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Privacy-Preserving One-Class Support Vector Machine with Horizontally Partitioned Data

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Abstract

We propose a new algorithm of privacy-preserving one-class support vector machine (SVM) with horizontally partitioned data. Every participant holds a part of data with all the data attributes. They apply the same random matrix to establish their own kernel matrix. By sharing these partial kernel matrices, we generate a global kernel matrix and establish two privacy-preserving one-class SVM models, which include the linear model and the nonlinear model. Partial kernel matrix can protect the privacy of the participants, and the global kernel matrix can ensure the classification accuracy. Experimental results on benchmark data sets indicate the effectiveness of the proposed algorithms.

Keywords: *One-class SVM, random kernels, privacy-preserving, horizontally partitioned data*

1. Introduction

With the rising number of Internet users and the advent of Big Data, data mining has attracted wide attention. Data mining is a powerful technique which can extract the hidden and useful knowledge from the vast amounts of data. In practice, the data contains a lot of sensitive information, and for various reasons, owners do not want to make it public. So the data mining should be carried out under the condition of the privacy preservation.

Support vector machine (SVM) [1] is an effective data mining method to solve the classification problem and it has a significant impact on the statistical learning theory. Experts have gradually explored and improved the SVM theory, and they have studied different branches of the SVM algorithms. Among these algorithms, an important branch is the one-class SVM [2-3]. Schölkopf *et. al.,* [2] proposed a method to adapt the SVM algorithm for one-class SVM, which only uses examples from one class for training, instead of multiple classes. One-class SVM training algorithm works by finding the maximum margin separation between the training points and the origin. The approach introduces a favorable parameter $\nu \in (0,1]$, which can control the fraction of outliers and the fraction of support vectors. Support vector domain description (SVDD) [3] is another one class classification method which seeks for a hypersphere with minimum volume containing most of the target class data. In recent years, there have been more and more theory research and applications on one-class SVM [4-5]. One-class SVM has been used in various fields, such as ecological modeling [6], and text clustering [7].

Recently, privacy-preserving SVM (PPSVM) has been getting more and more attention in the academic research [8-12]. Hwanjo Yu *et. al.,* studied the Boolean data privacy preservation [8]. They obtained two Boolean vector inner product by a hash function so that they can find out gram matrix and get the SVM model. Based on reduced SVM (RSVM) [10-11] and random matrix, Mangasarian *et. al,* [9] have established PPSVM

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model. In addition, SVDD model has been successfully applied to the field of privacy-preserving [12].

In this paper, we propose a privacy-preserving one-class SVM based on horizontally partitioned data. We introduce the completely random matrix [13] to establish the new model. Each participant makes public only the matrix product of its privately held matrix multiplied by the transpose of the random matrix for linear kernels, and a similar kernel function for nonlinear kernels. At last, experiments verify the effectiveness of the proposed algorithm.

The paper is organized as follows. In Section 2, we introduce one-class SVM. In Section 3, we establish the linear and nonlinear privacy-preserving one-class SVM models. The experiments of the two proposed models on public datasets are described in Section 4. At last, we conclude the paper.

2. One-Class SVM

Before presenting our work, we first briefly review the standard one-class SVM algorithm. Consider training data $x_1, x_2, ..., x_l \in X$, where X is the input space. Denote $\phi : X \to F$ a feature map, which is a map into a dot product space F such that the dot product in the image of ϕ can be computed by evaluating some simple kernel $K(x, y) = (\phi(x), \phi(y))$, such as the Gaussian kernel $K(x, y) = \exp(-\| x - y \|^2 / 2\sigma^2)$. One-class ν -SVM needs to solve the following optimization problem:

$$\min_{w,\xi_i,\rho} \frac{1}{2} \| w \|^2 + \frac{1}{\nu l} \sum_{i=1}^{l} \xi_i - \rho$$

$$\text{s.t.} \quad w \cdot \phi(x_i) \geq \rho - \xi_i \tag{1}$$

$$\xi_i \geq 0, \quad i = 1, 2, ..., l$$

where w represents the model complexity, and the parameter $\nu \in (0,1]$ can control the fraction of outliers and the fraction of support vectors. In practice, the above optimization program is solved by its dual:

$$\min_{\alpha} \frac{1}{2} \sum_{i=1}^{l} \sum_{j=1}^{l} \alpha_i \alpha_j K(x_i, x_j)$$

$$\text{s.t.} \quad 0 \leq \alpha_i \leq \frac{1}{\nu l} \tag{2}$$

$$\sum_{i=1}^{l} \alpha_i = 1$$

The optimal normal vector is given by $w = \sum_{i=1}^{l} \alpha_i \phi(x_i)$, and then the optimal boundary can be determined by the support vector expansion:

$$f(x) = \text{sgn}(\sum_{i=1}^{l} \alpha_i K(x_i, x) - \rho) \tag{3}$$

where α_i is the solution of the dual problem and training samples x_i with non-zero α_i are support vectors. Select α_j^* from the components of α in the interval $(0,1/\nu l]$, then $\rho = \sum_{i=1}^{l} \alpha_j^* K(x_j, x_i)$.

3. Horizontally Privacy-Preserving One-Class SVM

In this section, we propose the horizontally privacy-preserving one-class SVM models, including horizontally linear privacy-preserving one-class SVM (HLPPOCSVM) and horizontally nonlinear privacy-preserving one-class SVM (HNPPOCSVM). First, we describe the notation. Denote A a real $l \times n$ matrix. In Figure 1, there are N data sets $A_1, A_2, ..., A_N$, where $A_i \in R^{l_i \times n}$ signifies a real $l \times n$ matrix, which denotes the i-th row or i-th block of rows of A. Then $A = (A_1^T, A_2^T, ..., A_N^T)^T$. A_i is the data that is held by the i-th participant, and the information of the data should be protected. $B = (z_1, z_2, ..., z_k)$ is an $n \times k$ random real matrix with $k < n$, and the rank of B is k. According to [13], we know that such B exists. When $x, y \in R^n$, $K(x, y)$ is a real number. $K(x, B)$ is a row vector in R^k, and $K(A, B)$ is an $l \times k$ matrix.

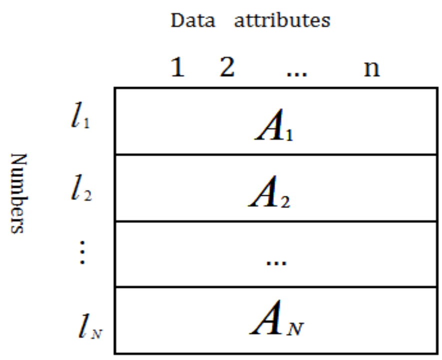

Figure 1. Horizontally Partitioned Data

3.1. Privacy-Preserving Linear One-Class SVM

In order to protect the private data, we propose a privacy-preserving linear one-class SVM.

Suppose $w = Bu$, we formulate the privacy-preserving model for one-class SVM as follows.

$$\min_{w,\xi,\rho} \; u^T B^T B u + \frac{1}{\nu l} e^T \xi - \rho$$

$$\text{s.t.} \quad ABu \geq \rho e - \xi \tag{4}$$

$$\xi \geq 0$$

where $e = (1,1,...,1)^T \in R^l$. In the above model, $AB = ((A_1B)^T, (A_2B)^T, ..., (A_NB)^T)^T$, and we can see that each participant makes public A_iB instead of the real data A_i.

In order to solve the optimization problem (4), we introduce the Lagrangian function:

$$L(u,\xi,\rho,\alpha,\beta) = u^T B^T Bu + \frac{1}{\upsilon l} e^T \xi - \rho - \alpha^T (ABu - \rho e + \xi) - \beta^T \xi \tag{5}$$

Minimize it with respect to u,ξ,ρ and maximize with respect to Lagrange multipliers $\alpha = (\alpha_1,\alpha_2,...,\alpha_l)^T \geq 0,\ \beta = (\beta_1,\beta_2,...,\beta_l)^T \geq 0$. The Lagrangian function meets Karush-Kuhn-Tucker (KKT) conditions:

$$\frac{\partial L}{\partial u} = B^T Bu - (AB)^T \alpha = 0 \tag{6}$$

$$\frac{\partial L}{\partial \xi} = \frac{1}{\nu l} e - \alpha - \beta = 0 \tag{7}$$

$$\frac{\partial L}{\partial \rho} = -1 + e^T \alpha = 0 \tag{8}$$

$$\alpha_j((AB)_j u - \rho + \xi_j) = 0,\quad j = 1,2,...,l \tag{9}$$

$$\beta_j \xi_j = 0,\quad j = 1,2,...,l \tag{10}$$

where $(AB)_j$ denotes the j -th row of AB. The dual of (4) is

$$\min_{\alpha}\ \frac{1}{2}\alpha^T AB(B^T B)^{-1}(AB)^T \alpha$$

$$\text{s.t.}\quad 0 \leq \alpha \leq \frac{1}{\nu l} e, \quad e^T \alpha = 1 \tag{11}$$

The optimal normal vector is given by

$$u = (B^T B)^{-1}(AB)^T \alpha \tag{12}$$

Select $\alpha_i \in (0,1/\upsilon l)$ from the components of α. By (7), the corresponding $\beta_i > 0$. Thus by (9) and (10), we have $(AB)_i u - \rho = 0.$. Hence, we get

$$\rho = (AB)_i u \tag{13}$$

The optimal boundary is the determined by the support vector expansion:

$$f(x) = \text{sgn}(x^T Bu - \rho) \tag{14}$$

Linear Algorithm

(1) All N participants agree on the same random matrix $B \in R^{n\times k}$ with $k < n$ for security reasons.

(2) Each participant makes public its linear kernel $A_i B$ and the full linear kernel can be calculated by $AB = ((A_1B)^T,(A_2B)^T,...,(A_NB)^T)^T$.

(3) Solve the quadratic programming (11) and get the optimal solution α^*.

(4) Calculate (12) and (13), and get u^* and ρ^*.

(5) For each new $x \in R^n$, compute $f(x) = \text{sgn}(x^T Bu^* - \rho^*)$.

Note that no participant reveals its real data set A_i. This is because each participant reveals only the $l_i k$ numbers constituting the matrix $P_i = A_i B$. For a given B with $k < n$, there are an infinite numbers of matrices A_i that satisfy P_i.

Proposition: Given the matrix product $P_i = A_i B$ where $A_i \in R^{l_i \times n}$ is unknown and B is a known matrix in $R^{n \times k}$ with $k < n$, there are an infinite numbers of solutions.

Proof: Consider the problem of solving for r-th row of A_i, that is $A_{ir} \in R^n, r \in \{1,...,l_i\}$. The r-th equation of $P_i = A_i B$ is

$$B^T A_{ir}^T = P_{ir}^T \tag{15}$$

Note that the rank of B^T is k. Hence the rank of $(B^T, P_{ir}{}^T)$ is k. Since $k < n$, the Equation (15) has an infinite numbers of solutions. Therefore, for a given B, there exist an infinite numbers of matrices A_i that satisfy P_i.

3.2. Privacy-Preserving Nonlinear One-Class SVM

In order to extend the linear model to the nonlinear case, we express w in terms of the mapped $B = (z_1, z_{2,...,} z_k)$ as follows.

$$w = \sum_{i=1}^{k} u_i \phi(z_i) \tag{16}$$

The privacy-preserving nonlinear one-class SVM is formulated as follows.

$$\min_{u,\xi,\rho} \ \frac{1}{2} u^T K(B^T, B) u + \frac{1}{\nu l} e^T \xi - \rho$$

$$\text{s.t.} \quad K(A, B) u \geq \rho e - \xi \tag{17}$$

$$\xi \geq 0$$

where

$$K(B^T, B) = \begin{bmatrix} (\phi(z_1) \cdot \phi(z_1)) & ... & ... & (\phi(z_1) \cdot \phi(z_k)) \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ (\phi(z_k) \cdot \phi(z_1)) & \cdots & \cdots & (\phi(z_k) \cdot \phi(z_k)) \end{bmatrix}$$

$$K(A, B) = \begin{bmatrix} K(A_1, B) \\ K(A_2, B) \\ \vdots \\ K(A_N, B) \end{bmatrix}$$

with

$$K(A_i,B)=\begin{bmatrix}(\phi(A_{i1})\cdot\phi(z_1)) & \cdots & \cdots & (\phi(A_{i1})\cdot\phi(z_k))\\ \vdots & & & \vdots\\ (\phi(A_{ir})\cdot\phi(z_1)) & \cdots & \cdots & (\phi(A_{ir})\cdot\phi(z_k))\\ \vdots & & & \vdots\\ (\phi(A_{il_i})\cdot\phi(z_1)) & \cdots & \cdots & (\phi(A_{il_i})\cdot\phi(z_k))\end{bmatrix}$$

The Lagrangian function of (17) is

$$L(u,\xi,\rho,\alpha,\beta)=\frac{1}{2}u^TK(B^T,B)u+\frac{1}{\nu l}e^T\xi-\rho-\alpha^T(K(A,B)u-\rho e+\xi)-\beta^T\xi \tag{18}$$

where $\alpha=(\alpha_1,\alpha_2,...,\alpha_l)^T$ and $\beta=(\beta_1,\beta_2,...,\beta_l)^T$ are Lagrange multipliers. Differentiating L with respect to u,ξ,ρ and setting the results to zero, we obtain

$$\frac{\partial L}{\partial u}=K(B^T,B)u-K^T(A,B)\alpha=0 \tag{19}$$

$$\frac{\partial L}{\partial \xi}=\frac{1}{\nu l}e-\alpha-\beta=0 \tag{20}$$

$$\frac{\partial L}{\partial \rho}=-1+e^T\alpha=0 \tag{21}$$

$$\alpha_i[(K(A,B))_iu-\rho+\xi_i]=0,\ \ i=1,2,...,l \tag{22}$$

$$\beta_i\xi_i=0,\ \ i=1,2,...,l \tag{23}$$

where $(K(A,B))_i$ denotes the i-th row of $K(A,B)$. The dual of (17) is

$$\min_{\alpha}\ \frac{1}{2}\alpha^TK(A,B)K(B^T,B)^{-1}K^T(A,B)\alpha$$

$$\text{s.t.}\quad 0\le\alpha\le\frac{1}{\nu l}e,\quad e^T\alpha=1 \tag{24}$$

The optimal normal vector is given by

$$u=K(B^T,B)^{-1}K^T(A,B)\alpha \tag{25}$$

Select $\alpha_j\in(0,1/\nu l)$ from the components of α. By (20), the corresponding $\beta_j>0$. Thus by (22) and (23), we have $(K(A,B))_ju-\rho=0$. Hence, we get

$$\rho=(K(A,B))_ju \tag{26}$$

The optimal boundary is the determined by the support vector expansion:

$$f(x)=\text{sgn}(K(x^T,B)u-\rho) \tag{27}$$

Nonlinear Algorithm

(1) All N participants agree on the same random matrix $B\in R^{n\times k}$ with $k<n$ for security reasons.

(2) Each participant makes public its nonlinear kernel $K(A_i, B)$ and does not reveal A_i. The full nonlinear kernel matrix can be calculated by

$$K(A,B)=[K^T(A_1,B),K^T(A_2,B),\cdots,K^T(A_N,B)]^T$$

(3) Solve the optimization problem (24) and get the optimal solution α^*.

(4) Calculate (25) and (26), and get u^* and ρ^*.

(5) For each new $x \in R^n$, compute $f(x)=\text{sgn}(K(x^T,B)u^*-\rho^*)$.

4. Experiments

In the experiment, we compare Privacy-Preserving One-class SVM with One-class SVM. All the experimental data sets are from UCI Machine Learning Repository which includes Ionosphere, Heart, Bupa, WDBC, Pima and German. Table 1, reports the information of these data sets.

Table 1. Benchmark Data Sets

Data set	Instances	Features	Positive instances	Negative instances
Ionosphere	351	34	225	126
Heart	270	13	150	120
Bupa	345	6	200	145
WDBC	569	30	357	212
Pima	768	8	500	268
German	1000	24	700	300

We select positive instances of each data set as the training set and apply the ten-fold cross validation method for parameter optimization. We randomly divide all the positive instances into 10 disjoint subsets $s_1, s_2, ..., s_{10}$, each subset of roughly equal size, and then we operate 10 iterations. The i-th iteration process is that $s_1, s_2, ..., s_{i-1}, s_{i+1}, ..., s_{10}$ form the training sets, s_i and all negative instances are contained to be the test sets. On i-th iteration, we get the misclassification number of points m_i and calculate the error ratio $R_i = {}^{m_i}\!/_{m}$, where m is the number of the test instances. After 10 iterations, we get $R_1, R_2, ..., R_{10}$ and their average ratio,

$$r=\sum_{i=1}^{10}\frac{R_i}{10}, \tag{28}$$

where r is an evaluation index of model. In addition, we also use G-means as the evaluation index.

$$\text{G-means}=\sqrt{acc^+ \times acc^-} \tag{29}$$

$$acc^+ = \frac{TP}{TP+FN}$$

$$acc^- = \frac{TN}{TN+FP} \tag{30}$$

TP is the number of positive instances which are predicted positive instances; FN is the number of positive instances which are predicted negative instances; TN is the numbers of negative instances which are predicted negative instances; FP is the numbers of negative instances which are predicted positive instances. The higher G-means is, the higher acc^+ and acc^-. The parameter υ is selected from $\{0.1, 0.2, ..., 1\}$ in the models. In nonlinear model, the Gauss kernel function $K(x, x') = \exp(-\|x - x'\| / 2\sigma^2)$ is employed. The optimal parameters (υ^*, σ^*) are from $\{0.1, 0.2, ..., 1\} \times \{2^{-6}, 2^{-5}, ..., 2^6\}$

Table 2, shows the results of r and G-means of HLPPOCSVM and linear OCSVM. The error ratios of the two linear models are almost consistent, and the same to G-means. That suggests the HLPPOCSVM model has high accuracy. Table 3, shows the experimental results of r and G-means of HNPPOCSVM and nonlinear OCSVM. Comparing the HNPPOCSVM with the nonlinear OCSVM, we conclude that the HNPPOCSVM model not only preserves the privacy but also has high accuracy.

Table 2. Experimental Results of HLPPOCSVM and Linear OCSVM

Data sets	Model	ν^*	r	G-means
Ionosphere	OCSVM	0.9	0.1480	0.7348
	HLPPOCSVM	1.0	0.1486	0.7290
Heart	OCSVM	1.0	0.1113	0.7235
	HLPPOCSVM	1.0	0.1101	0.7275
Bupa	OCSVM	1.0	0.0760	0.7271
	HLPPOCSVM	1.0	0.0870	0.7166
WDBC	OCSVM	1.0	0.5601	0.4506
	HLPPOCSVM	1.0	0.5656	0.4659
Pima	OCSVM	1.0	0.1601	0.7360
	HLPPOCSVM	1.0	0.1712	0.7290
German	OCSVM	1.0	0.2021	0.7325
	HLPPOCSVM	1.0	0.1949	0.7421

Table 3. Experimental Results of HNPPOCSVM and Nonlinear OCSVM

Data sets	Model	ν^*	σ^*	r	G-means
Ionosphere	OCSVM	0.5	2^3	0.1480	0.6970
	HNPPOCSVM	1.0	2^2	0.1905	0.7349
Heart	OCSVM	0.2	2^4	0.1111	0.7320
	HNPPOCSVM	1.0	2^4	0.1244	0.7210
Bupa	OCSVM	0.2	2^3	0.0650	0.7190
	HNPPOCSVM	1.0	2^4	0.1930	0.6992
WDBC	OCSVM	0.5	2^3	0.1417	0.7336
	HNPPOCSVM	1.0	2^5	0.1413	0.7235
Pima	OCSVM	0.1	2^{-2}	0.1572	0.7367
	HNPPOCSVM	0.8	2^5	0.1513	0.7306
German	OCSVM	0.9	2^3	0.1892	0.7431
	HNPPOCSVM	0.1	2^2	0.2132	0.7239

In order to make comparisons more clearly, we give the comparison figures of error ratios and G-means of these models. In Figure 2, the point under the line L which is 45 degree line indicates that the error ratio of the linear OCSVM is higher, and the point above the line L suggests that the error ratio of the HLPPOCSVM is higher. From Figure 2, we can see that all points are near the line L, which indicates that the two linear models have almost the same accuracies. In Figure 3, the points under the line R, which is 45 degree line, show that the G-means values of the linear OCSVM is higher. From Figure 3, we can see that almost all points are near the line R. Through Figure 2 and Figure 3, we can conclude that the accuracies of HLPPOCSVM and linear OCSVM are consistent. We note that unlike OCSVM model, in HLPPOCSVM model, the data multiplied by random matrix instead of real data was used to do experiments. The HLPPOCSVM model has the effect of privacy protection.

Figure 4 and Figure 5 show similar results for nonlinear models HNPPOCSVM and OCSVM. From Figure 4, we can see that the error ratios of nonlinear OCSVM are lower than those of HNPPOCSVM on four data sets, and almost the same on two data sets. From Figure 5, we notice that most of the points are under the line D and we conclude that the G-means values of nonlinear OCSVM are higher than those of HNPPOCSVM. From the above two figures, we can conclude that the accuracy of the nonlinear OCSVM is higher than that of HNPPOCSVM. But we notice that the original data is not directly used in HNPPOCSVM model to solve the one class classification problem. In HNPPOCSVM, we use kernel matrix protected by the random matrix to solve problems, which impacts the classification accuracy of HNPPOCSVM. However, the gap is small.

5. Conclusion

In this paper, we have proposed two privacy-preserving models HLPPOCSVM and HNPPOCSVM to solve one class classification problems. The same random matrix is applied to calculate each participant's kernel matrix. Each participant makes public only the data multiplied by the random matrix instead of the real data. By sharing these partial kernel matrices, a global kernel matrix can be generated. Partial kernel matrix can protect the privacy of the participants, and the global kernel matrix can ensure the classification accuracy. Experimental results show that the two proposed models not only ensure the classification accuracy, but also realize the data privacy preservation.

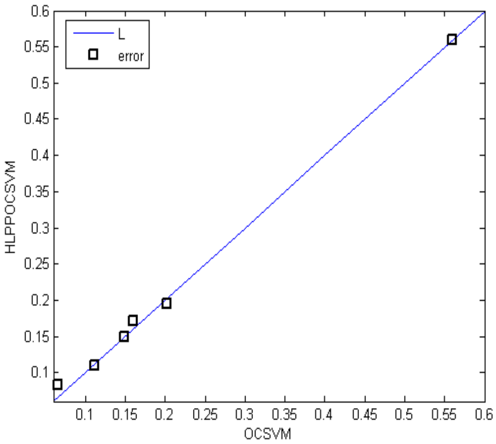

Figure 2. Error Ratios of HLPPOCSVM

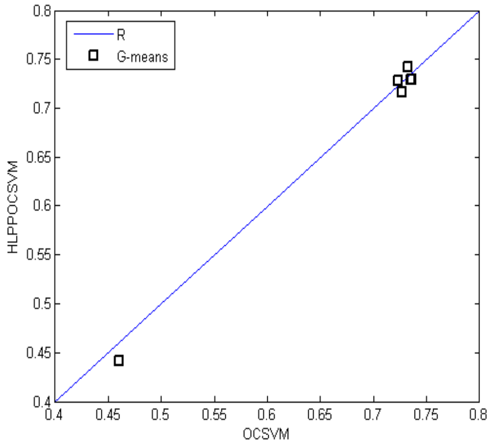

Figure 3. G means of HLPPOCSVM and Linear OCSVM and Linear OCSVM

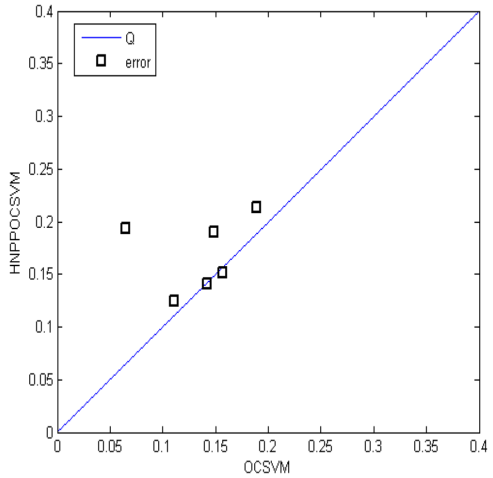

Figure 4. Error Ratios of HNPPOCSVM

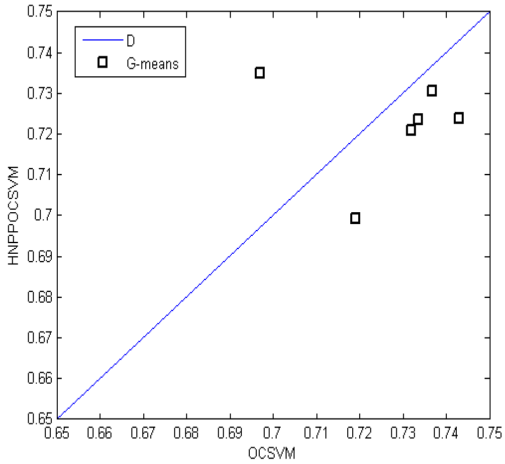

Figure 5. G-Means of HNPPOCSVM and Nonlinear OCSVM and Nonlinear OCSVM

Acknowledgments

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References

[1] V. N. Vapnik, "The Nature of Statistical Learning", Berlin: Springer, **(1995)**.

[2] B. Schölkopf, R. C. Williamson and A. J. Smola, "Support Vector Method for Novelty Detection", Neural Information Processing Systems, vol. 12, **(1999)**, pp. 526-532.

[3] D. Tax and R. Duin, "Support Vector Domain Description", Pattern Recognition Letters, vol. 20, no.11, **(1999)**, pp. 191-199.

[4] W. Zhu and P. Zhong, "A New One-class SVM Based on Hidden Information", Knowledge-Based Systems, vol. 60, **(2014)**, pp. 35-43.

[5] Z. Zhao, P. Zhong and Y. Zhao, "Reduced Least Squares One-class SVM in Empirical Feature Space for Imbalanced Data", ICIC Express Letters, vol. 5, no. 11, **(2010)**, pp. 4115-4121.

[6] G. Foody, A. Mathur and H. C. Sanchez, "Training Set Requirements for Classification of A Specific Class", Remote Sensing of Environment, vol. 104, no. 1, **(2006)**, pp. 1-14.

[7] L. Manevitz and M. Yousef, "One Class SVMs for Document Classification", Journal of Machine Learning Research, vol. 2, **(2002)**, pp. 139-154.

[8] H. Yu, X. Q. Jiang and J. Vaidya, "Privacy-Preserving SVM Using Nonlinear Kernels on Horizontally Partitioned Data", Proceedings of the 2006 ACM Symposium on Applied Computing, ACM, **(2006)**, pp. 603-610.

[9] O. L. Mangasarian and E. W. Wild, "Privacy-Preserving Classification of Horizontally Partitioned Data via Random Kernels", Proceedings of the 2008 International Conference on Data Mining, **(2008)**, pp. 473- 479.

[10] Y. J. Lee and O. L. Mangasarian, "RSVM: Reduced Support Vector Machines", Proceedings of the First SIAM International Conference on Data Mining, **(2001)**, pp. 57-64.

[11] K. Lin and C. J. Lin, "A Study on Reduced Support Vector Machines", IEEE Transactions on Neural Network, vol. 45, no. 2, **(2003)**, pp. 199- 204.

[12] W. Hu and W. Wang, "Privacy Protection of SVM Classification Method", Electronic Journals, vol. 2, no. 2, **(2012)**, pp. 281-286.

[13] X. Feng and Z. Zhang, "The Rank of Random Matrix", Applied Mathematics and Computation, vol. 2, no. 1, **(2007)**, pp. 689-694.

