed by CIM-OSA [20].

The workflow SNS model must be defined in most workflow modeling paradigms. However, the conflict constructs are inevitable in complex workflow SNS model. Table 2 shows three typical conflict constructs.

Table 2. Graphical Expression of Typical Conflict Constructs

Conflict constructs	Graphical expressions
Deadlock	
Lack of synchronization	
Infinite cycle (livelock)	

A deadlock refers to a situation in which a workflow instance gets into a stalemate such that no activity can be executed. This conflict construct results from the dissatisfied interrelation *and-join*. For example, *and-join* is used to join *or-split*. In this case, the Boolean expression is $F_4(a_1, a_2) \wedge F_3(a_2, a_4) \wedge F_4(a_1, a_3) \wedge F_3(a_3, a_4)=1$.

Lack of synchronization refers to a situation in which the *and-split* are joined by an *or-join*, resulting in unintentional multiple executions of a_4. The Boolean expression is $F_2(a_1, a_2) \wedge F_5(a_2, a_4) \wedge F_2(a_1, a_3) \wedge F_5(a_3, a_4)=1$.

An infinite cycle (livelock) is a conflict construct that causes come activities in a workflow model to be repeatedly executed forever. For instance, table II illustrates a deterministic infinite cycle in which after a_3 is executed, both a_2 and a_4 are executed. The execution of a_2 triggers a_3 and itself again, thus causing an infinite cycle. The Boolean expression is $F_5(a_1, a_3) \wedge F_2(a_3, a_4) \wedge F_2(a_3, a_2) \wedge F_5(a_2, a_3)=1$.

5.3. Verification Algorithm of Workflow SNS Model

In order to find the conflict constructs, the workflow SNS model is needed to reduce. The workflow SNS model without conflict constructs can only be reduced completely, whose reduction result is that start node joins by a virtual node with end node.

For the simple workflow SNS model, the conflict constructs are easy to be found and eliminated. However, for the complex workflow SNS model, it is necessary to establish the reduction rules based on PG to find and eliminate conflict constructs.

The verification algorithm of workflow SNS model is shown in Figure 2.

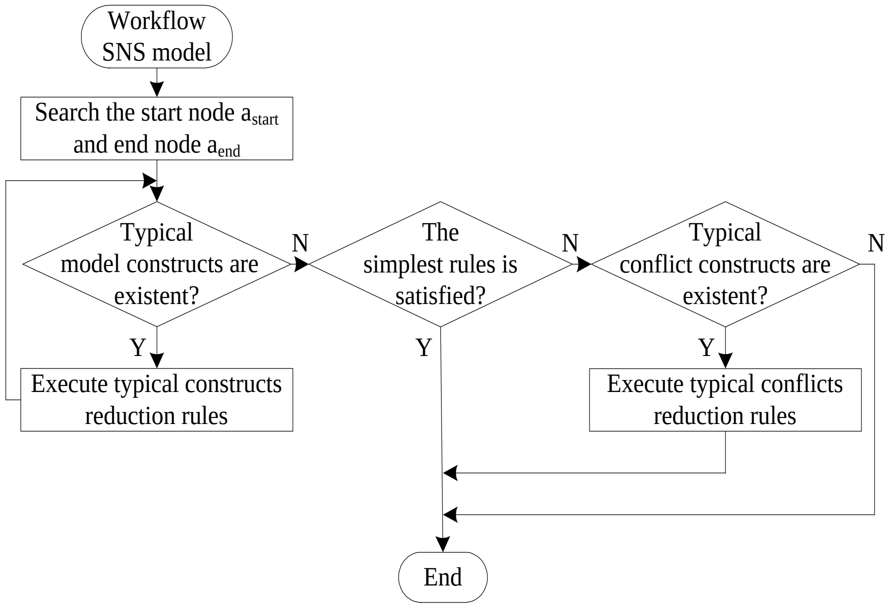


Figure 2. Verification Algorithm of Workflow SNS Model

The verification algorithm pseudo-codes of workflow SNS model are listed as follows:

BEGIN

// PG_{SNS} is workflow SNS model, a_{start} and a_{end} are start node and end node, and C_i and R_i (i=1,2,…,7) are reduction conditions and reduction rules of sequential, parallel, choice, iterative, deadlock, lack of synchronization and infinite cycle (livelock).

```
Initialize PG_SNS; a_start; a_end
WHILE (TRUE)
{
    WHILE C_1 DO R_1
    WHILE C_2 DO R_2
    WHILE C_3 DO R_3
    WHILE C_4 DO R_4
    IF F_1(a_start,V)∧F_1(V,a_end)=1
        BREAK
    ELSE IF C_5
        R_5, BREAK
    ELSE IF C_6
        R_6, BREAK
    ELSE IF C_7
        R_7, BREAK
    ELSE
        BREAK
}
END
```

6. A Case

The service-oriented manufacturing is an important development trend of manufacturing mode and a lot of manufacturing enterprises are paying attention to improve the after service level.

Figure 3 shows a business process of the after service. It includes 16 nodes. The nodes are explained in Table 3.

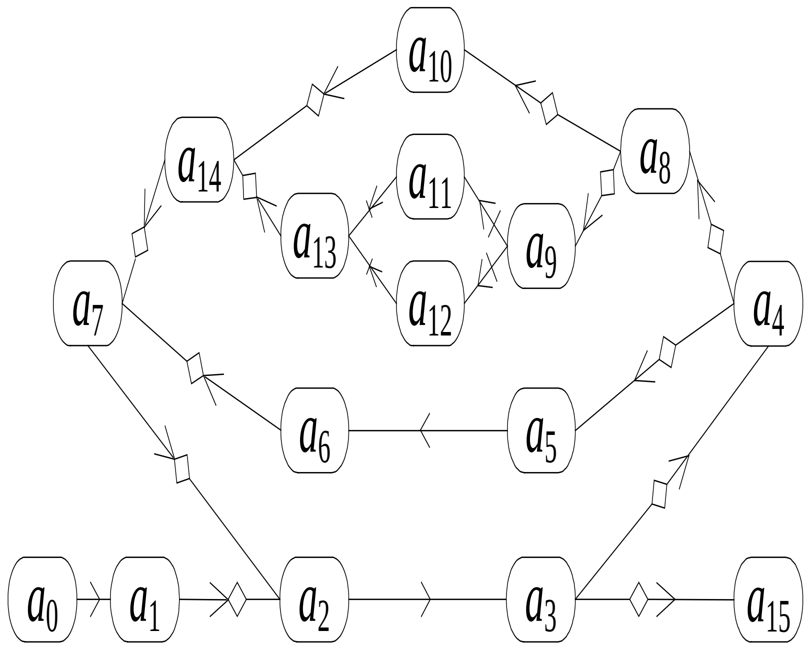


Figure 3. Business Process of After Service

Table 3. Explanation of Nodes

Node	Explanation	Node	Explanation	Node	Explanation
a_0	Start node	a_6	Returning processing report	a_{12}	Maintenance by self
a_1	Product marketing	a_7	Virtual node	a_{13}	Virtual node
a_2	Deliver to customer	a_8	Determining service type	a_{14}	Deliver to service department
a_3	Feedback information by customer	a_9	Filling in maintenance card	a_{15}	End node
a_4	Filling in service card	a_{10}	Filling in exchange card		
a_5	Rejecting requisition	a_{11}	Maintenance by other enterprise		

According to the business process of the service-oriented manufacturing enterprise, the workflow SNS model is established by using developed prototype system, as shown in Figure 4.

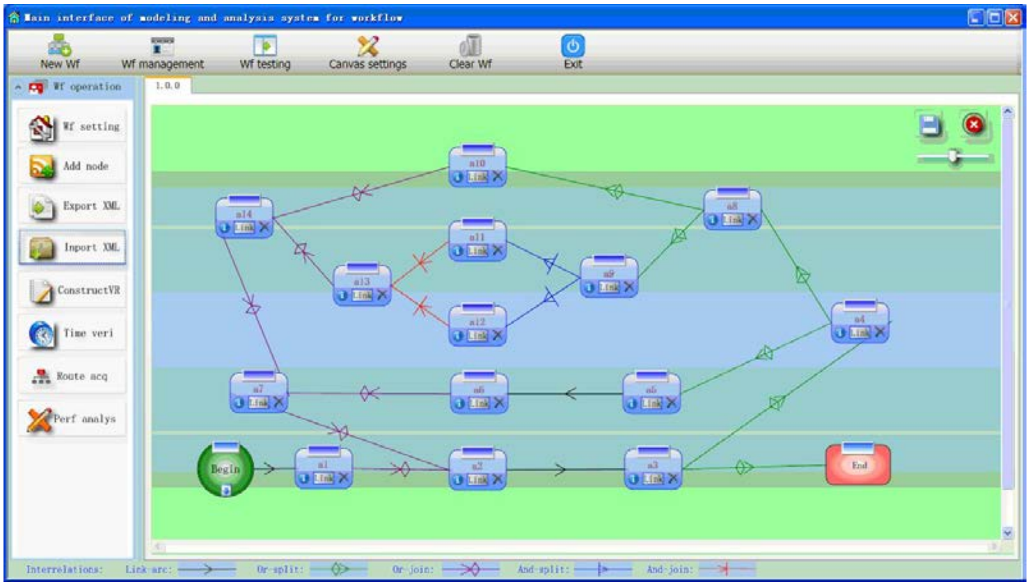


Figure 4. Workflow SNS Model of a Case

The verification algorithm is executed to verify the correctness of the established workflow SNS model and the verification results are shown in Figure 5.

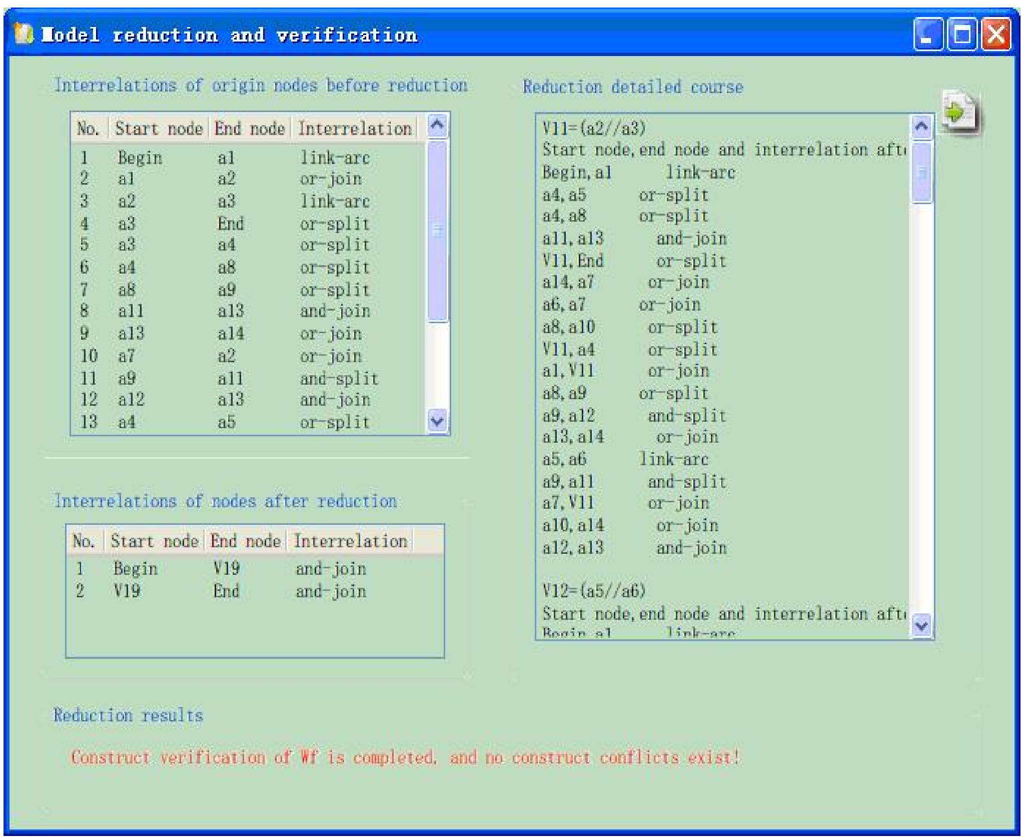


Figure 5. Verification Results of a Case

7. Conclusions

Net structure or graph structure has a wide application in engineering field. This paper proposes a special net structure (SNS) for workflow modeling. Based on polychromatic sets theory, the logical structure of SNS is described formally. The Hash table is used as a physical storage method of SNS in computer programming in order to reduce storage space, diminish the search region and decrease complexity of reduction algorithm The special net structure was applied to workflow modeling, and the verification algorithm of workflow special net structure model was proposed. The proposed SNS provides a new idea of workflow modeling and can be applied into other engineering fields.

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