

## A Novel 3D Model Retrieval Method Based On Shape Index Distribution

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### Abstract

*Feature extraction is a key technique in 3D model retrieval. In this paper, we use the local geometrical attribute of the 3D model surface to propose a new descriptor based on shape index distribution (SID). First, the sample points are obtained on the 3D model surfaces. The shape index of these points is obtained by their principal curvatures. Secondly, the surfaces surrounding these sample points are labeled for ten fundamental types by the value of shape index. Finally, 3D model is divided into 50 different Radius of spherical shells by taking the centroid as the center. SID is constructed by computing the statistical numbers of shape label of the sample points between the different spherical shells. Experiments show that our method has a good retrieval performance.*

**Keywords:** *principal curvature; shape index distribution; similarity measures; 3D model retrieval*

### 1. Introduction

With the development of computer technology, the 3D model has been widely used, and has generated a large number of 3D model databases. Science building a high fidelity 3D modeling is very time consuming, the use of existing 3D model will save a lot of expenses. Therefore it is necessary to propose a convenient 3D model retrieval algorithm to help people obtain satisfying model. The essence of 3D model retrieval is how to determine the similarity between the different models. The core issue is how to extract features from the existing model, in order to facilitate the geometry and topology features of 3D model for characteristic quantitative description. Content-based retrieval methods extracted the characteristics of 3D model to build a descriptor, the advantages of them are the objectivity and efficiency. Elad *et. al.*, [1] use the moment descriptions to measure the similarity of 3D models. The disadvantage of this method is more sensitive to the model's attitude changing, and does not have the translation, rotation and scale invariance. Motofumi *et. al.*, [2] implemented a 3D model retrieval system based on the feature descriptor which is combined with the tension, normal vector, volume, vertices and triangular meshes of 3D model. Osada *et. al.*, [3] suggested five shape functions to compare the feature of 3D model. Liu *et. al.*, [4] proposed a shape descriptor "Thickness Histogram" by uniformly estimating thickness of a model using statistical methods. Wang *et. al.*, [5] computed the angles between the normal vectors of the points and the line into three sets. For each set, they calculate shape distribution histogram associated with D2 function. Horn [6] proposed the Extended Gaussian Image (EGI). Each mesh of the model has been mapped to an extended Gaussian sphere vectors, the vector direction is same as normal vector of the mesh, the norm of vector is equal the area of the mesh. Zaharia [7] presented 3D shape Spectrum Descriptor (3DSSD) which provided an object intrinsic

shape index (SI) to describe according to some local geometric properties of the model surface. SI is a function of the principal curvatures of a point, the distribution of SI in the whole model is 3DSSD which is represented by the histogram. Tangelder [8] used the Gauss curvature and normal vector change rate sets as the statistics of histogram.

In this paper, we present a novel retrieval algorithm based on shape index distribution. As we known, the principal curvatures have a rotation and translation invariance which is the important property to describe the shape feature of 3D model. We present the principal curvatures and the statistical distance as the feature descriptor. The rest of paper is organized as follows: In Section 2, our method is described in details. The experiments and the results are reported in Section 3, Section 4, gives the discussion and the conclusion.

## 2. Method

From differential geometry [9], the principal curvatures are important attribute which have the rotation and translation invariance. The shape index is defined as a function of the principal curvatures. Based on shape index, 3D model can be subdivided into nine elementary shapes. We get the random points on the 3D model surface and compute the Euclidean distance between sample points and the model's centroid. Then the shape index distribution of 3D model is constructed by counting the number of shape types in different distance interval.

### 2.1. Principal Curvatures Estimation

Taubin [10] used the tensor analysis and eigenvector matrix analysis method to compute the principal curvatures. The advantage of this approach is that is linear in time and space, it has the characteristics of the faster speed and the smaller storage space. The details as follows:

Let  $T = t_1 T_1 + t_2 T_2$ , be some unit length tangent vector at P. The normal curvature along the T direction at P is:

$$\bar{k}_p(T) = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \begin{pmatrix} k_p^{11} & k_p^{12} \\ k_p^{21} & k_p^{22} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \quad (1)$$

When  $k_p^{12} = k_p^{21} = 0$ ,  $T_1$  and  $T_2$  are the two principal directions at point P of surface S,  $k_p^{11}$  and  $k_p^{22}$  are the two principal curvatures. We can get a three orthogonal basis  $\{N, T_1, T_2\}$  by introducing the normal vector N. The normal curvature can be expressed as:

$$\bar{k}_p(T) = \begin{pmatrix} n \\ t_1 \\ t_2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & k_p^1 & 0 \\ 0 & 0 & k_p^2 \end{pmatrix} \begin{pmatrix} n \\ t_1 \\ t_2 \end{pmatrix} \quad (2)$$

Let  $T_\theta$  be unit length tangent vector at P.

$$T_\theta = \cos(\theta)T_1 + \sin(\theta)T_2 \quad (-\pi \leq \theta \leq +\pi) \quad (3)$$

$$k_p(T_\theta) = k_p^1 \cos^2 \theta + k_p^2 \sin^2 \theta \quad (4)$$

Where  $\theta$  is the angle between  $T_1$  and  $T_2$ .  $k_p^1$  and  $k_p^2$  are the principal curvatures in the direction  $\{T_1, T_2\}$ . The symmetric matrix  $M_p$  is defined as:

$$M_p = \frac{1}{2\pi} \int_{-\pi}^{+\pi} k_p(T_\theta) T_\theta T_\theta' d\theta \quad (5)$$

Since the unit length normal vector  $N$  at  $P$  is an eigenvector of  $M_p$  associated with the eigenvalue zero. Let  $T_{12} = \{T_1, T_2\}$  be the  $3 \times 2$  matrix constructed by concatenating the column vectors  $T_1, T_2$ .  $M_p$  can be expressed as:

$$M_p = T_{12} \begin{pmatrix} m_p^{11} & m_p^{12} \\ m_p^{21} & m_p^{22} \end{pmatrix} T_{12}' \quad (6)$$

$$\text{Due to } m_p^{12} = m_p^{21} = 0, \quad m_p^{11} = \frac{3}{8}k_p^1 + \frac{1}{8}k_p^2, \quad m_p^{22} = \frac{1}{8}k_p^1 + \frac{3}{8}k_p^2.$$

The principal curvatures can be obtained as functions of the nonzero eigenvalues of  $M_p$ .

$$k_p^1 = 3m_p^{11} - m_p^{22}, \quad k_p^2 = 3m_p^{22} - m_p^{11} \quad (7)$$

## 2.2. Shape Type Labeling

The shape index that is first introduced by Koenderink [11], is a local geometrical attribute of a 3D surface. It expresses as the angular coordinate of a polar representation of the principal curvature vector [7] and represents the concave and convex shapes of 3D surface [12].  $P$  is a vertex on 3D model surface.  $k_p^1$  and  $k_p^2$  are the principal curvatures of  $P$ . Then the shape index  $I_p$  is defined as follow:

$$I_p = \frac{1}{2} - \frac{1}{\pi} \arctg \frac{k_p^1 + k_p^2}{k_p^1 - k_p^2} \quad (k_p^1 \geq k_p^2) \quad (8)$$

Its value is mapped into the interval [0-1].  $I_p$  reflects the local geometrical properties surrounding point  $P$ .

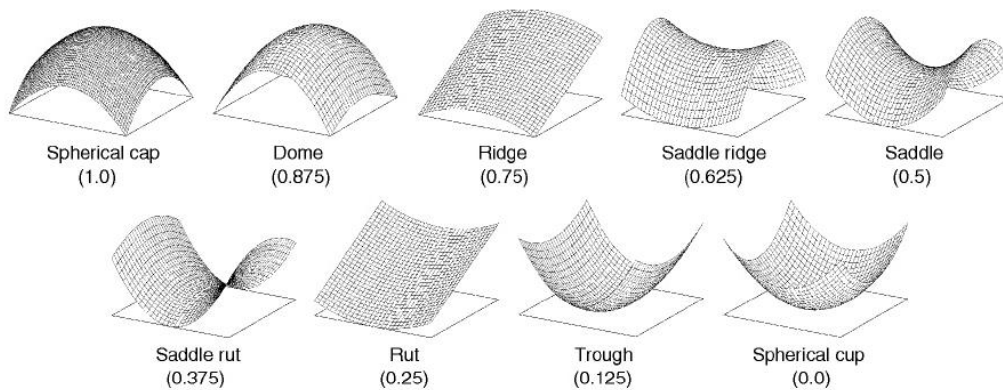


Figure 1. Nine Shape Types and Their Values on Shape Index

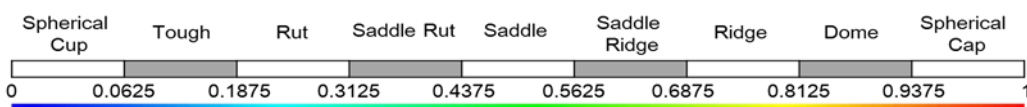
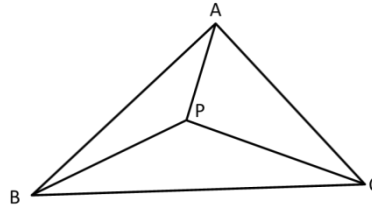


Figure 2. The Color and SI Values of Nine Shape Types

It provides the nine fundamental shape types (shown in Figure 1), except flat. In the actual calculation, the value of  $k_p^1$  and  $k_p^2$  is the rare occurrence of zero. So we set  $k_a = \sqrt{(k_p^1)^2 + (k_p^2)^2}$ . When  $K_a$  is less than a threshold T, we consider it as flat shape and color it with white. The color of nine shape types is shown in Figure 2. All of ten shapes are rotation, translation and scale invariance.

### 2.3. The Principal Curvature of Sample Point



**Figure 3. Principal Curvature of Sample Point P**

We apply D2 method to obtain the random points. The principal curvature of the point is calculate as:

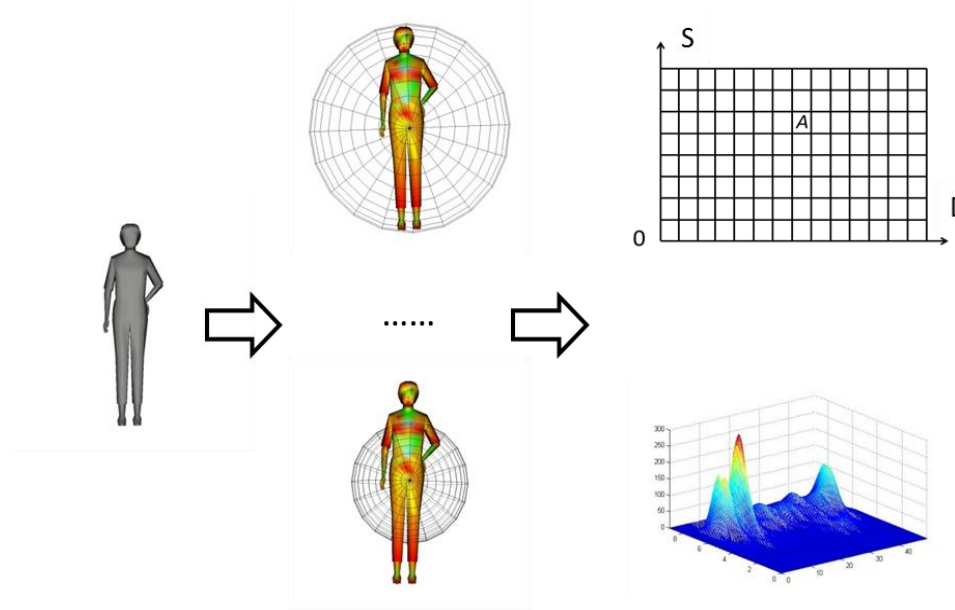
$$k_p = \omega_A k_A + \omega_B k_B + \omega_C k_C \quad (9)$$

As Figure 3 shown,  $k_A, k_B, k_C$  are the principal curvatures of A,B,C.  $\omega_A, \omega_B, \omega_C$  are the weights of a triangle area and defined in the Equation (10).

$$\omega_A = S_{\triangle ABP} / S_{\triangle ABC}, \omega_B = S_{\triangle BCP} / S_{\triangle ABC}, \omega_C = S_{\triangle CAP} / S_{\triangle ABC} \quad (10)$$

### 2.4. Construct Shape Type Distribution (SID)

The SID represents the distribution of ten types of shape in the different distances between the spherical shells. 3D model centroid  $p_0$  is calculated as:  $p_0 = \frac{1}{N} \sum_{i=1}^N p_i$  is the vertex coordinates,  $N$  is the total number of 3D model vertices. Distance  $d_i$  between  $p_i$  and  $p_0$  is calculated as:  $d_i = \sqrt{(x_i - x_o)^2 + (y_i - y_o)^2 + (z_i - z_o)^2}$ . Let  $p_0$  be the center of the sphere and let the maximum distance  $d_{max}$  be the bounding sphere Radius. We divide  $d_{max}$  into  $n$  parts  $\tilde{d} = d_{max} / n$  ( $n=50$ ). The sphere is divided into  $n$  concentric spherical shells with increments of  $\tilde{d}$ . We take the 10000 sample points to construct a  $10 \times 50$  matrix  $M(D,S)$  by calculating the number of shape types of random points between each spherical shell.  $D$  is the distance between the sample point and the centroid point.  $S$  is the ten types of shape.  $A=M(D, S)$  denotes the number of  $S$  types of shape in the distance  $D$  (shown in Figure 4).



**Figure 4. Shell Decomposition and a 3D People\_m1 Model SID**

### 2.5. Similarity Computation


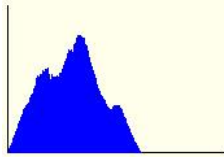
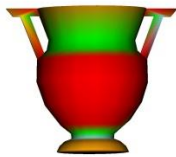
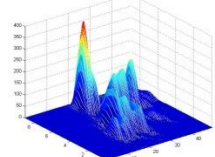

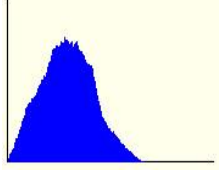
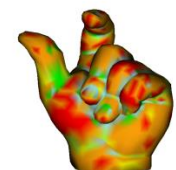
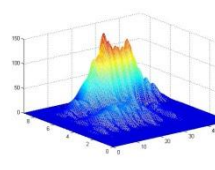

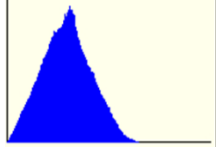
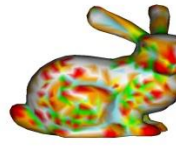
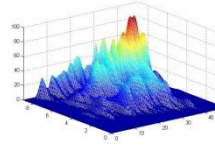
By extracting the local shape information, each 3D model is represented by  $10 \times 50$  shape index distribution matrix (SID). The similarity measure between two models usually employs Euclidean distance. But the classic Euclidean distance is easy to produce deformation and distortion. To overcome these limitations, we adopt the Manhattan distance [13] to measure the similarity between the two models. The distance between the query model  $Q$  and the matching model  $M$  is defined as follows:

$$D(Q, M) = \sum_{i=1}^{10} \sum_{j=1}^{50} |q_{ij} - m_{ij}| \quad (11)$$

## 3. Experiments

In this section, we apply some experiments to test the performance of our method. The database that we employ for the experiments is the Princeton Shape Benchmark (PSB) [14]. PSB contains 1814 models and consists of a training set (907 models in 90 classes) and a test set (907 models in 92 classes). We select 50 classes from the model base, total 600 models to experiment. All experiments are performed on the windows 7 using 2.70GHz AMD A6 CPU with 2GB of memory. Table 1, shows the D2 and SID of three models. We can find that the shape of model 1, 2 and 3 are quite different, but their D2 histograms are the similar. This is because the D2 histograms only extract the spatial coordinate information of 3D model, It is not able to effectively represent the geometrical information of 3D model surface. SID uses the local geometric properties of surface to represent 3D model, so SID has the better performance to distinguish the 3D models than D2.

**Table 1. Comparison between D2 and SID**

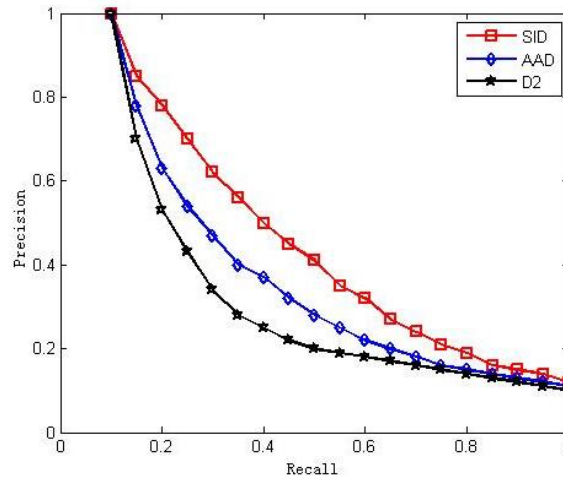
	3D models	D2	Surface types	SID
1				
2				
3				

We used the Precision–Recall curve to evaluate our approach. Precision and Recall are well known in the literature of content-based search and retrieval. They are computed as follows:

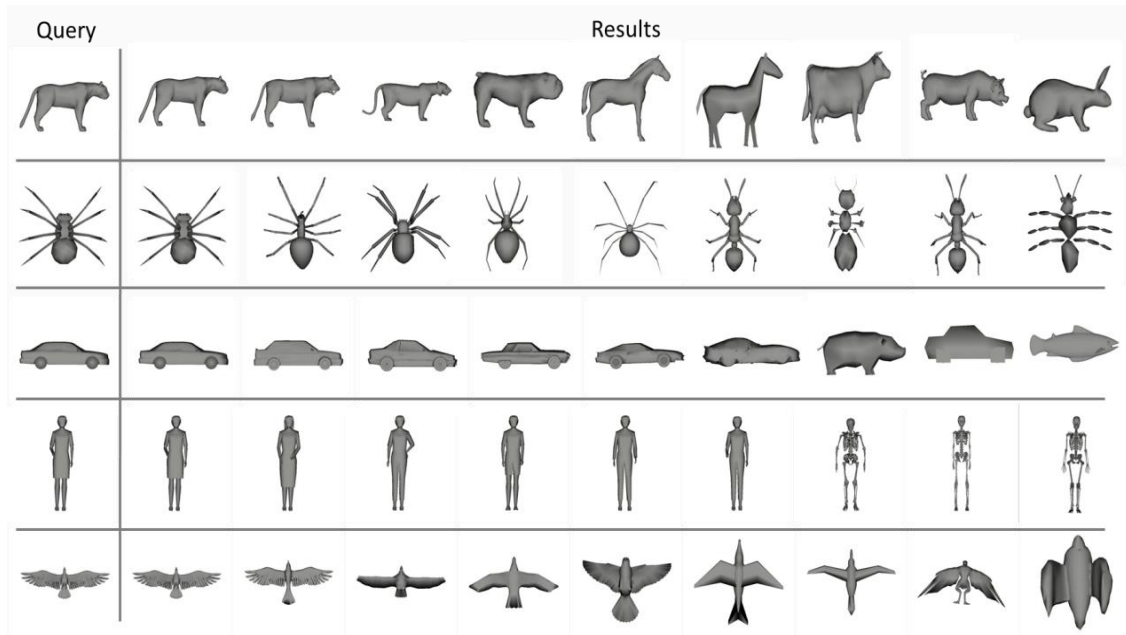
$$Precision = \frac{K}{C} \quad , \quad Recall = \frac{K}{N} \quad (12)$$

Where  $K$  is the number of relevant models in the retrieved result,  $C$  is the total number of relevant models in the database,  $N$  is the total number of retrieved models. The high Precision or Recall value means a better retrieval performance.

We compared the performance of the SID method with the D2 and AAD [15] descriptors. AAD is an enhancement of D2. It uses the distance between the sample points and the angle formed by the surface normal vectors at the sample points as a descriptor. According to Figure 5, we can find that the performance of SID is better than D2 and AAD. This is because the SID is the combination feature of the curvature information and distance information of random points on the surface, it contains more local geometry features than D2 and ADD. In terms of feature computational cost, the feature extraction costs more for the SID than D2 and AAD. In the cost of distance computation, the time complexity of Manhattan distance is  $O(n^2)$ . But SID is  $10 \times 50$  matrix. Therefore, the SID costs less to compare than the D2 and AAD due to its smaller feature size. The modest increase in cost is well justified considering the performance advantage of the SID. Figure 6 shows retrieval examples for querying the PSB by the “leopard”, ”spider”, “human”, “car” and “bird” models. The retried models are ranked from left to right in decreasing order similarity.



**Figure 5. Precision-Recall Curve for Retrieving Shapes in the PSB**



**Figure 6. Examples of Queries from the PSB Database and the Corresponding Top 9 Retrieved Models Using SID Method. The Retrieved Models are Ranked from Left to Right in Decreasing Order Similarity**

#### 4. Conclusion

In this paper, we proposed a novel approach to retrieve 3D models based on the shape index distribution (SID). SID is constructed by computing the statistic data of the sample point and their shape label in different spherical shells. It is the rotation, translation and scale invariance. It can effectively reflect the complex shapes of 3D model and is good at computation and comparison time. We have experimentally evaluated the method by using PSB. The results show that our method has a good retrieval performance.

#### Acknowledgment

This work was supported by the National Natural Science Foundation, China(No. 61373117).

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