METANET Model Improvement for Traffic Control

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Abstract

Variable Speed Limits (VSL) and Coordinated Ramp Metering (CRM) are the main control strategies for freeway traffic. Model-based control design for CRM requires density (or flow) dynamics with the onramp inflow rate as the control variable; and the VSL design requires speed dynamics with the desired speed as the control variable. The METANET model provides a suitable structure for this control design, but previous work with METANET parameterized the speed control variable to be highly nonlinear, which caused difficulty in control design and implementation. Furthermore, the previous METANET model could not catch quick and significant changes in traffic dynamics. Therefore, the dynamics model itself needs improvement. This paper suggests improvement in two aspects: (a) drop the nonlinear parameterization in the speed control variable and directly use linear control, which greatly simplifies the control design; and (b) propose several alternatives for the convection term of the speed dynamics to improve model matching. Model calibration using Berkeley Highway Lab (BHL) field data and simulation for comparison are presented to show the effectiveness of the improvements.

Keywords: freeway traffic modeling; second-order model; METANET model; model improvement; model calibration; traffic control

1. Introduction

Freeway Active Traffic Management (ATM) is a large-scale systems challenge, because it requires combining several means to control traffic for better performance. Variable Speed Limits (VSL) and Coordinated Ramp Metering (CRM) are the backbone of ATM. CRM directly controls the demand into the freeway and, therefore, the average density immediately downstream of the onramps. Once a driver gets onto the freeway, the traffic characteristics are determined mainly by driver behavior. VSL influences driver behavior and, therefore, could be used to regulate traffic flow. The two approaches are complementary to each other. Due to the highly stochastic property of traffic, as well as sensor measurement limits and traffic state estimation errors, for control design, it is desirable to have a model that predicts
the traffic conditions for a particular period to reduce time delays and to improve control performance.

The goal of this paper’s model improvements is to support design of freeway traffic control strategies combining VSL and CRM. To achieve this, it is necessary to accurately represent both speed and density (occupancy) dynamics. It is well known from the control systems theory that robust control design methods allow some model mismatches, although the performance of the control will degrade as the model mismatch increases. Reducing model mismatching will directly improve control performance. In traffic control design, there are four main functions of the traffic dynamics model:

- Simulation: representing traffic dynamics with a given set of initial and boundary conditions. Simulation is essential in control design, for evaluating the performance of the controllers to select the most promising one;
- Prediction in real-time control implementation: for real-time control, one has to predict at least one step ahead based on previous information and current time step measurement; for Finite Time Horizon Predictive Control, one needs to predict several steps (Finite Time Horizon) ahead;
- Control synthesis: to construct control variables based on measurement and current state estimation;
- Optimal state estimation: the model could be used in conjunction with current measurements to provide an optimal estimation of current state variables to feed into the control.

To design a model-based controller to achieve good performance, there are two model requirements:

- Capture the traffic dynamics in the relevant traffic situations, including free flow and all the congested equilibrium states and non-equilibrium transitions between them.
- Be simple enough to incorporate the real-time control implementation. An excessively complicated (highly nonlinear) model, which could capture the traffic dynamics better, may not be suitable for control implementation since the formulated control problem could have no mathematical solution, or there may be no efficient numerical solution even if the solution can be proven to exist.

Therefore, there is often a subtle trade-off between model complication and faithfulness, subjected to the control performance requirement. This paper shows that simplifications in the METANET model does not degrade the model match with the traffic dynamics.

The paper is organized as follows: Section 2 is a brief literature review; Section 3 presents some details about the METANET model; Section 4 reports the main results for model improvement; Section 5 uses field data for model calibration and comparison with simulated data; the paper is ended with some conclusions and future work in Section 6.

2. Literature Review

In the literature, there are at least three macroscopic traffic modeling approaches:

(1) Based on the physics of fluid flow, traffic is considered as compressible flow. Representative work in this approach is the well-known Lighthill and Whitham (LWR) model [20, 21] and Richards [40], and later developments by Newell [30, 31, 32]
Daganzo [9, 10, 11] and Zhang [44, 45, 46, 47]. A good collection and review of the kinematic wave models and its development history can be found in Gartner et al., [15].

(2) Based on driver behaviour and intuitive understanding of traffic behaviour, such as prediction and delay. Representative work in this direction is the (METANET) model by Payne [37], which is in a form suitable for control purposes. It was used for traffic state parameter estimation, and for ramp metering control by Payne et al., [38] and Isaksen [19]. The model was later improved by Papageorgiou [33]. It is essentially a second-order model with coupled density and speed dynamics. The model has been further improved in Papageorgiou et al., [35] by introducing the weaving effect due to lane changing. Since the authors intended for ramp meter control and different ramp meter strategy evaluation only, there was only one control variable – the ramp meter rate. The speed dynamics is not intended to be independent; instead, it generates a reference speed based on a static density-speed relationship – the Fundamental Diagram (FD) – and went back to the loop to affect the density dynamics indirectly through coupling. In this sense, the second-order model represents the dynamics of density with some driver behaviour added through the speed dynamics. This model was used in Hegyi et al., [18] for combined Variable Speed Limit (VSL) and Ramp Metering for reducing shockwave. A good reference is Nagel [29] for understanding several second-order traffic models based on fluid dynamics. The explanation is compelling, particularly the car following model and the Payne-Papageorgiou model.

(3) Based on obtaining a second order model by aggregating microscopic traffic models, such as the car following model (Darbha [12]; Tyagi et al., [42]). This approach guarantees that the obtained macroscopic model is compatible with the underneath microscopic model, which is not obvious for other macroscopic models. Aggregation of a set of dynamical systems must follow particular rules: (a) their initial conditions must belong to the same set; (b) the dynamics should have a similar structure; and (c) considering the trajectories and Math Flow, there exists a monic and an epic between two categories. For the collection of vehicle trajectories on freeways, the monic and epic exist. However, the second-order model deduced and discretized in Tyagi et al., [42] is different from any other second-order model in the literature. Zhang [43] considered the consistency of micro and traffic models for the improvement of the second-order model to remove the upstream traffic effect on downstream flow, which is similar to a gas-flow behaviour. The first-order model is the conservation law; and the second-order model considers the dynamics of speed, which is a conservation of momentum:

\[ v_s + vv_s = -c(\rho)v_s \]

which was deduced from the car following model:

\[ \tau(s_v(t)) \dot{x}(t) = \dot{x}_{a-1}(t) - \dot{x}_a(t) \]

\[ s_v(t) = x_{a-1}(t) - x_a(t) \]

\[ \tau(s_v(t)) \] – following distance

\[ \tau(s_v(t)) \] – driver response time
\(-c(\rho) = -\rho V_{\text{s}}(\rho) = \frac{v}{\tau(s)}\) is an introduced concept – traffic shock wave propagation speed.

However, the deduction simply dropped the index for individual vehicles, which is different from the principles suggested in Darbha [12].

The most popular first-order model for traffic control purpose is the Cell Transmission Model (CTM) from Daganzo [8, 9]. To understand the equivalence of the model to the first order LWR model, it is necessary to look at traffic states and their units in the dynamics. The original LWR dynamics is:

\[
\frac{\partial q(x,t)}{\partial x} = -\frac{\partial \rho(x,t)}{\partial t}
\]

or in a different form:

\[
\frac{\Lambda}{\Delta x} \left( \min \{ v \rho \cdot q_{\text{max}}, v(\rho_j - \rho) \} \right) = \frac{\rho(x,t + \Delta t) - \rho(x,t)}{\Delta t}
\]

If the equation is discretized by introducing the concept of cell as in Daganzo [8, 9], it can be written as:

\[
\rho(i,k + 1) - \rho(i,k) = \frac{1}{v(i,k)} \left[ Y(i,k) - Y(i+1,k) \right]
\]

\(Y_i(i,k) = \min \{ v(i,k) \cdot \rho(i,k), q_{\text{max}}, w(i,k) \cdot (\rho_j - \rho(i,k)) \} \)  \hspace{1cm} (2.1)

The first equation can be equivalently represented as,

\[
\rho(i,k + 1) = \rho(i,k) + \frac{T}{L} \left[ Y(i,k) - Y(i+1,k) \right]
\]  \hspace{1cm} (2.2)

where \(i\) is the cell index and \(k\) is the time index;

\(\rho(i,k)\) – density in cell \(i\) at time period \(k\);

\(v(i,k)\) – distance mean speed in cell \(i\) at time period \(k\);

\(w(i,k)\) – shockwave back-propagation speed in cell \(i\) at time period \(k\);

\(q_{\text{max}}\) – maximum flow of the cell to be identified as a constant parameter – the capacity;

\(\rho_j\) – jam density, a known constant parameter.

Model (1.2) is not a closed dynamical system since it contains speed variable \(v(i,k)\), which is not supposed to be in the dynamics; although, the shockwave speed \(w(i,k)\) is a known constant for traffic with high enough density. To overcome this difficulty, the FD is assumed, which is a static flow-density relationship \(q = q(\rho)\) or speed-density relationship \(v = v(\rho)\). Once the relationship is modeled for each cell, the first and the third term in the expression of

\[
Y_i(i,k)
\]

can be evaluated. Refined quantitative behaviour of the traffic is needed at a reasonable resolution in both time and space to achieve high control performance, because both time
delays and model discrepancies degrade control performance. With higher aggregation of traffic data, larger time delays are introduced.

As recognized in recent years, ramp metering can only control the demand from onramps and the average density immediately downstream, while indirectly affecting the traffic upstream by the back propagation of congestion. Ramp metering does not influence driver behavior on the main lane. To control main lane flow, VSL is one possible measure. Several implementations have been conducted in the UK, France, Germany and the Netherlands using VSL to harmonize traffic, mainly for safety rather than for mobility improvement. VSL attempts to control the collective vehicle speed (or driver behavior) of main lane traffic is complementary to RM. Generally, it is accepted that crashes and incidents can be reduced between 25-40% by using VSL [43] in recent years, VSL has been investigated in both theory and practice for mobility improvement (Hegyi et al., [18]; Carlson et al., [3, 4]).

3. The METANET Model

The METANET model was selected for further study because: (a) control design would be more convenient using ordinary differential equations; (b) control variables in the system correspond to practical control measures in highways – in this case, the onramp flow and desired speed or VSL; (c) the control variables are well-posed in the model for convenient control design (to select feedback control). Highly nonlinear control parameters will cause control design to be very difficult or even impossible. Even if one can theoretically design the controller, practical implementation cannot be conducted simply because there is no efficient numerical solution. It is obvious that to design a combined VSL and CRM, one needs to consider speed dynamics as well as density dynamics. This is the main motivation for investigating the second-order model involving both density and speed dynamics.

The following assumptions are made throughout the rest of the discussion:
- A freeway corridor is divided into links;
- Each link contains exactly one onramp meter;
- Each link may have any number of off-ramps;
- Each link could be divided into cells if necessary; for simplicity, each link is considered as a cell;
- Each link contains at least one traffic sensor;
- \( m \) – link (or cell) index;
- \( T \) – time step for model update;
- \( L_m \) – length of link \( m \);
- \( m \lambda \) – number of lanes in link \( m \);
- \( (\rho_m (k), v_m (k), q_m (k)) \) – density, distance mean speed and flow for link (cell) \( m \) at time \( k \);
- \( s_m (k) \) – onramp and off-ramp \( m \) flow, measured;
- \( r_m (k) \) – off-ramp flow, to be determined;
- \( u_m (k) \) – speed control variable, to be determined.

Payne [37] deduced the speed dynamics based on the four assumptions:
- Distance mean speed \( v(x,t) \) at space-time point \( (x,t) \) depends on the density downstream with distance increment \( \Delta x \).
There is a time delay due to driver’s response in speed adjustment; therefore, 
\[ v(x + \tau, t) \] was used instead;

Drivers can predict the traffic speed of the next cell; although, this assumption was
not necessarily true in the past, it is reasonable in a VSL and CRM strategy since the
traffic situation downstream is taken into account in the control design. This process
implicitly helps the driver to predict and incorporate traffic downstream;

There is a static relationship between speed and density described as

\[
v(x, t + \tau) = V \left( \rho \left( x + \Delta x, t \right) \right)
\]  
(3.1)

which could be interpreted as: the density in the \( v - \rho \) relationship of the FD has been
predicted ahead over distance \( \Delta x \), but the average driver’s response has been delayed by \( \tau \) in time. Using Taylor series expansion on both sides of the equation with respect to \( t \) and \( x \)
respectively and discretizing it, the following speed dynamics equation is obtained:

\[
v_m(k + 1) = v_m(k) + \frac{T}{\tau} (v_m(k) - v_{m-1}(k)) - \frac{T}{L_m} v_m(k) (v_{m-1}(k) - v_{m-1}(k)) - \frac{\nu T}{\tau L_m} \frac{\rho_{m-1}(k) - \rho_m(k)}{\rho_m(k) + \kappa}
\]  
(3.2)

where \( T \) - time step length
\( L_m \) - cell length
\( \tau, \nu, \kappa \) are parameters to be calibrated from field data.

A quantitative ingredient is implicitly introduced here: since both the cell length and time
step are involved in the model, and time step is usually fixed a priori, not all the vehicles of
cell \( m \) will fit into the link \( m+1 \) after each time step. Instead, only those vehicles with the
running distance of that time period greater than the cell length will get into the next cell. This
is significantly different from the assumption in CTM, in which it was assumed that all the
vehicles in the current cell will be in the cell downstream at the next time step. This
assumption is reasonable for CTM since no speed dynamics were involved.

Each term on the right side of the model (3.2) could be interpreted as follows:

1) **Relaxation term:** It is a high gain filter \( \frac{1}{\tau} \), the high gain with small \( \tau \) from a dynamical
systems viewpoint. The collective of drivers is trying to achieve the desired speed
\( v(\rho_m(k)) \) - the control variable. The selection of the desired speed is critical to reflect
driver behavior. The \( \tau \) can be interpreted as the average response delay of the driver to
the desired speed. The smaller \( \tau \), the faster \( v_m(k) \) approaches the desired speed (or, the
faster the driver response is).

2) **Convection term:** the effect of the traffic into the downstream cell from upstream cell,
i.e., the speed increase/decrease caused by in-flow and out-flow vehicle speeds. It can be
modified by adding a factor \( sat(\rho_{m-1}/\rho_m) \) where \( sat(\cdot) \) is the saturation function to
address the driver speed change with respect to density variation between the two
consecutive cells Cremer and Papageorgiou [7]. This term is the most sensitive one in
speed dynamics and will be discussed further.
(3) Density gradient term: when downstream density increases/decreases, the speed in the current cell will decrease/increase;

\[
\frac{-v T}{\tau L_m} \rho_{u-1}(k) - \rho_u(k) = -\frac{1}{\tau L_m} \left( \frac{v T}{\rho_{u-1}(k) - \rho_u(k)} \right) + \frac{1}{\tau L_m} \left( \frac{v T}{\rho_u(k) + \kappa} \right)
\]  

(3.3)

where \( \tau \) is the time delay for the response of a collective of drivers to the perception of the traffic density (basically, what each driver could observe is the inter-vehicle distance in the immediate vicinity, which could be interpreted as the driver version of local density); \( v \) is a sensitivity factor. The part in the bracket expresses the effect of downstream cell density: the higher the downstream density, the lower the speed for the current cell. \( \rho_u(k) \) is the denominator is for normalization. The parameter \( \kappa > 0 \) is added for two purposes:

- To force the model to only work for medium to high density;
- To avoid the singularity or the sensitivity of the term to the model in low density situations.

The physical meaning of the three terms including the parameters was also explained in detail related to driver behavior by Cremer and Papageorgiou [7]. Those explanations could be used as the basis for parameter identification.

Since \( v_u(\rho(k)) \) is basically the speed control parameter to be designed, it could be parameterized with any other value instead of density \( \rho_u(k) \) or without parameterization. For example, it could be parameterized with flow \( q(k) \). Doing this is just a matter of coordinate transformation. However, parameterization with density directly links the control design to the shape of the FD.

Putting together the density dynamics and speed dynamics, the METANET model is obtained as follows (Payne [37]; Messmer and Papageorgiou [27]; Papageorgiou et al [35]):

\[
\rho_u(k + 1) = \rho_u(k) + \frac{T}{L_m} (\rho_{u-1}(k) - \rho_u(k) v_u(k)) + r_u(k) - s_u(k)
\]

\[
v_u(k + 1) = v_u(k) + \frac{T}{\tau} (V(\rho_u(k), b_u(k)) - v_u(k)) + \frac{T}{V_u(k)} (v_{u-1}(k) - v_u(k)) - \frac{1}{\tau L_m} \left( \frac{v T}{\rho_{u-1}(k) - \rho_u(k)} \right) + \frac{1}{\tau L_m} \left( \frac{v T}{\rho_u(k) + \kappa} \right)
\]  

(3.4)

Parameters \( (\tau, v, \kappa) \) are to be determined in model calibration using field data. In the equation (3.4), the control variable \( V(\rho_u(k), b_u(k)) \) was re-parameterized in (Papageorgiou [33]; Messmer and Papageorgiou [27]; Hegyi [17]; Hegyi et al [18]) as:
\[ V \left( \rho_m (k), b_m (k) \right) = v_f \left[ b_m (k) \right] \cdot \exp \left[ - \frac{1}{a \left[ b_m (k) \right]} \left( \frac{\rho_m (k)}{\rho_v \left[ b_m (k) \right]} \right)^{a (v)} \right] \]

\[ v_f \left[ b_m (k) \right] = v_f^* \cdot b_m (k) \Rightarrow b_m (k) = \frac{v_f \left[ b_m (k) \right]}{v_f^*} ; \quad (3.5) \]

\[ \rho_v \left[ b_m (k) \right] = \rho_v^* \left[ 1 - 2 A \left( 1 - b_m (k) \right) \right] \]

\[ a \left[ b_m (k) \right] = a^* \left[ E - (E - 1) b_m (k) \right] \]

where \((v_f^*, \rho_v^*, a^*, E, A)\) are to be determined from field data in the model calibration process.

- \(v_f^*\) - the free-flow speed;
- \(\rho_v^*\) - critical density for traffic to break down from free flow or from a homogeneous flow;
- \((a^*, E, A)\) - model parameters for FD in the form of speed-density relationship.

The default values of the parameters used in the METANET model are:

- \(v_f^* = 115 \text{ km/h}\);
- \(\rho_v^* = 28.2 \text{ veh/km/ln}\);
- \(a^* = 2.15\);
- \(E = 1.9\);
- \(A = 0.7\)
- \(\tau = 18 \text{ s}\);
- \(\kappa = 400 \text{ veh/lane}\);
- \(v = 60 \text{ km/h}\);
- \(T = 10 \text{ s}\)

The parameterization in the METANET model is designed to link the speed control variable \(b_m (k)\) to the speed and density in the FD relationship. The purpose of the parameterization could be explained as:

- Desired speed and density are restricted to follow the FD curve;
- Control parameter \(b_m (k)\) determines the shape of the FD; therefore, selection of control strategy determines the shape of desired FD;
- The objective of VSL control is to make the speed and density follow the desired FD.

The authors believe that the model needs improvement in the following aspects:

- The control variable \(b_m (k)\) appears highly nonlinear in the dynamics of (3.4); if the constraints of the control variable are not set properly, the feasible set (decision parameter domain) could be very small, which is likely to cause problems for the numerical optimization process;
- Density and, thus, ramp meter rate through the density dynamics (3.4) are tightly and nonlinearly coupled with the speed control variable \(b_m (k)\) (3.5), which further complicates the control design, particularly the Model Predictive approach;
- Most importantly, the model is deduced from the FD, which is intrinsically a static relationship. Therefore, the model may not be able to capture fast transition phases of the traffic dynamics very well, which can be observed in the simulation.

Those points are the main motivation of this paper.
4. Model Improvement

Model improvement in this paper includes both simplification and modification:

- Dropping the re-parameterization of the speed control variable in the METANET model and directly using the desired speed as the control variable. This removes the restriction of speed and density strictly following the FD curve;
- Using a modified CTM to represent density dynamics, which is essentially the conservation of flow with some practical constraints; this removes some anomalous behaviour of the METANET model (Daganzo [10]);
- Moving the nonlinear terms in the density dynamics to the constraints;
- Determining the feasible set of the system by adding extra constraints on the speed-density plane based on traffic speed drop probability analysis (Chow et al [5]);

The model can now be written as:

\[
\begin{align*}
\rho_n(k+1) &= \rho_n(k) + \frac{T}{L_n}(\rho_{n-1}(k) - \rho_n(k)) + r_n(k) - s_n(k) \\
v_n(k+1) &= v_n(k) + \frac{T}{\tau}(u_n(k) - v_n(k)) + \frac{T}{L_n}v_n(k)\{v_n(k) - v_n(k)\} - 1 \left( \frac{v_n(k) - \rho_{a-1}(k)}{L_n} \right) \rho_n(k) + \kappa \\
\end{align*}
\]

Where \( u_n(k) \) is the speed control variable. The following constraints are proposed to address the system dynamics property and traffic characteristics:

\[
\begin{align*}
0 &\leq \rho_n(k) \leq Q, \\
0 &\leq v_n(k) \leq V, \\
0 &\leq u_n(k) \leq \psi_n\left(\rho_n(k)\right), \text{ or } u_n(k) \leq \rho_n(k) \\
0 &\leq u_n(k) \leq V, \\
r_n(k) &\leq s_n(k) + v_n(k) \rho_f, \\
r_{n,\text{ana}} &\leq r_n(k) \leq r_{n,\text{ana}} \\
\end{align*}
\]

where \( \rho_j \) – jammed density; \( V_j \) – flow speed. The first three constraints in (4.2) are constraints on state variables; and the fourth and fifth constrains are for VSL; and the sixth and seventh are on the ramp metering rate.

The following discussion justifies the constraints. It is clear that the second, third, and fifth are just simple physical limits. In the CTM (Daganzo [8]), the following equality is based on physical limits of the traffic flow:

\[
q_n(k) = \min\{v_{n-1}(k) \rho_{n-1}(k), Q_n, v_n(k)(\rho_j - \rho_{n-1}(k))\} \tag{4.3}
\]

If we take into account the flow from onramp and off-ramp, (4.3) becomes

\[
q_n(k) = \min\{v_{n-1}(k) \rho_{n-1}(k) + r_n(k) - s_n(k), Q_n, v_{n-1}(k)(\rho_j - \rho_{n-1}(k))\} \tag{4.4}
\]
\( q_m (k) \) assumes one of the three possible values in braces. If it assumes the first value, then
\[ q_m (k) = v_m (k) \rho_m (k) = v_{m-1} (k) \rho_{m-1} (k) + r_m (k) - s_m (k) \]  
(4.5)

It is the conservation of the flow, or the first dynamics in (4.1). If
\[ q_m (k) = Q_m \]
it is the first inequality of (4.2). If
\[ q_m (k) = v_{m-1} (k) (\rho_j - \rho_{m-1} (k)) \]

then
\[ \rho_m (k) \cdot v_m (k) = v_{m-1} (k) (\rho_j - \rho_{m-1} (k)) \Rightarrow \]
\[ r_m (k) - s_m (k) = v_{m-1} (k) \rho_j \Rightarrow \]
\[ r_m (k) = s_m (k) + v_{m-1} (k) \rho_j \]
in which (4.5) is used. This justifies the sixth constraint in (4.2). Here the \( v_{m-1} (k) \) can be predicted based on the speed dynamics of the last time step. If \( s_m (k) \) is measured or predicted, then the constraint is again linear with respect to the control variable \( r_m (k) \).

The last three constraints are for the control variables – VSL and ramp meter rate added for traffic operation. \( (r_{m,m}, r_{m,x}) \) are minimum/maximum ramp meter rates. \( \rho_m (k) = \sigma (a_m (k)) \) is the curve of a specified traffic speed drop probability contour as indicated in Figure 1, with three flow contours for reference. For a given acceptable traffic drop probability, the contour gives an upper bound for the feasible region (Chow et al [5]). The contour is obtained as follows: a bivariate Weibull distribution is adopted to model the probability of breakdown as a function of the combination of mean speed and occupancy (converted to density) of the incoming traffic. Data collected from a section of I-80W Freeway in West Berkeley, California was aggregated to test the performance of the methodology. The contour can be expressed in two forms: as a function of density with respect to speed, or a function of speed with respect to density as in (4.2). Whichever is taken depends on control design requirements. For example, for speed control design \( a_m (k) \leq \psi_m (\rho_m (k)) \) would be more convenient if density is known since it is a linear constraint.

The advantages of dropping the FD constraint are as follows:

- Speed control variable appears linearly, which significantly simplifies the control design problem;
- The model has two degrees of freedom for control design: both VSL and CRM rate are free in the feasible region bounded by the constraint (4.2) in the speed-density plane;
- Model mismatch caused by discrepancies between field data and the FD curve could be avoided.
4.1. Model Improvement on Convection Term of Speed Dynamics

Analyzing the convection term in speed dynamics:

\[
\frac{T}{L_m} v_m(k) (v_{m-1}(k) - v_m(k))
\]

indicates that if the upstream (link m-1) speed in the last time step is greater than the speed of link m, then the speed of link m will likely increase; otherwise, it will decrease. This may be true if the downstream (link m+1) is free flow and the link m is not saturated yet. However, if the downstream (link m+1) is congested and/or the link m is saturated, this may not be correct. This motivates us to consider other possible convection terms in the speed dynamics of the METANET model. The following alternatives are proposed:

(a)

\[
\frac{T}{L_m} v_m(k) [\alpha \cdot v_{m-1}(k) + (1 - \alpha) \cdot v_{m+1}(k) - v_m(k)], \quad 0 \leq \alpha \leq 1
\]

or,

\[
\frac{T}{L_m} v_{m-1}(k) [\alpha \cdot v_{m-1}(k) + (1 - \alpha) \cdot v_{m+1}(k) - v_m(k)], \quad 0 \leq \alpha \leq 1
\]

which means that speed further upstream is used to predict the current cell speed. This is expected to lead to less time delay in speed dynamics.

(b)

\[
\frac{T}{L_m} \left( \sqrt{v_{m-1}(k) \cdot v_m(k)} \right) \cdot v_m(k)
\]

The idea here is to use the geometric mean of current and upstream speed to replace upstream speed, reducing the significance of the upstream speed effect on the speed of the current link, as well as the sensitivity of the term.

(c)
\[
\frac{T}{L_w} \left\{ \frac{v_{n-1}^2(k) + v_n^2(k)}{2} - v_n(k) \right\} \cdot v_n(k) \tag{4.10}
\]

This also reduces the effect of upstream link speed on the speed of the current link.

\[
\frac{T}{L_w} \left\{ \frac{v_{n-1}^2(k) + v_n^2(k)}{2} - v_n(k) \right\} \cdot v_{n-1}(k) \tag{4.11}
\]

This term reduces the effect of the upstream link speed due to the first factor, but increases the significance of the upstream speed due to the second factor \(v_{n-1}(k)\). Model calibration and simulation results show that using this term improves model matching.

### 4.3. Model Application

The model has been applied to control design in several ways:

- Assume known ramp metering and then conduct VSL design (Lu et al., [24]);
- Design combined CRM and VSL with coupled density and speed dynamics (4.1-4.2);
- Design VSL at each step before CRM – the former linearized model, which leads to a linear model for CRM design (Lu et al [24]; the control problem can be formulated as an Linear Programming (LP) at each time step with the Model Predictive Control approach.

From these works, the benefits of model simplification and improvement can be observed. However, there is still a long way to go for model simplification and improvement for convenient control design.

### 5. Model Validation and Simulation Using Field Data

The original and modified METANET models are validated and simulated for comparison in model matching using field data, which includes the following cases: with and without FD, with convection term (4.6) and (4.11). The model has the following common parameters to be identified: \((\tau, \nu, \kappa)\). Other parameters are assumed to have the values suggested by Hegyi [17]. The parameter searching ranges are also referred to in Cremer and Papageorgiou [7]; Papageorgiou et al., [35]; and Sanwal et al., [41].

#### 5.1. Field Data

The Berkeley Highway Lab (BHL) (Figure 2) dual loop data with a one-second update rate and 60 Hz loop on/off information is used for the calibration (May et al., [26]). Twenty days of data was processed by cleaning, imputation and correction, and then averaged across all lanes and aggregated into 20 second (s) and 60s traffic state variables in distance mean speed, density and flow, and then used for model calibration. The calibrated model is then simulated with the following setup:

- 8 sections (links) of I-80 East and West in the BHL are used;
- Onramp and off-ramp flows are ignored since the measurements are not available;

About onramp and off-ramp flow: arbitrarily imputed onramp or off-ramp data may not be appropriate. In principle, one can add onramp flow to the model; however, it is not reasonable
to add flow to the field data since the flow-speed-density relationship will be changed by
doing so.

In the simplified METANET (4.2), the relaxation (equilibrium speed) term is removed in
model calibration and simulation. This implicitly assumes that drivers follow the desired
speed reasonably well. This can also be justified from a dynamical systems identification
viewpoint; i.e., the speed tracks the desired speed quickly with high gain \(1/\tau\) (small enough
\(\tau\)). This is another reason why the parameter searching region in the model calibration has a
limit of \(0 \leq \tau \leq 1.8\).

![BHL with Sensor Coverage, Location and Distance](image)

**Figure 2. BHL with Sensor Coverage, Location and Distance**

Most recently, BHL real-time data at a time step of 30s is available online with speed,
occupancy and flow for the eight stations in two directions.

### 5.2. Model Calibration Method

Objective function: The following objective function is selected for minimizing both speed
and flow errors.

\[
Z = \sum_m \sum_k \left[ (\hat{v}_n(k) - v_n(k))^2 + \sigma (q_n(k) - q_n(k))^2 \right]
\]

Here, \(m\) is the link index, and \(k\) is the time steps in the calibration time period (e.g., 24
hours). \(v_n(k)\) and \(q_n(k)\) are estimated by the model, \(\hat{v}_n(k)\) and \(\hat{q}_n(k)\) are the corresponding
values from field data. The weighting parameter \(\sigma = 0.15\) is used.

Inserting the estimated state from (4.1), it can be observed that model calibration is a
nonlinear optimization problem. For a given region, there may be multiple solutions for a
given convergence threshold; therefore, the search region for optimization is very important.
For this reason, several parameter search ranges have been conducted. The one that is
believed to be most suitable for control design is presented:

Parameter searching range: Let \(\beta = [\tau, \kappa, \nu]\) be the vector of parameters. The search
interval \([\beta_{\text{min}}, \beta_{\text{max}}]\) is suggested as follows: \(\beta_{\text{min}} = [0, 30, 7]\), \(\beta_{\text{max}} = [1.8, 38, 9]\). Such
setup is based on the following considerations: \(0 \leq \tau \leq 1.8\) — the average driver response
delay. Small \(\tau\) will force the speed dynamics to track the desired speed fast, which represents
the practical control situation. Larger $\tau$ might lead to better model matching for some data sets, which explains the driver behaviour without control, but that is not suitable for control since the speed will not track the desired speed and, therefore, will not be adopted. The next two are based on simulation and on previous work (Cremer and Papageorgiou [7]; Papageorgiou et al., [35]; Sanwal et al., [41]).

The performance measure is the averaged Root Mean Relative Square Error (RMRSE), which is defined for time sequence $x$ and its estimation $\hat{x}$ as follows:

$$
RMRSE(x) = \frac{1}{M} \sqrt{\sum_{m=1}^{M} \sum_{k=1}^{n} \left( x_m(k) - \hat{x}_m(k) \right)^2}
$$

where $M$ is the total number of links, and $k$ is the overall number of time steps in the calibration time period.

The searching algorithm is described as follows:

**Optimization method:** 2-D Golden Section Search to find the minimum overall points in the region;

**Calibration algorithm:**

**Step 1:** Calculate the initial solution $\beta^{(0)} := \frac{1}{2}(\beta_{\text{min}} + \beta_{\text{max}})$

**Step 2:** Set the iteration counter, $n := 1$

**Step 3:** Set $p := 1$, where $p$ is the index of the elements (parameters) in vector $\beta$

**Step 4:** While $p \leq |\beta|$, where $|\beta|$ is the number of elements in vector $\beta$, set the following:

1. $[\beta(p)]_{\text{upper}}^{(n)} := [\beta(p)]^{(n)}_{\text{max}}$
2. $[\beta(p)]_{\text{lower}}^{(n)} := [\beta(p)]^{(n)}_{\text{min}}$
3. $[\beta(p)]_l^{(n)} := (1 - 0.618)\left( [\beta(p)]_{\text{upper}}^{(n)} - [\beta(p)]_{\text{lower}}^{(n)} \right) + [\beta(p)]_{l}^{(n)}$
4. $[\beta(p)]_z^{(n)} := 0.618 \left( [\beta(p)]_{\text{upper}}^{(n)} - [\beta(p)]_{\text{lower}}^{(n)} \right) + [\beta(p)]_{z}^{(n)}$

**Step 5:** Evaluate $Z(\beta_1)$ and $Z(\beta_2)$, where $\beta_1$ and $\beta_2$ are the same as $\beta$ except that the $p^{th}$ entry of the two vectors is replaced by $[\beta(p)]_l^{(n)}$ and $[\beta(p)]_z^{(n)}$ respectively.

**Step 6:** Update the search interval:

If $Z(\beta_1) < Z(\beta_2)$, then

1. $[\beta(p)]_{\text{upper}}^{(n)} := [\beta(p)]_z^{(n)}$
2. $[\beta(p)]_{\text{lower}}^{(n)} := [\beta(p)]_l^{(n)}$
3. $[\beta(p)]_{l}^{(n)} := (1 - 0.618)\left( [\beta(p)]_{\text{upper}}^{(n)} - [\beta(p)]_{\text{lower}}^{(n)} \right) + [\beta(p)]_{l}^{(n)}$

else

1. $[\beta(p)]_{\text{lower}}^{(n)} := [\beta(p)]_l^{(n)}$
2. \([\beta(p)^{(n)}]_2 := [\beta(p)]^{(n)}_2\)

3. \([\beta(p)]^{(n)}_2 := [\beta(p)]^{(n)}_{\text{lower}} + 0.618 \times ([\beta(p)]^{(n)}_{\text{upper}} - [\beta(p)]^{(n)}_{\text{lower}})\)

**Step 7:** Check terminating criterion:

If \([\beta(p)]^{(n)}_{\text{upper}} - [\beta(p)]^{(n)}_{\text{lower}} \leq \epsilon\) and \(p = |\beta|:\)

1. \([\beta(p)]^{(n)} := \frac{1}{2} ([\beta(p)]^{(n)}_{\text{upper}} - [\beta(p)]^{(n)}_{\text{lower}}),\)

2. proceed to Step 8.

Else, go to Step 5.

**Step 8:** Check if all elements in \(\beta\) have been updated:

If \(p < |\beta|,\)

1. \(p := p + 1\)

2. go to Step 4.

Else proceed to Step 9.

**Step 9:** Check overall terminating criterion:

If \(\|\beta^{(n)} - \beta^{(n-1)}\| < \epsilon\) or \(n > N_{\text{max}},\)

STOP and return \(\beta := \frac{1}{2} (\beta^{(n)} + \beta^{(n-1)})\) as the final solution;

Else \(n := n + 1\) and go to Step 3.

The error tolerance is set as \(\epsilon = 0.1\). The maximum number of iterations, \(N_{\text{max}}\), is set to be 10.

**5.3. Simulation Results**

After model calibration, the obtained model parameters were inserted into the model for simulation of the dynamical behavior of the model. The following setups for initial and boundary conditions are adopted:

- **Initial conditions:** Flow, density and speed at the first time step for all the links assume the field measured values, but after that, they were determined by the model dynamics.

- **Boundary conditions:** Flow, density and speed at Cells 1 and 8 assumed measured value all the time; therefore, only the 6 links in between the two terminal links are simulated.

The following tables collect the modeling and simulation results, with and without FD, and using the default convection term (4.6) in Table 1 or using the convection term (4.11) in Table 2.
Using different search ranges, the coefficients are different. It can be observed from the tables that:

(a) model matching with FD and without FD does not make a significant difference on average. Therefore, since a linear control variable is highly desirable in control design, it is better to omit the FD;
(b) Using convection term (4.11) in speed dynamics improves the model matching for both cases, with and without FD. This is obvious by looking at the last line for the average error over the five datasets.

Typical data plots of the simulation results using calibrated model parameters compared with field data (speed, flow and density) are shown in the Figure 3-5. Due to page limit, other plots have been omitted. Simple observation and analysis of simulation results indicate the following points:

- The model matches are reasonable for some datasets but not for all datasets;
- Simulated speed is slightly higher than the field measured speed; this coincides with the observation that simulated flow is slightly higher than field measurements;
- Simulated traffic states did not catch the traffic dynamics in the transition phases very well. This may be due to several reasons:
  - Data quality: the sensor data may not be good enough quality to represent the real traffic characteristics;
  - In data processing, efforts have been made to keep the original data characteristics as much as possible. However, due to some data missing, data logging errors, loop detector faults and communication faults, some data filtering processing has to be applied to get reasonable speed and flow data series, which might distort the data;
  - Density estimation based on point sensors is very difficult so it is calculated from the flow and speed relationship $\rho(k) = q(k) / v(k)$. Due to the point time-mean speed estimation error, the distance mean speed estimation obtained from the harmonic mean will also have estimation error, which will, in turn, lead to errors in the density value;
- The model dynamics itself needs improvement to fully capture the transitions between traffic phases.

<table>
<thead>
<tr>
<th>EB</th>
<th>Without FD</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau, \kappa, \nu$</td>
<td>Average RMRSE</td>
<td>$\tau, \kappa, \nu$</td>
<td>Average RMRSE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data set</td>
<td>$\tau, \kappa, \nu$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12/01/2005</td>
<td>0.0410</td>
<td>37.9474</td>
<td>8.5836</td>
<td>0.0126</td>
<td>0.0256</td>
<td>0.0325</td>
<td>0.1261</td>
<td>30.0526</td>
<td>8.9869</td>
</tr>
<tr>
<td>12/02/2005</td>
<td>0.0410</td>
<td>33.0432</td>
<td>8.0426</td>
<td>0.0136</td>
<td>0.0253</td>
<td>0.0358</td>
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<td>8.9869</td>
</tr>
<tr>
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<td>37.9474</td>
<td>8.7903</td>
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<td>0.0243</td>
<td>0.0393</td>
<td>0.1606</td>
<td>30.0526</td>
<td>8.9869</td>
</tr>
</tbody>
</table>

Table 1. Comparison of Modelling and Simulation Results with Field Data with FD and Without FD; using Default Speed Convection Term (4.6)
<table>
<thead>
<tr>
<th>Date</th>
<th>EB</th>
<th>Without FD</th>
<th>With FD</th>
<th>Average RMRSE Without FD</th>
<th>Average RMRSE With FD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \tau, \kappa, \nu )</td>
<td>( v )</td>
<td>( q )</td>
<td>( \rho )</td>
<td>( v )</td>
</tr>
<tr>
<td>12/01/2005</td>
<td>0.0360, 30.0526, 8.1966</td>
<td>0.0119</td>
<td>0.0243</td>
<td>0.0305</td>
<td>0.3328, 30.0526, 8.9869</td>
</tr>
<tr>
<td>12/02/2005</td>
<td>0.0279, 30.0526, 8.5836</td>
<td>0.0123</td>
<td>0.0236</td>
<td>0.0330</td>
<td>0.4230, 30.0526, 8.9869</td>
</tr>
<tr>
<td>12/03/2005</td>
<td>0.0279, 34.7215, 8.0426</td>
<td>0.0113</td>
<td>0.0225</td>
<td>0.0364</td>
<td>0.2295, 30.0526, 8.9869</td>
</tr>
<tr>
<td>12/04/2005</td>
<td>0.0279, 30.2879, 7.0131</td>
<td>0.0144</td>
<td>0.0222</td>
<td>0.0449</td>
<td>0.0491, 30.0526, 8.9869</td>
</tr>
<tr>
<td>12/05/2005</td>
<td>0.0197, 37.5017, 7.0395</td>
<td>0.0120</td>
<td>0.0249</td>
<td>0.0333</td>
<td>0.2558, 30.0526, 8.9869</td>
</tr>
<tr>
<td>Average over 5 days</td>
<td>0.0279, 32.5233, 7.7751</td>
<td>0.0124</td>
<td>0.0235</td>
<td>0.0356</td>
<td>0.2580, 30.0526, 8.9869</td>
</tr>
</tbody>
</table>
Figure 3. Data Date 12/05/2005, Speed for Cell 2-4 without FD

Figure 4. Data Date 12/05/2005, Density for Cell 2-4 without FD
6. Concluding Remarks

The freeway traffic network is a large, complicated and stochastic system. To handle the stochastic properties, traffic state parameters are aggregated over time and space at appropriate levels. Based on the experience with our simulations and the field practice in Europe, this level is somewhere between 10-30s. With aggregated data, the second-order METANET model is adopted due to its control system structure. Previous work used the METANET model for freeway traffic control design; however, the speed control variable in the model was parameterized nonlinearly due to the FD assumption, which causes difficulty in control design and efficient numerical calculation. Besides, the METANET model itself needs improvement to capture the transition phase dynamics of the traffic to reduce model mismatching. This paper suggests two basic improvements: (a) drop the nonlinear parameterization in the speed control variable and directly adopt the linear desired speed as the control variable; and (b) several alternatives for the convection term of the speed dynamics for better model matching. Model calibrations using BHL field data have been conducted, with results presented for comparison. As the results show, the original METANET model and its simplified counterpart are comparable in model matching, but the suggested new convention term in speed dynamics improves the model matching. However, the model still could not always capture the traffic dynamic accurately based on comparison of simulations with field data. This may be due to several reasons including: (1) data quality – whether the data accurately represents the traffic dynamics for the freeway section; (2) the model needs further improvement. For example, during simulation, we recognized that the density gradient term in the speed dynamics is sluggish most of the time. Besides, the convection term in the speed dynamics needs further improvement. Further model improvement in those aspects is underway and will be reported in the future.
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References


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