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# An Evaluation Method for the Connectivity Reliability Based on the Transportation Network of Critical Links

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***Abstract***

*This article introduces the concepts of the closeness and neighborhood of the transportation network node, and defines the important degree of the node by combining with the traffic volume loaded by the transportation network node. The duality theory is also quoted to identify the critical links. Assessment methods and algorithms for the important degree of critical links on transportation network based on connect reliability which was formulated in this paper, and proved effective by a numerical example. The evaluation method can be served as a reference for the transportation planning and management of the transportation network.*

***Keywords:*** *Connect reliability of transportation network, Critical links, Traffic volume*

## 1. Introduction

The sound development of the social economy is inseparable from a stable and reliable transportation network. There exists an increasing demand for a stable and reliable service of transportation network. The reliability of transportation network has become one research hotspots. Current researches on the reliability of urban transportation network can be divided into three categories: capacity reliability, travel time reliability and connect reliability. As one of earliest research areas in transportation network reliability, connect reliability has began to attract widespread spotlights since the Kobe earthquake in Japan and gradually becomes one of the most popular branches of transportation network reliability. The connect reliability is a key index for the reliability of urban transportation network and it plays an important role in the reliability analysis of the transportation network.

Connect reliability was originally used to analyze the degree of connectivity between two points within the transportation network under the conditions of disasters and evaluate the network's transportation function. The analysis of traditional connect reliability, confined by huge quantity of calculating data, is difficult to be applied in the urban transportation network of large scale and high density. In the urban transportation network, different links of the network hold various status, which in return to focus on critical sections and improve the reliability of the entire transportation network.

## 2. The Review of Connect Reliability

In 1982, Japan's Mine and Kawai first proposed the concept of connect reliability, which reflected the probability of the transport network to maintain connect between any two nodes, it generally only focused on the 0/1 two states of the link or node, namely connected or disconnected.(Mine and Kawai, 1982). In 1989 Iida and Takayama supplemented the study of Mine and Kawai, extending connect reliability from two points to the k-point connect reliability and network connect reliability (Mine and Kawai, 1989). Subsequently, Iida and Bell studied the method calculating the connect reliability based on their research on graph theory method in 1992 and 1997(Bell and Iida, 1999); Their

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researches only considered two running state, namely, traffic unimpeded and completely disrupted, link status 0 (disconnected) or 1 (connected),regardless of the link capacity time-varying.

The traditional connect reliability evaluation method is mainly based on the analysis of the transportation network graph theory, But the transportation network of high density and large scale not only requires a tremendous amount of calculation, but also presents an insensitive reflection of the real situation. Different links of the transportation network have various degrees of influence and possibilities for the function of the whole network. The most influential links with the most possibilities to be destroyed was vital for connect reliability of the transportation network. Hence, to facilitate the connect reliability of the whole network, the improvement of the key links should be put on top of the agenda. The evaluation methods for connect reliability based on above ideas are put forward by some scholars. Nicholson and Du, Berdica have defined two key points about vulnerability concept, using the method of sensitivity analysis and reliability analysis with the identification of the key unit of transportation network and provides the reliability of network system by improving the key unit(Nicholson and Du, 1994;Berdica, 2000). D'Este and Taylor proposed a reliability evaluation method for the key point of a transportation network whose small portion was disastrous (D'Este and Taylor, 2001). Hossain referred the Birnhaum probability importance and defined the route importance based on the index of consumers' surplus reliability with which he used to improve network performance through heuristic algorithm as well, but still an enormous calculation for a large scale net(Hossain and Sated, 2005). According to Sanso, the most important system state should be prioritized to evaluate the reliability of transportation network .It is vital importance to find out the key link to facilitate the network and optimize the performance with the consideration of random accidents .The so-called critical link refers to those road with an enormous impact on network performance once they closed, especially under the conditions of natural disasters(Sanso and Milot, 1999). Sanchez defined accessibility index node as the objective of network performance optimization. But the accessibility index cannot reflect the change of demand elasticity(Sanchez and danielsm, 2005).In 2006 Liu Haixu and Pu yun ,based on the key links, raised transportation network optimization theory holding that the key link for the probability of being selected is bigger than a predetermined threshold section. Liu Hai-Xu built a transportation network reliability improvement model based on key links using the heuristic method which can identify key links proposed by Taylor(Taylor, 2003), and the improved model is established based on the key links of the transportation network reliability without measure methods on critical link(Liu Haixu, Pu yun, 2006); Gao Yong combined the city basic traffic data and started from the actual operation, putting forward a kind of connect reliability evaluation method based on key links and managed to avoid too much computation, but this method did not analyze the influencing factors, which caused the result. It is a kind of afterward result according to experience for improvement to certain extent but not for prediction for transportation network performance(Gao Yong, 2008). Hou Liwen considered direct measurement of the relative importance was not feasible, and then he figured out the solution to find key links based on connect reliability. But the computation of network connect reliability is difficult. (Hou Liwen and Jiang Fu, 2004).

Connect reliability analysis method based on the key point is a new way for researching connect reliability, but the key point identification method is not yet mature and it is still in the earlier research stage. Recent years, the rise and development of complex network science provided the theoretical support for varieties of networks existence and also provided a way for the study of the transportation network reliability. To distinguish it from general networks, transportation network is weighted network. The network with the same structure but the traffic volume is different, the important degree obviously vary (Chen Jing and Sun Lin fu, 2009).We can make use of the dual graph theory to identify the key link with the identification of the key point. Therefore, this article evaluates the node importance with the consideration of traffic volume in the network.

## 3. A Evaluation Model of the Important Degree of Transportation Network Node

Urban transportation network structure is mainly composed of nodes and links. In general, a transportation network topology graph is represented by $G(V,E)$, $V=(v_1,v_2,\cdots v_i,\cdots v_n)$ represents the set of nodes in the transportation network, $v_i$ for network node; $E=(e_1,e_2,\cdots e_i,\cdots e_m)$ represents a collection of links in the transport network. So the transportation network graph $G$ has formed by $n$ nodes and $m$ links, The adjacency matrix of network graph $G$ is $A=[a_{ij}]$:

$$a_{ij}=\begin{cases}1, \text{node i and node j connected to a link} \\ 0, \text{node i and node j without link connected}\end{cases}$$

The meaning of other symbols are defined as follows.

$W$ :the collection of OD pairs in the transportation network;

$w$ : an element of the collection $W$ ;

$P_w$ : collection of paths for the OD $w$ ;

$P$ :collection of all paths in the transport network, namely $P=\cup_{w\in W} P_w$;

$q_w$ :traffic volume between OD $w$ ;

$q$ :vector representation of all ODs' traffic volume, namely $q=(q_w : w\in W)$;

$x_{ij}$ :traffic flow on the links of $(v_i, v_j)$;

$x$ :vector representation of all sections of the traffic flow, namely $x=(x_a : a\in E)$;

$f_{wr}$ : traffic flow on the path $r$ between the OD pair $w$ ;

$f$ : vector representation of all paths' traffic flow, $f=(f_{wr} : w\in W, r\in P_w)$;

$\delta_{ar}$ :0-1variable,if path $r$ use section $a$ ,then $\delta_{ar}$ is 1,otherwise, $\delta_{ar}$ is 0;

$\delta_{war}$ : 0-1variable, if the path $r$ between OD pair $w$ use the section $a$ ,then $\delta_{war}$ is 1,otherwise,is 0;

$\hat{f}$ :vector representation of all types of traveler route traffic flow $\hat{f}=(f_{wri} : w\in W, r\in P_w, i\in I)$.

Definition1:Node closeness. Assuming that $l(v_i,v_j)$ indicates the shortest path length from the starting point $v_i$ to the end node $v_j$, so the closeness of $v_i$ is $D(i)$,

$$D(i)=1/\sum_{j=1}^{n} l(v_i,v_j), i\neq j \tag{1}$$

According to definition 1, the higher the node closeness degree $D(i)$ is, the closer the node resides in the network center, and the more important the node is in the global network.

Definition 2: The neighborhood of the node $v_i$ .Adjacent nodes with $v_i$ constitute a collection $F_{ki} = \{v_j \mid a_{ij} = 1\}$ ; the degree of the node $v_i$ is the number of edges which associated with $v_i$ , namely, $k = \left|F_{ki}\right| = \sum_{v_j \in F_{ki}} a_{ij}$ .

Definition 3: Node's key degree. The node degree of $v_i$ is $C(i)$ ,in its neighborhood $F_{ki}$ ,if $k \geq 2$,suppose node $v_i$ of the shortest path between any node pair is $S(i)$ ,Without going through the number of the shortest path to the node $v_i$ is $B(i)$ ,so the key degree of node $v_i$ is $C(i)$ **.**

$$C(i) = S(i) / (S(i) + B(i)) \tag{2}$$

The larger the node's key degree $C(i)$ is, the more the shortest paths through the node are, the more important the node is in its neighborhood $F_{ki}$; The smaller the node's key degree $C(i)$ is, indicating the less the shortest paths through the node are, the less important the node is in its neighborhood $F_{ki}$ ,and for the whole network, the less important the node is.

Definition 4: In the transportation network's neighborhood of $F_{ki}$ ,traffic flow through the node $v_i$ is $c_i$ ,all ODs traffic flow is $Q_F$ .The node's key degree in transportation network is $K_i$ .The $K_i$ is difined according to the following formula(3).

$$K(i) = S(i)c_i / (S(i) + B(i))Q_F \tag{3}$$

$c_i$ , $Q_F$ can be solved as follows:

$$\sum_{w \in W} \sum_{r \in P_w} f_{wr} \delta_{ar} = x_{ij} , \quad a \in A , f_{wr} \in f \tag{4}$$

$$\sum_{w \in W} x_{ij} = c_i \quad x_{ij} \in x \tag{5}$$

$$\sum_{w \in W} q_w = Q_F , \quad q_w \in q \tag{6}$$

$$f_{wr} \geq 0 , \quad w \in W , r \in P_w \tag{7}$$

Definition 5 **:**Key field. The key domains of node $v_i$ is

$$G_i = \left\{ v_s \mid v_s \in (F_{v_s s} \bigcap_{\forall v_s, v_t \in F_{ki}} F_{v_t t}) \bigcup F_{ki} \right\} .$$

According to definition 5 ,in the $F_{ki}$ , the shortest paths set for any two nodes between $v_s$ and $v_j$ , $P(v_s, v_j) = \{v_s, v_j\} \bigcup \{v_s, v_i, v_j\} \bigcup \{(v_s, v_i^{'}, v_j) \mid v_i^{'} \in F_i, v_i^{'} \neq v_i\}$ .Assuming that the number of the shortest paths $w_{sj}$ from node $v_s$ to $v_j$ ,Then $S(i)$ and $B(i)$ can be calculated according to the following formula:

$$S(i) = \sum_{F_i} s(i) , s(i) = \begin{cases} w_{sj}, v_i \in P(v_s, v_j) \\ 0, v_i \notin P(v_s, v_j) \end{cases} \tag{8}$$

 

$$B(i) = \sum_{F_i} b(i) \, , b(i) = \begin{cases} 1, v_i \in P(v_s, v_j) \\ 0, v_i \notin P(v_s, v_j) \end{cases} \tag{9}$$

The formula (3) can be applied to calculate node's key degree $K_i$ .

After step (4)~(8), the node $v_i$ 's network key degree can be calculated with formula (3).

Let $T(i) = D(i)K(i)$ (10) as node $v_i$ important degree. The bigger $T(i)$ is, the more important the node is in the network.

The importance degree of nodes in a transportation network first depends on the position of the nodes in the network. For instance, the importance degree of "the peripheral node" and "non-peripheral node" in the transportation network is clearly not consistent, so is the importance degree of "the central node" and "the non-central node". Secondly, the importance degree of the node in the transportation network is also dependent on its connectivity. In other words, the more the shortest paths through the node are, the more important the node is in the network, and the more influential it is to the network connect. Finally, the importance of the node in the network is closely related to traffic load of the node. Apparently, those nodes with a larger flow are more important in real life.

## 4. Evaluation Algorithm of the Importance Degree of a Transportation Network Node

According to the above definitions, the importance degree of a network node depends on the node's location and the traffic flow loaded in complex transportation networks.

The evaluation algorithm for the connect reliability based on the key links of the transportation network is given as follows:

Step1:for $i$ =1 to $n$

Step2:calculate the shortest paths $l(v_i, v_j)$ from node $v_i$ to any other node $v_j$ in the network;

Step3: calculate the closeness of the node $v_i$ with formula (1);

Step4:calculate node $v_i$ 's neighborhood $F_{ki}$ and key field $G_i$ ;

Step5:for(each pair of nodes( $v_i$ , $v_j$ ) in $F_{ki}$

{find the shortest path set P( $v_i$ , $v_j$ )

calculate $c_i$ and $Q_F$ with the formula (4~7);

calculate $S(i)$ and $B(i)$ with the formula(8,9); }

Step6: calculate $K_i$ with the formula (3)

Step7: calculate $T(i)$ with the formula (10)

From the steps of the above algorithm, the time complexity of the algorithm depends on the calculation of $l(v_i, v_j)$ in step (2) and node $v_i$ 's key degree $K_i$ in step (5). If the complexity of the algorithm in step (2) is $o(n^\alpha)$ , and the algorithm complexity of step (5) is $o(k^3)$ , the whole complexity of the algorithm is $o(n^\alpha + k^3)$ . In the real network, node degree $k$ is far less than the entire network of nodes n, therefore, under normal circumstances, the time complexity of the algorithm is not bigger than $o(n^\alpha)$ , that is to say, the complexity of the algorithm designed by this paper depends on the calculation method of the shortest path.

## 5. Case Study

As shown in Figure 1, a transportation network with red lines. The Figure 2 is corresponding topology map. We can use the dual graph method to obtain the urban transportation network's dual graph topology map through the transformation of edges and nodes (Wan Xu jun, Hu An zhou, 1998) in Figure 3. The nodes traffic information and the degree of importance indicators are shown in Table 1. For calculation purposes, it is assumed that the length of each side is equal, according to the node evaluation algorithm, the result of calculating the degree of importance of each node in the network is in Table 1.

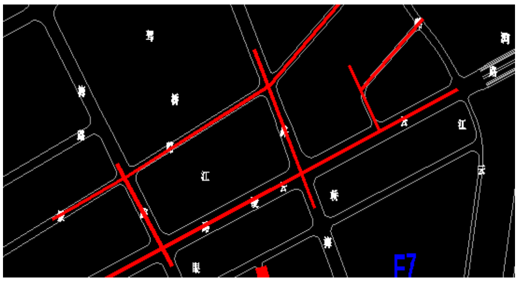

**Figure 1. A Transportation Network**

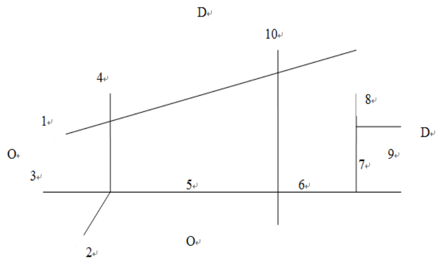


**Figure 2. A Topology Map**

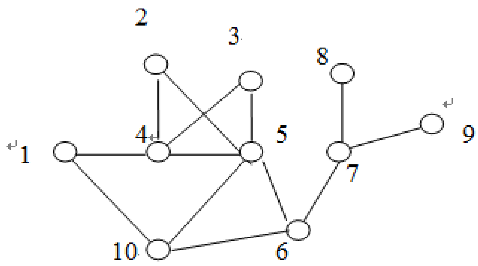


**Figure 3. The Dual Topology Map**

**Table 1. Index Information of Nodes**

| Order | $c_i$ / $Q_F$ | Closeness | Key degree | Importance degree |
|---|---|---|---|---|
| 1 | 0.267 | 0.048 | 1.000 | 0.013 |
| 2 | 0.200 | 0.043 | 02.000 | 0 |
| 3 | 0.267 | 0.048 | 0.000 | 0 |
| 4 | 0.187 | 0.053 | 0. 667 | 0.007 |
| 5 | 0.280 | 0.067 | 0. 667 | 0.014 |
| 6 | 0.307 | 0.071 | 0. 667 | 0.014 |
| 7 | 0.307 | 0.053 | 1.000 | 0.016 |
| 8 | 0.160 | 0.037 | 0.000 | 0 |
| 9 | 0.147 | 0.037 | 0.000 | 0 |
| 10 | 0.267 | 0.059 | 0. 667 | 0.010 |

From the calculation results of table 1, the importance of the peripheral nodes2, 3,8 and node 9 is consistent. The importance degree of node 7 occupies first place, the importance degree of node 5 and 6 come second, which is consistent with the actual results.

## 6. The Conclusion

With the growing demand of the urban traffic, the scale of urban transportation network is expanding, which makes the research of urban transportation network reliability more and more complex and difficult. In the urban transportation network, the influence of different links on reliability varies due to their different positions in the whole network and various traffic sharing rate and thus bears different stakes in the reliability. To improve the reliability of the whole network through key links' facilitation is a necessary method.

Combined with the complex network theory and the traffic load, this paper puts forward the evaluation method for the importance of transportation network nodes, which can identify the key links and combine the whole and partial importance of nodes in the complex transportation network. Moreover, it overcomes existing problems of the complexity of deleting nodes calculates the number of nodes and simplifies the calculation of connect reliability of transportation network. And the example analysis shows the effectiveness of the proposed method. This method can provide a reference for transportation planning and management. Traffic is a dynamic process. So the further research should consider the random variation of the transportation network and discussing more reasonable evaluation method for links.

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