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Chaos Theoretic Approach for Travel Demand Estimation

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Abstract

This paper attempts to present a chaos theoretic approach for travel demand estimation. First, we have analyzed and identified if the counted traffic data have chaotic characteristics using a Fourier power spectrum analysis. Second, we have applied a chaos entropy control method to estimate travel demand using traffic counts on a small freeway example. We have found chaotic characteristics in the numerical analysis of the counted traffic data and the travel demand estimation problem. In particular, in the travel demand estimation, each of different initial points has many local optima but has some common attractors converged in the end, which is a key characteristic in the chaos theory. Therefore, the chaos entropy control method has been used to stabilize the local optima using different initial starting points. We have obtained the boundary of attraction points using these tests. We find that the proper selection of an initial point is very important because of chaotic property in this problem.

Keywords: *Chaos Theory, Attraction Theorem, Fourier power spectrum analysis, Travel Demand Estimation, Chaos control*

1. Introduction

Transportation systems are complex and have chaotic characteristics (see Kockelman *et. al.*, 2004). Chaos is defined as the study of complex, nonlinear and dynamical systems. Complex indicates the high complexity, nonlinear recursion and non-periodic nature. Casdagli *et al.*, (1992) presented an effective introduction to chaos theory and analysis techniques. Hilborn (2001) offers a broad but detailed examination of chaos, while Argyris *et al.*, (1994) provide more mathematically complex coverage. Abarbanel (1996) offers an excellent presentation of chaotic data analysis techniques. Thus chaos theory is, in general, to study on constantly changing complex systems based on mathematical concepts of recursion, whether it is in the form of a recursive process or a set of differential equation modeling.

Chaos theory attempts to explain that complex and unpredictable results can be obtained in systems that are sensitive to their initial conditions. The attractor theorem in chaos theory is very important in this sense. It is because some dynamical systems are chaotic, and each initial condition can lead to each of different results. However, the chaotic behavior can take place on a common attractor, which means the dynamical systems can be stabilized in these attractors.

For example, a simple, three-dimensional model of the Lorenz (1993) weather system gives rise to the famous Lorenz attractor. The Lorenz attractor is one of the well-known chaotic systems, because it gives rise to a very interesting pattern which looks like the wings of a butterfly. Another such attractor is the Rössler map (see Abarbanel, 1996), which shows a period-two doubling route to chaos.

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Especially, in the study on the stability of chaos, Ott, Grebogi and Yorke (OGY) presented a perturbing method for chaos control by linearizing the nonlinear map of the system. The OGY method was successfully applied for chaos control in some economic systems. Pyragas also presented a method for chaos control by using delayed feedback signal.

This paper attempts to present chaos theoretic approach for travel demand estimation. First, we have analyzed and identified if the counted traffic data have chaotic characteristics using a Fourier power spectrum analysis. Second, we have applied a chaos entropy control method to estimate travel demand using traffic counts on a small freeway example. We have found chaotic characteristics in the numerical analysis of the counted traffic data and the travel demand estimation problem. In particular, in the travel demand estimation, each of different initial points has many local optima but has some common attractors converged in the end, which is a key characteristic in the chaos theory. Therefore, the chaos entropy control method has been used to stabilize the local optima using different initial starting points. We have obtained the boundary of attraction points using these tests. We find that the proper selection of an initial point is very important because of chaotic property in this problem.

In Section 2, we have analyzed the counted traffic data on Korea freeways to see if the Fourier spectrum technique can identify the existence of the chaotic phenomena in this analysis. In Section 3, mathematical frameworks for the existence of local optima are presented. In Section 4, entropy control methods are introduced to see if the method can be used to estimate the travel demand using traffic counts on a small freeway example. In Section 5, we draw some findings

2. Chaotic Phenomenon on Traffic Data

In order to apply the chaos control method, an important step is determining the presence of chaotic behavior. To check the existence of chaos phenomenon, we can apply a Fourier power spectrum technique for a traffic data on Korea freeway from Yangjae I.C. to Seocho I.C., which is a similar analysis as in Kockelman *et al.*, (2004) applied in US Freeways.

Kockelman *et al.*, (2004) used the following Fourier power spec-trum to see if traffic data have a chaotic character:

$$P(w) = \frac{1}{N^2}\left|\sum_{t=0}^{N-1} s(t)(e^{-i(2\pi/N)tw})\right|^2 \quad (1)$$

Where N is the number of complex number, and s(t) is discrete complex number. This is the discrete Fourier power spectrum, since we are assuming that this is applied to real and discrete data. For periodic data, the power spectrum will spike at frequencies that characterize the system, and lie close to zero for all others.

For determining the presence of chaos, we use the Fourier trans-form using time series data of 1 hour interval traffic counts on Korea expressway. The whole 5 day of count data in the interval 1 hour, March 5 through 9, can be plotted in Figure 1. We see that there is a morning peak and a broader evening peak, corresponding to the commuting times. This periodic structure remains, and this will be important for the analysis.

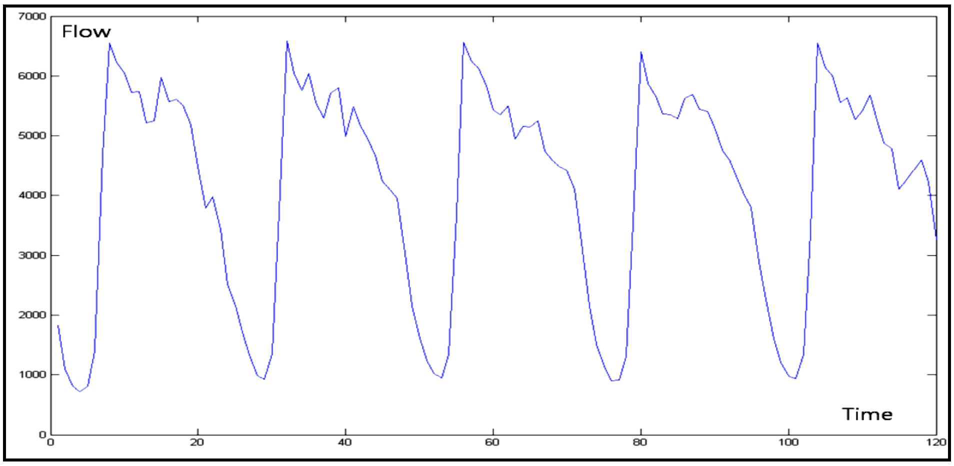


Figure 1. Time Series of 1-hour Traffic Counts on Kyung-Bu Expressway (2007/03/05~2007/03/09)

In Figure 2, the Fourier power spectrum technique is used to check for chaotic structure. Even though time series traffic counts data seem to have a regular form in its pattern, Fourier power spectrum after Fourier transformation leads to identify its irregular forms. This means the traffic counts data look like a regular form but it has a chaotic irregular characteristic in its detail. This is a key characteristic in chaos theory. And note that the broadband power spectrum is characteristics of chaotic data, which is also shown in Kockelman *et al.*, (2004).

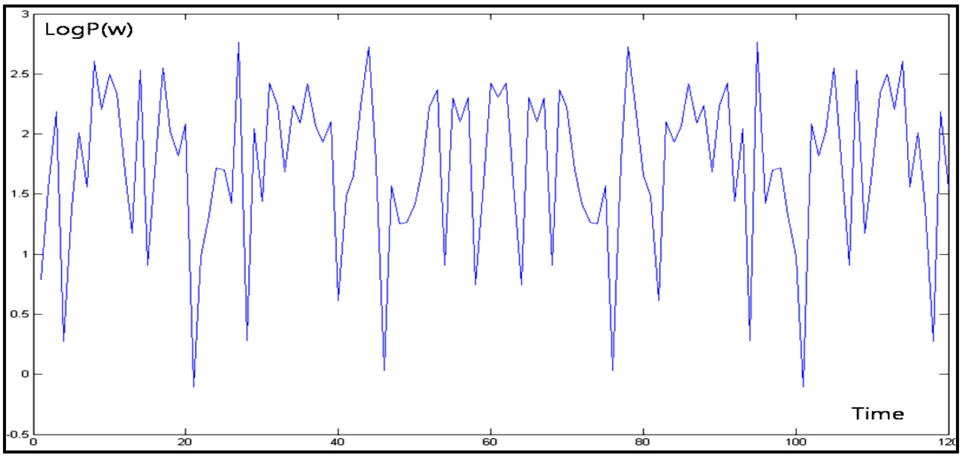


Figure 2. Fourier Power Spectrum for Weekday Traffic Flow Data on Korea Freeways

3. Existence of Local Optima

One of the most interesting problems dealing with chaotic systems is a chaos control with the aim of stabilizing unstable periodic trajectory inside the strange attractors of chaotic systems (Ott *et al.*, 1990). For a brief review of the minimum entropy control method, we consider a nonlinear map that represents a chaotic dynamical system:

$$x(n+1) = f(x(n), u(n)) \tag{2}$$

Where x(n) is the state vector, u(n) is the control action, and f is the nonlinear systems. It is assumed that the function f is unknown but the system states are accessible. The main goal is to design the control u(n) for quenching chaos phenomenon in the system by stabilizing one of its unstable fixed points. To this end the control action u(n) is determined in a way that the system entropy converges to zero (see Ott *et al.*, 1990).

In a dynamical system such as equation (2), the entropy in the sense of Shannon is defined as: (Hassan and Salarieh. 2007)

$$E(u) = -\int_{x\in\Omega} p(x,u)\ln p(x,u)dx \tag{3}$$

Where E is the entropy of the system, p is the probability density function and Ω is the region in the phase space occupied by the state variables.

If p(.,.) is not known, it can easily be estimated by numerical methods. In this case it divides X into N sub-regions denoted by Xi, i = 1,…, N. The following equation estimates the probability density function of the system:

$$p(x,u) = p_i = \frac{N_i}{N_t} \quad \text{if } q \in \Omega_i \tag{4}$$

Where, N_i is the number of points (obtained by iteration) that lie in sub-region Ω_i, and $N_t = \sum_{i=1}^{N} N_i$ is the total number of points or iterations.

Here, we can apply this concept to the travel demand estimation developed by Wilson (1970).

First, we apply the Shannon Entropy (see Hassan and Salarieh, 2007) and consider the concept of strange attractor.

In this case we repeat the Shannon entropy model as follows:

$$E(u) = -\sum_{ij=1}^{N} T_{ij}(x,u)\ln T_{ij}(x,u) \tag{5}$$

It is assumed that the parameter u can be changed to minimize E. To obtain a local minimum for E, which is defined in equation (5), by a recursive algorithm, the gradient descent method may be applied as:

$$u(n+1) = u(n) - \gamma \left.\frac{\partial E}{\partial u}\right|_{u=u(n)} \tag{6}$$

Where γ >0 indicates the rate of convergence but it may have an optimum value. Here an arbitrary positive value for γ is set. Applying equation (5) to equation (6) yields:

$$u(n+1) = u(n) - \gamma \sum_{ij=1}^{N} (1+\ln T_{ij})\frac{\partial T_{ij}}{\partial u} \tag{7}$$

And using the definition of equation (4) we can obtain the following relation:

$$\frac{\partial T_{ij}}{\partial u} = \frac{N_t \frac{\partial N_i}{\partial u} - N_i \sum_{j=1}^{N} \frac{\partial N_j}{\partial u}}{N_t^2} = \frac{1}{N}\frac{\partial N_i}{\partial u} - \frac{N_i}{N_t^2}\sum_{j=1}^{N}\frac{\partial N_j}{\partial u} \tag{8}$$

The definition of T(x,u) and equation (4) and equation (3) is rewritten as:

$$u(n+1)=u(n)-\gamma\sum_{ij=1}^{N}(1+\ln(\frac{N_i(n)}{N_t(n)}))(\frac{1}{N_t(n)}\frac{\partial N_i(n)}{\partial u(n)}-\frac{N_i(n)}{N_t^2(n)}\sum_{j=1}^{N}\frac{\partial N_j(n)}{\partial u(n)} \tag{9}$$

Where $\Delta N_i(n)=N_i(n)-N_i(n-1)$ and $\Delta u(n)=u(n)-u(n-1)$ are the variations of $N_i(n)$ and u(n) between the nth and (t-1)th iterations, respectively. Note that $\Delta N_t(n)=N_t(n)-N_t(n-1)=1$ if the nth iteration point lies in Ω_i. Otherwise, $N_t(n)-N_t(n-1)=0$. In addition to $N_t(n)=n$, equation (9) may be modified as:

$$u(n+1)=u(n)-\frac{\gamma}{n}\frac{1}{u(n)-u(n-1)}\left[(1+\ln(\frac{N_k(n)}{n}))-\sum_{i=1}^{N}(1+\ln(\frac{N_i(n)}{n}))\frac{N_i(n)}{n}\right] \tag{10}$$

Where k is the index of the region, *i.e.*, Ω_k, where the nth iteration point lies. To show that E(u(n)) becomes a decreasing sequence with respect to t, the second order Taylor approximation of E(u(n+1)) around u(n) is used:

$$\begin{aligned}E(u(n+1))-E(u(n))&=\left[\frac{\partial E}{\partial u}\right]_{u=u(n)}\\&=(u(n+1)-u(n))+\frac{1}{2}\left[\frac{\partial^2 E}{\partial u^2}\right]_{u=u(n)}(u(n+1)-u(t))^2+H.O.T.\end{aligned} \tag{11}$$

Where H.O.T denote higher order terms in the Taylor series. Using equation (9) we can obtain:

$$\begin{aligned}E(u(n+1))-E(u(n))&=-\gamma\left[\frac{\partial E}{\partial u}\right]^2_{u=u(n)}+\frac{1}{2}\left[\frac{\partial^2 E}{\partial u^2}\right]_{u=u(n)}\left[\frac{\partial E}{\partial u}\right]^2_{u=u(n)}+H.O.T.\\&=-\gamma(1-\frac{1}{2}\gamma\left[\frac{\partial^2 E}{\partial u^2}\right]_{u=u(n)})\left[\frac{\partial E}{\partial u}\right]^2_{u=u(n)}+H.O.T.\end{aligned} \tag{12}$$

So if

$$0<\gamma<2\left|\left[\frac{\partial E}{\partial u}\right]^2_{u=u(n)}\right|^{-1}$$

And $\left|\frac{\partial E}{\partial u}\right|$ is sufficiently small, it is concluded that :

$$E(u(n+1))-E(u(n)\le 0\rightarrow E(u(n+1))\le E(u(n)) \tag{13}$$

Hence, E(u(n)) is a descending function, and by applying some standard solution algorithms such as a gradient descent algorithm, a local minimum, which is a fixed point with zero entropy, may be obtained.

4. Entropy Control in Travel Demand Analysis

To apply the minimum entropy control by using a feedback control, the control action is set to be:

$$u(n) = \Phi(T_{ij}(n) - T_{ij}(n-1), \mu) \tag{14}$$

Where, $\Phi(.)$ is a function with the property of $\Phi(0, \mu) = 0$ and μ is an unknown parameter which must be adaptively determined to achieve the minimum entropy objective. In this case the feedback system has the form of:

$$T_{ij}(n+1) = f(T_{ij}(n), \Phi(T_{ij}(n) - T_{ij}(n-1), \mu)) \tag{15}$$

Here, μ plays the same role as u in equtions (5), hence a similar algorithm may be obtained to determine μ in each iteration:

$$\mu(n+1) = \mu(n) - \gamma \left.\frac{\partial E}{\partial \mu}\right|_{\mu=\mu(n)} \tag{16}$$

Equation (10) is re-obtained as:

$$\mu(n+1) = \mu(n) - \frac{\gamma}{n}\frac{1}{\mu(n)-\mu(n-1)}\left[(1+\ln(\frac{N_k(n)}{n})) - \sum_{i=1}^{N}(1+\ln(\frac{N_i(n)}{n}))\frac{N_i(n)}{n}\right] \tag{17}$$

5. Numerical Example

For a numerical example, consider the small freeway network that has four ramps (ramp 1 and ramp 2, ramp 3 and ramp 4) and two-way links. We assume that each link has observed traffic volumes and all ramps have a limit capacity that is 5 veh/hr.

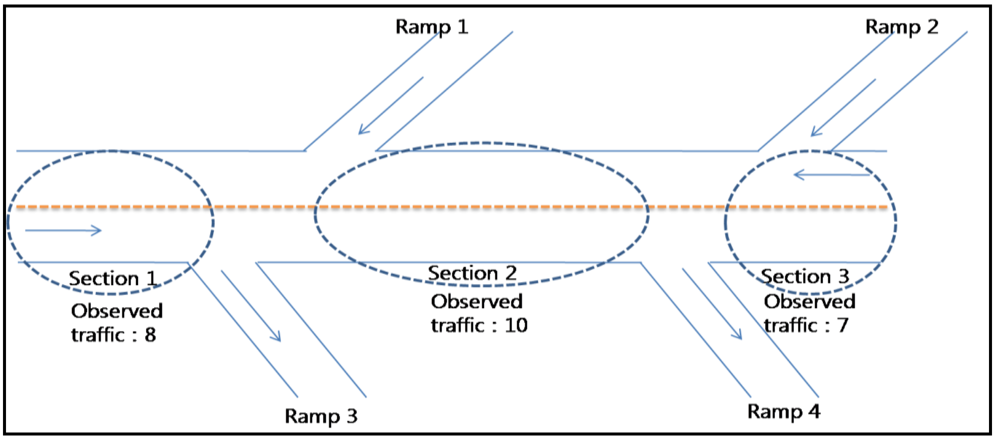


Figure 3. The Simple Freeway Network with Traffic Counts

We can solve the traffic counts of each ramp. It is because we know link traffic volumes and capacities of all ramps. For solving ramp traffic volumes, possible solutions are deducible as in Table 1.

Table 1. Possible Traffic Volumes on Ramps

Case \ Ramp	Ramp 1	Ramp 3	Ramp 2	Ramp 4
Case 1	0(veh/hr)	2(veh/hr)	3(veh/hr)	0(veh/hr)
Case 2	1(veh/hr)	3(veh/hr)	4(veh/hr)	1(veh/hr)
Case 3	2(veh/hr)	4(veh/hr)	-	-

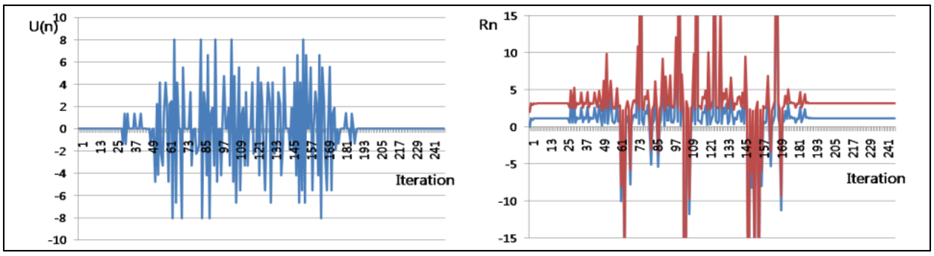
To satisfy the observed traffic volumes in section 1, possible traffics on ramp 1 and ramp 3 can be in three cases. Likewise, to satisfy the flow conservation in section 2, two cases are possible on ramp 2 and 4.

For this case, the control action is applied to the following form without loss of generality. The control law is applied directly to the production quantity in the form on Section 1:

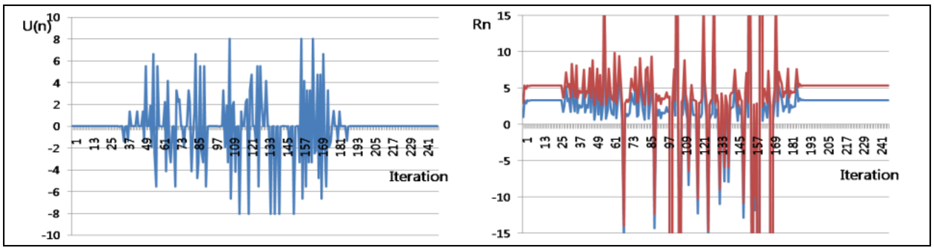
$$R_1(n+1) = \alpha_1 + \beta R_2(n)$$
$$R_2(n+1) = \alpha_2 + \beta R_1(n) + u(n) \qquad (18)$$

Where, Ri is the ramp traffic volume, n is a case number. And u(n) is the control action. The parameter of equation (9) are set to be N=6, and γ =0.05. Here the sub-regions are $\Omega_i = [\frac{i-1}{N}, \frac{i}{N}]$. In numerical simulations, $\beta = 0.37$.

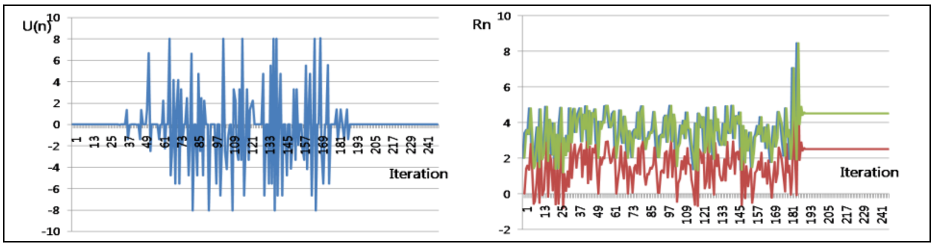
The results of the simulations of Section 1 are shown in Figure 4. The section 1 is subject to 8, thus possible solutions of {R1, R2} are three. All these cases are possible, and the control action is not applied to the system for the initial iteration. As it is seen after about 200 iterations the convergence to the fixed points, which means the stabilization of the fixed points, are achieved irrespective of different initial starting solutions. However, some strange attractors are obtained among different initial starting solutions.



Stabilizing of Initial Volume (0,2)



Stabilizing of Initial Volume (1,3)

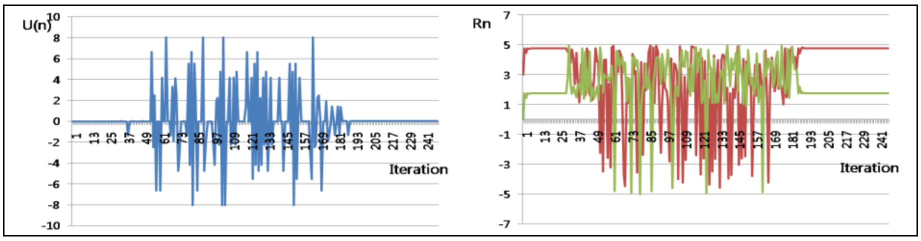


Stabilizing of Initial Volume (2,4)

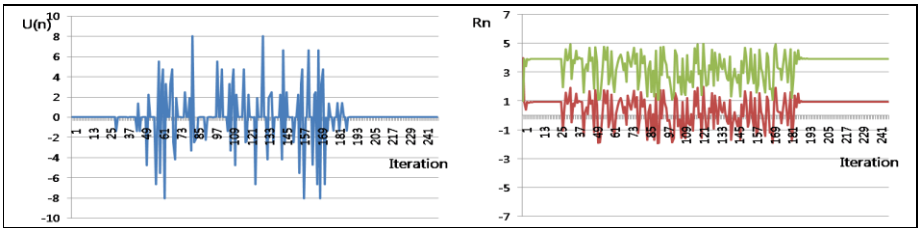
Figure 4. The Stability of Entropy Minimization in Production Total Section 1

This method of entropy minimization is a local solution algorithm, so the results of control implementation depend on the initial volume. There may exist some local minima for the entropy, initial volumes of (0, 2) → (1.17, 3.17) and (1, 3) → (3.35, 5.35), (2, 4)→(2.52, 4.52). And then, equation (9) is apply to the section 2. In this case, for finding stabilizing the ramp volumes, the production quantity formulation uses the equation (9) and parameter setting is the same case as in the Section 1.

First, the results of simulations on the Section 2 are shown in Figure 5. The Section 2 is subject to 10, thus possible initial volumes of {R2, R4} are two. *i.e.*, {3, 1} and {4, 1}. All the case of possible initial volumes, for the first iterations the control action is not applied to the system. As it is seen after about 200 iterations, the convergence to the fixed points is appeared, which means the stabilization of the fixed points is achieved from the initial points of (3, 0)→(4.76, 1.76) and (4, 1)→(3.93, 0.93) respectively.



Stabilizing of Initial Volume (3,0)



Stabilizing of Initial Volume (4,1)

Figure 5. The Stability of Entropy Minimization in Production Total Section 2

6. Conclusion

This paper presented a chaotic characteristic of traffic flow data in Korea Freeways using the Fourier power spectrum technique. We have found a chaotic characteristic in this traffic flow data analysis. It is because daily regular patterns of the traffic data can be decomposed into hourly irregular patterns by the Fourier transformation. This is a key characteristic in the chaos theory. In addition, we introduced the attraction theory in the chaos theory, and the control algorithm to stabilize the fixed points of chaotic maps. The algorithm used in this paper is based on minimizing the entropy control method in the case of the travel demand estimation problem. In order to apply the entropy control method for the travel demand estimation problem, we proved the entropy control mathematically, and determined the importance of the initial point in the travel demand estimation problem.

In the mathematical process, the chaos control method has been assumed that the system equations are not known priori, and the feedback gain is adjusted by minimizing the entropy number of systems. An entropy reduction regulates the behavior of the systems and this is used for the chaotic noise elimination. It is shown that one can use this technique to stabilize the unstable fixed points of this complex problem. The numerical simulation shows that proposed control algorithm successfully quenches the chaotic transportation systems, and stabilizes the fixed point when the system equations are not known. Also the stabilizing points are determined by initial points. And we determined the chaotic character in transportation systems in which each initial point has many local optimum points such as the attractors in the chaos theory, and confirmed the applicability of this new approach for the travel demand estimation problems (See Lee *et. al.*, 2008). The entropy control method was used to stabilize the local optima using different initial starting points. The boundary of the attraction points was obtained using these tests. In conclusion, the minimum entropy control using the property of the chaos theory in transportation systems is a new approach for the method of travel demand estimation problems. And we have found that a proper selection of an initial point is very important because of the chaotic property in this problem.

Acknowledgments

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