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AFS Theory based Integrated Multiple Criteria Decision Making Model for Transportation Mode Selection

Sunil Pratap Singh[1*] and Preetvanti Singh[2]

[1]*Bharati Vidyapeeth's Institute of Computer Applications and Management, India*
[2]*Dayalbagh Educational Institute (Deemed University), India*
[1]*sunil_pratap@rediffmail.com*, [2]*preetvantisingh@gmail.com*

Abstract

This paper presents the development of a novel hybrid decision support model aimed to solve the complex problem of transportation mode selection in multiple criteria decision making environment. The proposed model integrates Analytic Hierarchy Process (AHP) to determine relative importance of transportation mode selection attributes; Axiomatic Fuzzy Set (AFS) theory for description of decision alternatives; and Simple Additive Weighting (SAW) technique to obtain the final ranking of alternatives. The axiomatic fuzzy logic is incorporated into the model to overcome the uncertainty and ambiguity in human decision making process. The main advantage of the proposed model is that it processes the linguistic values using axiomatic fuzzy logic that overcomes the ambiguity in human decision making process and copes with the inconsistency caused by different types of fuzzy numbers and normalization methods. The proposed methodology is illustrated by considering a case of transportation mode selection. A comparative analysis with an established technique shows the effectiveness and validity of the proposed model.

Keywords: *Hybrid Method, Transportation Mode Selection, Travel Mode, Multiple Criteria Decision Making, Axiomatic Fuzzy Set (AFS) Theory, Simple Additive Weighting (SAW)*

1. Introduction

In everyday life, people face decision situations in their professional as well as private lives. In general, the decision-making process involves a sequence of logical steps to resolve a complex problem. The process begins with recognizing the decision problem and ends with determining a best alternative from among multiple alternatives for the purpose of attaining a goal or goals [6].

The multiple criteria decision making (MCDM) is one of the most well-known branch of decision making that deals with the complex problems under the presence of a set of decision criteria. The term MADM (multiple attribute decision making) is used interchangeably with MCDM. The development of MCDM techniques and their applications in solving complex decision problems have shown a remarkable growth in last few years [3].

Generally, a MCDM problem can be concisely expressed in matrix form as:

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* Corresponding Author

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$$
\begin{array}{c|cccc|}
 & C_1 & C_2 & \cdots & C_n \\
\hline
A_1 & a_{11} & a_{12} & \cdots & a_{1n} \\
A_2 & a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & a_{m1} & a_{m2} & \cdots & a_{mn} \\
\hline
\end{array}
$$

$$W = [w_1, w_2, \dots, w_n]$$

where $A = \{A_i \mid i = 1, 2, \dots, m\}$ is a set of possible alternatives among which decision makers have to choose; $C = \{C_j \mid j = 1, 2, \dots, n\}$ is a set of criteria with which alternative's performances are measured, a_{ij}, $i = 1, \dots, m$; $j = 1, \dots, n$ is the rating of alternative A_i with respect to criterion C_j, $j = 1, \dots, n$ and w_j is the weight of criterion C_j.

In practice, it is common to find complex decision-making situations involving both, quantitative and qualitative aspects. The quantitative aspects are generally assessed by means of precise numerical values, but qualitative aspects are complex to assess with precise and exact values. It is not possible to model such imprecise situations using traditional MCDM approaches and requires combining these with fuzzy logic and other techniques in a hybrid manner to deal with the qualitative aspects and uncertainty in decision-making process [13]. However, the fuzzy set theory deals with fuzzy numbers and the use of these different fuzzy numbers given by decision makers in different knowledge areas lead to different results. It may also be difficult to select an appropriate method for normalising the decision matrix (converting the criteria values into the dimensionless form) since a lot of normalisation methods have been developed and the choice of different methods might change the final selection and ranking for a specific problem [1].

The main objective of this research study is to develop a hybrid decision support model, for assisting the travellers in intelligent decision-making activities, by combining multicriteria decision-making and fuzzy logic techniques. The AFS theory is incorporated into the MCDM techniques to model fuzziness in human knowledge representation and reasoning process. The developed hybrid model integrates AHP, SAW and AFS theory to contribute a new way for solving complex problems under MCDM environment. It takes advantages of normalisation of decision matrix under the AHP calculation framework and inconsistency caused by different types of fuzzy numbers.

The presentation of our work on developing the hybrid decision support model in this paper is organised as follows. In Section 2, the background of MCDM techniques (AHP and SAW) is provided. The concept of AFS theory is also discussed in Section 2. Section 3 outlines the methodological steps of the proposed model. Section 4 presents the utilisation of developed decision support model in a real life example of transportation mode selection. In Section 5, a comparative analysis and validation is reported, and finally, Section 6 concludes the paper.

2. Theoretical Background

Although the theoretical backgrounds of AHP, AFS theory, and SAW have been documented in the literature, a brief outline of these methods is included below in the context of the present work.

2.1. Analytic Hierarchy Process (AHP)

AHP, developed by Saaty [19], is a mathematical technique which addresses how to determine the relative importance of a set of criteria in a decision problem. The ability to incorporate judgments on intangible qualitative criteria alongside tangible quantitative criteria makes AHP an ideal methodology for prioritisation problems having a set of potentially conflicting criteria. The literature on AHP applications is very rich [10]. In recent research, AHP has been applied to solve many complex decision problems [11], [2], [18], [17] [5].

The process of AHP can be summarised in four steps: construct the decision hierarchy, determine the relative importance of criteria and sub-criteria, evaluate each alternative and calculate its overall weight in regard to each criterion, and check the consistency of the subjective evaluations [19]. In the first step, the problem is organised as a hierarchical structure of decision elements where the top level represents the goal of decision, second and subsequent levels represent criteria and sub-criteria, and the lowest level represents the decision alternatives. In the second step, the decision maker is asked to subjectively evaluate pairs of criteria based on Saaty's [19] standardised scale of nine levels, as given in Table 1. In the third step, a relative importance value (weight) is computed for each criterion (and sub-criterion) based on pairwise comparisons. The logical consistency in comparison process is tested in the last step.

Table 1. Scale for Pairwise Comparison (Saaty [19])

Intensity of Importance	1	3	5	7	9	2, 4, 6, 8
Definition	Equal	Moderate	Strong	Demonstrated	Extreme	Intermediate Value

If we wish to compare a set of n criteria pairwise according to their relative importance, then the result of pairwise comparisons can be summarised in an $(n \times n)$ algebraic matrix P, as shown:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \quad (1)$$

where p_{ij} is the relative importance for i to j, $p_{ji} = 1/p_{ij}$ and $p_{ij} = 1$ if $i = j$.

The process commences to normalise the pairwise comparison matrix and obtain the relative weights. The relative weights are given by right eigenvector (w) corresponding to the largest eigenvalue (λ_{max}), as:

$$Pw = \lambda_{max} w \quad (2)$$

In order to ensure the consistency of subjective perception in pairwise comparison, two indices, consistency index (CI) and consistency ratio (CR) are suggested. The consistency index for the matrix of order n is expressed as:

$$\mathrm{CI} = \frac{(\lambda_{max} - n)}{(n-1)} \quad (3)$$

The CR is calculated as the ratio of CI and random index (RI), as indicated:

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} \quad (4)$$

where RI refers to a random consistency index derived from a randomly generated pairwise comparison matrix. The random indices with respect to different size matrices are shown in Table 2. If the calculated CR of a pairwise comparison is less than 0.1, the decision maker's judgment is consistent and acceptable; otherwise the evaluation procedure is considered inconsistent and needs revision to improve consistency.

Table 2. Random Consistency Indices (Saaty [19])

n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.46	1.49

2.2. Axiomatic Fuzzy Set (AFS) Theory

In everyday life, we come across complex decision situations, the nature of which is ambiguous and imprecise. It is not possible to formally describe such imprecise situations using classical set theory and bivalent logic. The AFS theory, proposed by Liu [23], is a mathematical framework that aims to explore how fuzzy set theory and probability can be made to work in concert, so that the uncertainty of randomness and of imprecision can be treated in a unified and coherent manner. It provides an effective tool to convert the information in observed data into the membership functions and logic operations of fuzzy concepts. The AFS theory is based on the AFS algebra – a kind of semantic methodology of fuzzy concepts and AFS structure – a kind of mathematical description of data structures. The recent literature of AFS studies and their applications ([27], [28], [24], [25], [14], [17], [26]) reveals that it is a flexible and powerful framework for representing human knowledge and studying intelligent systems in real world applications.

2.2.1. AFS Algebra: Liu [23] defined a family of completely distributive lattices, referred to as AFS algebras and applied them to study the semantics of expressions and representations of fuzzy concepts. In a multiple criteria decision problem, let $X = \{x_1, x_2, \dots, x_5\}$ be a set of alternatives, $M = \{m_1, m_1', \dots, m_5, m_5'\}$ be a set of fuzzy attributes on X, where m_1: "attribute 1 is good", m_1': "attribute 1 is not good", …, m_5: "attribute 5 is good", m_5': "attribute 5 is not good". For each set of concepts $A \subseteq M$, $\prod_{m\in A} m$ represents conjunction of the concepts in A. $\sum_{i\in I}(\prod_{m\in A_i} m)$, a formal sum of $\prod_{m\in A_i} m$, $A_i \subseteq M, i \in I$, is the disjunction of the conjunctions represented by $\prod_{m\in A_i} m$'s (*i.e.*, the disjunctive normal form of a formula representing a concept). For instance, we may have $m_1m_5 + m_1m_3 + m_4$ which translates as "*attribute 1 and attribute 5 are good*" or "*attribute 1 and attribute 3 are good*" or "*attribute 4 is good*" (the "+" sign represents disjunction of concepts). For $A_i \subseteq M, i \in I$, $\sum_{i\in I}(\prod_{m\in A_i} m)$ has a well-defined meaning as discussed above. The semantics of the logic expressions such as "*equivalent to*", "*or*", and "*and*", as expressed by $\sum_{i\in I}(\prod_{m\in A_i} m)$, can be formulated in terms of the AFS algebra (EM^*), defined as:

$$EM^* = \left\{\sum_{i\in I}\left(\prod_{m\in A_i} m\right) \mid A_i \subseteq M, i \in I, I \text{ is an non-empty indexing set}\right\} \quad (5)$$

Definition 1 [23]: Let M be a non-empty set, a binary relation R on EM^* is defined as follows:

$\forall \sum_{i\in I}(\prod_{m\in A_i} m)$ and $\sum_{j\in J}\left(\prod_{m\in B_j} m\right) \in EM^*$,

$\left[\sum_{i\in I}(\prod_{m\in A_i} m)\ R\ \sum_{j\in J}\left(\prod_{m\in B_j} m\right)\right] \Leftrightarrow$ (i) $\forall A_i (i \in I)$, $\exists B_h (h \in J)$ such that $B_h \subseteq A_i$, and (ii) $\forall B_j (j \in J), \exists A_k (k \in I)$ such that $A_k \subseteq B_j$.

It is obvious that R is an equivalence relation and the quotient set (EM^*/R) is denoted by *EM*. The notation $\sum_{i\in I}(\prod_{m\in A_i} m) = \sum_{j\in J}\left(\prod_{m\in B_j} m\right)$ means that $\sum_{i\in I}(\prod_{m\in A_i} m)$ and $\sum_{j\in J}\left(\prod_{m\in B_j} m\right)$ are equivalent (represents same semantics) under relation R.

Theorem 1 [23]: Let M be a non-empty set, then (EM,∧,∨) forms a completely distributive lattice under the binary compositions ∧ and ∨, defined as follows:

$\forall \sum_{i\in I}(\prod_{m\in A_i} m), \sum_{j\in J}\left(\prod_{m\in B_j} m\right) \in EM^*$,

$$\sum_{i\in I}(\prod_{m\in A_i} m) \wedge \sum_{j\in J}\left(\prod_{m\in B_j} m\right) = \sum_{i\in I, j\in J}\left(\prod_{m\in A_i\cup B_j} m\right) \quad (6)$$

$$\sum_{i\in I}(\prod_{m\in A_i} m) \vee \sum_{j\in J}\left(\prod_{m\in B_j} m\right) = \sum_{k\in I\sqcup J}(\prod_{m\in C_k} m) \quad (7)$$

where for any $k \in I \sqcup J$ (the disjoint union of I and J, i.e., every element in I and every element in J are always regarded as different elements in $I \sqcup J$); $C_k = A_k$ if $k \in I$ and $C_k = B_k$ if $k \in J$. $(EM,\wedge,\vee)$ is called the EI (expending on M) algebra over M.

2.2.2. AFS Structure: An AFS structure represented by a triple (M, τ, X), gives rise to membership functions and fuzzy logic operations of the concepts in EM.

Definition 2 [22], [23]: Let X, M be sets and $\tau: X \times X \to 2^M$. (M, τ, X) is called an AFS structure if τ satisfies the following axioms:

(a1). $\forall (x_1, x_2) \in X \times X, \tau(x_1, x_2) \subseteq \tau(x_1, x_1)$ (8)

(a2). $\forall (x_1, x_2), (x_2, x_3) \in X \times X, \tau(x_1, x_2) \cap \tau(x_2, x_3) \subseteq \tau(x_1, x_3)$ (9)

X is called the universe of discourse, M is called the concept set and τ is called the structure. In real world applications, τ can be constructed from a linearly ordered relation $(\geq_m)$ as follows:

$$\tau(x, y) = \{m | m \in M, x \geq_m y\} \subseteq 2^M \quad (10)$$

where , $x \geq_m y$ implies that the degree of x belonging to simple concept m is greater than or equal to y.

Definition 3 [22]: Let X and M be sets, (M, τ, X) be an AFS structure and (M, σ, m) be a measure space, where m is a finite and positive measure, $m(X) \neq 0, A_i^\tau \in \sigma,\ x \in X, i \in I$. For the fuzzy concept $\eta = \sum_{i\in I}(\prod_{m\in A_i} m) \in EM$, the membership function of η is defined as follows:

For any $x \in X$,

$$\mu_\eta(x) = sup_{i\in I} \frac{m(A_i^\tau(x))}{m(X)} \quad (11)$$

where $A_i^\tau(x) = \{y \in X: x \geq_m y,$ for any $m \in A_i\}$. In other words, A_i^τ is the set of all elements in X whose degree of belonging to concept $\prod_{m\in A_i} m$ are less than or equal to that of x.

2.3. Simple Additive Weighting (SAW)

The simple additive weighting (SAW), also known as weighted sum model, is one of the most popular and simple multicriteria decision analysis technique for evaluating a set of alternatives in terms of a set of criteria. In a multiple criteria decision problem with m alternatives and n criteria, the performance score of each alternative can be derived using following equation:

$$A_i^* = \sum_{j=1}^{n} a_{ij} w_j \quad (12)$$

where a_{ij} (for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$) represents the normalized rating of alternative i with respect to criterion j and w_j represents the normalized weight of criterion j. The alternative with maximum performance score (A_i^*) is considered as best alternative.

3. Proposed Model: Hybrid Decision Support Model

The subjective and qualitative nature of a multiple criteria decision-making problem requires expressing the strength or weakness of preferences using linguistic terms. In practice, the linguistic values are represented by different types of fuzzy numbers (triangular fuzzy number, trapezoidal fuzzy number, *etc.*,) which may lead to different results. The proposed methodological framework utilizes AFS theory in AHP and SAW to develop a hybrid model to solve complex MCDM problems. The linguistic values are processed with AFS theory to cope with the inconsistency caused by different types of fuzzy numbers. The hybrid model consists of five basic stages:

Stage 1: Identification of decision criteria and linguistic assessment of alternatives with respect to criteria using modified-delphi method.

Step 1: Identify all possible decision alternatives: A_i $(i = 1, \dots, m)$ and criteria: C_j $(j = 1, \dots, n)$.

Step 2: Establish a judgment matrix $[a_{ij}]_{m\times n}$ for linguistic assessment of alternatives in terms of decision criteria. The matrix element a_{ij} represents assessment of i^{th} alternative in terms of j^{th} criteria using linguistic terms.

Stage 2: Computation of relative weights (w_j) of decision criteria.

Step 1: Establish a comparison matrix $[p_{ij}]_{n\times n}$ by performing pairwise comparisons (using 9-point scale) between each pair of criteria.

Step 2: Compute the weights of criteria (w_j) using AHP.

Stage 3: Fuzzy description (ζ_{A_i}) of each decision alternative using AFS theory. According to Liu and Pedrycz [21], the fuzzy description for each alternative is determined by the following steps:

Step 1: Find the set of fuzzy attributes in M, defined as:

$$B_{A_i}^{\varepsilon} = \{m_k \in M: \mu_{m_k}(A_i) \geq \mu_v(A_i) - \epsilon\} \quad (13)$$

Step 2: Find the set $\bar{B}_{A_i}^{\varepsilon}$, defined as follows:

$$\bar{B}_{A_i}^{\varepsilon} = \left\{\prod_{m\in A} m : \mu_{\prod_{m\in A} m}(A_i) \geq \mu_v(A_i) - \varepsilon, A \subseteq B_{A_i}^{\varepsilon}\right\} \quad (14)$$

Step 3: Select the best fuzzy description $\zeta_{A_i} \in \bar{B}_{A_i}^{\varepsilon}$ for the alternative A_i as follows:

$$\zeta_{A_i} = arg\ min_{\zeta\in\bar{B}_{A_i}^{\varepsilon}} \left\{\sum_{x\in X, x\neq A_i} \mu_\zeta(x)\right\} \quad (15)$$

Stage 4: Establish a decision matrix by rating each alternative A_i $(i = 1, 2, \dots, m)$ over each decision criterion C_j $(j = 1, 2 \dots, n)$.

Step 1: Perform pairwise comparisons (using 9-point scale) between each pair of decision alternatives according to benefit and cost criterion in their best fuzzy descriptions (ζ_{A_i}). For benefit criterion, the alternatives are compared based on C_j (criterion C_j is good) and for cost criterion, the comparison process is carried out based on C_j' (criterion C_j is not good).

Step 2: Compute the performance scores of alternatives over decision criteria using AHP and establish a normalised decision matrix where each element (r_{ij}) represents the performance score of i^{th} alternative over j^{th} criteria. Since the performance scores (r_{ij}) are obtained using AHP, these are considered as normalised under AHP calculation framework and therefore there is no need to further normalise them explicitly.

Stage 5: Ranking the decision alternatives.

Step 1: Establish a weighted normalised decision matrix and evaluate the performance of each alternative using Equation (12) and rank accordingly.

4. Application of Proposed Model in Transportation Mode Selection

The proposed model is applied to a real world application of transportation mode selection in Agra, a historic city in northern region of India. The city has an international importance in tourism and industrial sector which makes it flooded with naïve commuters. The selection of best mode of transportation in advance may help the travelers in meeting their personalized preferences and making journey efficient. In following section, the application of proposed decision support model is described in reference to selecting best mode of transportation based on personalized preferences. There are various modes of transportation in Agra, this study considers only six most commonly used modes of transportation - A_1: auto rickshaw, A_2: cycle rickshaw, A_3: electric rickshaw, A_4: taxi, A_5: bus, and A_6: train. The following section illustrates the step by step process of the developed hybrid model in evaluating best means of transportation based on a set of decision criteria.

Stage 1: Identification of decision criteria and linguistic assessment of alternatives with respect to criteria using modified-Delphi method.

This phase begins with forming a team of 5 experts responsible for identifying a set of selection criteria important to determine best mode of transportation. A list of selection criteria was prepared based on expert's opinion and literature survey ([20], [15], [12], [4], [16], [8], [7]) and a calibration process was applied to narrow down the list to include only those criteria the travelers' feel pertinent to select a transportation mode. This process resulted in a set of eight criteria - C_1: safety, C_2: speed, C_3: travel cost, C_4: interchange, C_5: accessibility, C_6: comfort, C_7: environment friendliness, and C_8: capacity. Next, the expert's perceptions (linguistic assessment) of transportation modes with respect to selection criteria are elicited. Table 3 shows the result of the elicitation process.

Table 3. Linguistic Assessment of Transportation Modes

Modes	Decision Criteria							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	H	H	H	L	VH	H	ML	ML
A_2	ML	L	MH	MH	VH	ML	H	L
A_3	L	MH	MH	MH	H	ML	H	ML
A_4	H	H	VH	VL	MH	VH	MH	MH
A_5	MH	MH	ML	VH	ML	MH	L	H
A_6	MH	MH	L	VH	MH	MH	ML	VH

The meaning of linguistic ratings of different criteria in Table 3 can be defined as: VL = Very Low, L = Low, ML = Medium Low, M = Medium, MH = Medium High, H = High, VH = Very High.

Stage 2: Computation of relative weights (w_j) of decision criteria.

In this phase, a pairwise comparison matrix $P = (p_{ij})_{n\times n}$ is established based on traveler's preferences to decision criteria (see Table 4) which results in computation of criteria weights (w_j) using eigenvalue calculation framework.

Stage 3: Fuzzy description (ζ_{A_i}) of each decision alternative using AFS theory.

Let $X = \{A_1, A_2, A_3, A_4, A_5, A_6\}$ be the set of six transportation modes, $\epsilon = 0$, $C = \{C_1, C_1', C_2, C_2', \dots, C_8, C_8'\}$ be the set of decision criteria on X and $v = C_1 + C_1' + C_2 + C_2' + \cdots C_8 + C_8'$. By Table 3 and semantic meaning of decision criteria, we have following linearly ordered relations:

$C_1: A_3 < A_2 < A_5 = A_6 < A_1 = A_4$ $\quad$ $C_1': A_3 > A_2 > A_5 = A_6 > A_1 = A_4$

$C_2: A_2 < A_3 = A_5 = A_6 < A_1 = A_4$ $\quad$ $C_2': A_2 > A_3 = A_5 = A_6 > A_1 = A_4$

$C_3: A_6 < A_5 < A_2 = A_3 < A_1 < A_4$ $\quad$ $C_3': A_6 > A_5 > A_2 = A_3 > A_1 > A_4$

$C_4: A_4 < A_1 < A_2 = A_3 < A_5 = A_6$ $\quad$ $C_4': A_4 > A_1 > A_2 = A_3 > A_5 = A_6$

$C_5: A_5 < A_4 = A_6 < A_3 < A_1 = A_2$ $\quad$ $C_5': A_5 > A_4 = A_6 > A_3 > A_1 = A_2$

$C_6: A_2 = A_3 < A_5 = A_6 < A_1 < A_4$ $\quad$ $C_6': A_2 = A_3 > A_5 = A_6 > A_1 > A_4$

$C_7: A_5 < A_1 = A_6 < A_4 < A_2 = A_3$ $\quad$ $C_7': A_5 > A_1 = A_6 > A_4 > A_2 = A_3$

$C_8: A_2 < A_1 = A_3 < A_4 < A_5 < A_6$ $\quad$ $C_8': A_2 > A_1 = A_3 > A_4 > A_5 > A_6$

To distinguish A_i from other modes of transportation in X, the best fuzzy description of each mode (ζ_{A_i}) is obtained as follows:

By Equation (11),

$\mu_v(A_1) = \mu_v(A_2) = \mu_v(A_3) = \mu_v(A_4) = \mu_v(A_5) = \mu_v(A_6) = 1.0$

By Equation (11), (13), (14), and (15),

$\mu_{C_1}(A_1) = \mu_{C_2}(A_1) = \mu_{C_5}(A_1) = 1.0$

$B_{A_1}^0 = \{C_1, C_2, C_5\}$ and $\mu_{C_1C_2C_5}$ is the minimal element in $\bar{B}_{A_1}^{\varepsilon}$.

$\zeta_{A_1} = C_1C_2C_5$

The fuzzy description ζ_{A_1} can be interpreted as: "the transport mode A_1 has good safety, speed and accessibility".

Similarly,

$B_{A_2}^0 = \{C_5, C_7, C_2', C_6', C_8'\} \Rightarrow \zeta_{A_2} = C_5C_7C_2'C_6'C_8'$

$B_{A_3}^0 = \{C_7, C_1', C_6'\} \Rightarrow \zeta_{A_3} = C_7C_1'C_6'$

$B_{A_4}^0 = \{C_1, C_2, C_3, C_6, C_4'\} \Rightarrow \zeta_{A_4} = C_1C_2C_3C_6C_4'$

$B_{A_5}^0 = \{C_4, C_5', C_7'\} \Rightarrow \zeta_{A_5} = C_4C_5'C_7'$

$B_{A_6}^0 = \{C_4, C_8, C_3'\} \Rightarrow \zeta_{A_6} = C_4C_8C_3'$

Table 4. Pairwise Comparison Matrix and Criteria Weights

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	1.000	3.000	2.000	3.000	2.000	2.000	3.000	1.000
C_2		1.000	0.500	2.000	0.500	0.500	2.000	0.500
C_3			1.000	2.000	1.000	1.000	3.000	2.000
C_4				1.000	0.500	0.500	1.000	0.333
C_5					1.000	1.000	0.500	2.000
C_6						1.000	2.000	2.000
C_7							1.000	0.500
C_8								1.000
w_j	0.234	0.084	0.153	0.060	0.129	0.144	0.082	0.113

Stage 4: Establish a decision matrix by rating each alternative A_i $(i = 1, 2, \dots, m)$ over each decision criterion C_j $(j = 1, 2 \dots, n)$.

This phase starts with rating the transportation modes according to their fuzzy description. In this process, the transportation modes are rated over each criterion by considering $C_1, C_2, C_5, C_6, C_7, C_8$ as the benefit criteria and C_3, C_4 as the cost criteria. For illustration, the rating of all transportation modes over the criterion C_1 is presented in Table 5 with explanation as – since C_1 appears in ζ_{A_1} and ζ_{A_4}, hence A_1 and A_4 are extremely preferred over A_2, A_3, A_5 and A_6. Similarly, the weights of all transportation modes over each criterion $C_j: j = 1, 2, \dots, 8$ are obtained and a normalised (since the weights determined using AHP are already normalised, therefore no further normalisation is required) fuzzy decision matrix is obtained in Table 6.

Table 5. Rating of Transportation Modes over Criterion C_1

C_1	A_1	A_2	A_3	A_4	A_5	A_6	Weight
A_1	1	9	1	1	9	9	0.409
A_2	1/9	1	1/9	1/9	1	1	0.045
A_3	1	9	1	1	9	9	0.045
A_4	1	9	1	1	9	9	0.409
A_5	1/9	1	1/9	1/9	1	1	0.045
A_6	1/9	1	1/9	1/9	1	1	0.045

Table 6. Normalised Decision Matrix

Modes	Decision Criteria							
	C_1	C_2	C'_3	C'_4	C_5	C_6	C_7	C_8
A_1	0.409	0.409	0.071	0.071	0.409	0.071	0.045	0.071
A_2	0.045	0.045	0.071	0.071	0.409	0.071	0.409	0.071
A_3	0.045	0.045	0.071	0.071	0.045	0.071	0.409	0.071
A_4	0.409	0.409	0.071	0.643	0.045	0.643	0.045	0.071
A_5	0.045	0.045	0.071	0.071	0.045	0.071	0.045	0.071
A_6	0.045	0.045	0.643	0.071	0.045	0.071	0.045	0.643

In heading of Table 6, C'_3 and C'_4 indicate that these are cost criteria for which the minimum value will be preferred and hence A_i's are rated over C'_3 and C'_4 in order to minimise them.

Stage 5: Ranking the decision alternatives.

In this last phase, a weighted normalised decision matrix is established (Table 7) and the performance of each transportation mode (rank) is evaluated using Equation (12). The graphical representation of overall performance scores of transportation modes is shown in Figure 1.

Table 7. Weighted Normalised Decision Matrix and Final Ranking based on SAW

Modes	Decision Criteria									
	C_1	C_2	C_3'	C_4'	C_5	C_6	C_7	C_8	A_i^*	Rank
A_1	0.096	0.034	0.011	0.004	0.053	0.010	0.004	0.008	0.220	2
A_2	0.011	0.004	0.011	0.004	0.053	0.010	0.034	0.008	0.134	4
A_3	0.011	0.004	0.011	0.004	0.006	0.010	0.034	0.008	0.087	5
A_4	0.096	0.034	0.011	0.039	0.006	0.093	0.004	0.008	0.290	1
A_5	0.011	0.004	0.011	0.004	0.006	0.010	0.004	0.008	0.057	6
A_6	0.011	0.004	0.098	0.004	0.006	0.010	0.004	0.073	0.209	3

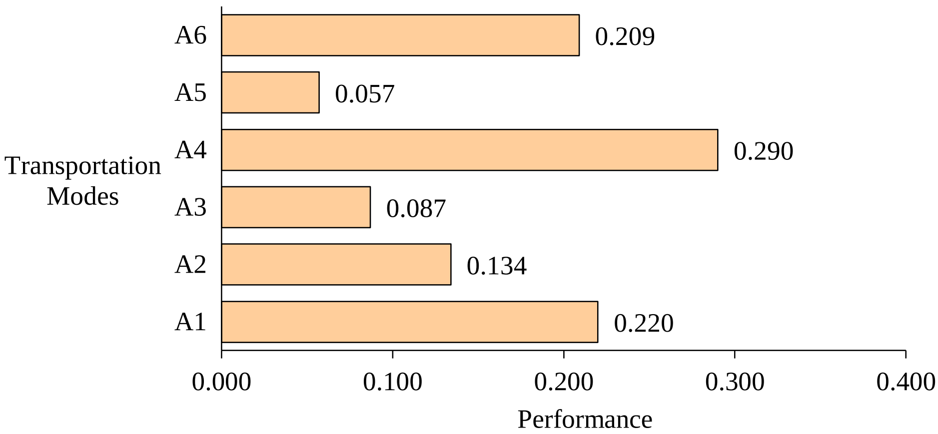

Figure 1. Overall Performance of the Transportation Modes

The values in Table 7 and the bars in Figure 1 show that A_4 (Taxi) is the best mode of transportation in order to meet the personalised preferences.

5. Comparative Analysis and Model Validation

In order to test the validity of the proposed model, we have applied it to an existing case study as given by [9] and a comparative analysis is carried out with the established MCDM technique. For the given case study, AFS theory is used to find the best fuzzy description of decision alternatives (companies X, Y and Z) and the final ranking are obtained using SAW approach.

The results (Table 8) obtained by the proposed model and that of established fuzzy MCDM approach are same that ensure the acceptability and validity of the developed decision support model.

Table 8. Comparative Study of Results

Alternatives	Using Proposed Model		Using Fuzzy MCDM Framework [9]	
	Composite Score (A_i^*)	Rank	Closeness Coefficient (CC_i)	Rank
Company X	0.3106	2	0.6946	2
Company Y	0.5713	1	0.8189	1
Company Z	0.1185	3	0.6886	3

6. Conclusion

This study focuses on development of a hybrid decision support model to solve complex multiple criteria decision problems. The developed model incorporates AHP as a multiple criteria decision-making method to determine priorities among decision criteria, and SAW methodology to obtain the ranking of decision alternatives. The best fuzzy description of each decision alternative is obtained using AFS theory and the performance scores of each alternative are determined using pairwise comparisons over cost or benefit criteria as described in alternatives' best fuzzy description. Finally, the resulting scores (decision matrix) are used to rank the decision alternatives using SAW methodology. The axiomatic fuzzy logic is incorporated into the model to overcome the uncertainty and ambiguity in human knowledge representation. The major advantage of the developed model is that it processes the linguistic values using axiomatic fuzzy logic that overcomes the ambiguity in human decision making process and copes with the inconsistency caused by different types of fuzzy numbers. An application of transportation mode selection has been presented to simulate the better understanding of the proposed methodology. As a result of the study, it is found that the proposed model is practical for ranking alternatives with respect to multiple conflicting criteria.

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Authors

Sunil Pratap Singh is an Assistant Professor at Bharati Vidyapeeth's Institute of Computer Applications and Management, New Delhi, India. He has completed his PhD in Computer Science from the Dayalbagh Educational Institute, Dayalbagh, Agra, India. His research interests include decision-making techniques and geographical information system (GIS).

Preetvanti Singh is a Professor in the Faculty of Science, Dayalbagh Educational Institute (Deemed University), Dayalbagh, Agra, India. She received her PhD in Operations Research from the Dayalbagh Educational Institute, Dayalbagh, Agra, India. Her current research interests include decision support system (DSS), geographical information system (GIS) and optimization techniques.

