International Journal of Signal Processing, Image Processing and Pattern Recognition
Vol.9, No.9, (2016), pp.25-32
http://dx.doi.org/10.14257/ijsip.2016.9.9.03

Reducing the Number of Sensors in a Linear Antenna Array by ℓ_p Norm Minimization Algorithm

Jinli Chen[1,2,3], Huasong Cao[1,2,3], Jiaqiang Li[1,2,3] and Junxiang Ge[1,2,3]

[1]*Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters, Nanjing University of Information Science and Technology, Nanjing 210044, China*
[2]*College of Electronic and Information Engineering, Nanjing University of Information Science and Technology,Nanjing 210044, China*
[3]*Jiangsu Key Laboratory of Meteorological Observation and Information Processing, Nanjing University of Information Science and Technology, Nanjing 210044, China*
chen820803@yeah.net

Abstract

The excitations and locations of sensors in the non-uniformly spaced array affect the array performance such as sidelobe level and spatial resolution. Consequently, finding the optimal excitation coefficients and sensor positions of the array to produce a desired beam pattern with the smallest number of sensors is of great importance in practice. With the aim of reducing the number of the sensors in a linear antenna array, a novel method based on ℓ_p (0<p<1) norm minimization for optimizing both excitation coefficients and sensor locations of the array is proposed. Compared with the reweighted ℓ_1 norm minimization (IRWL$_1$) method, the proposed method can reduce more array sensors by optimizing the objective functions that include the measurements of peak sidelobe level (PSL) and array sparsity denoted by the ℓ_p norm of excitations. Numerical experiments have proved the effectiveness and advantages of the proposed method in the reduction of the number of the sensors of the linear antenna array.

Keywords: *sparse array, array synthesize, ℓ_p norm minimization, iteratively reweighting*

1. Introduction

The antenna arrays have the characteristics of strong directionality, low sidelobe and easy scanning, which have been widely applied in fields of array radars, satellite communications and ultrasound imaging, *etc.*, [1-2]. When the inter-element spacing is larger than half of the signal wavelength, uniformly spaced arrays produce grating lobes in their beam pattern. In some applications, it may be desirable to require larger apertures and higher spatial resolutions of the antenna arrays. Then, the uniformly spaced arrays have to need large number of sensors. In order to reduce the number of array sensors while maintaining the higher quality of synthesized array beam pattern, non-uniformly sparse arrays are usually adopted [3].

The problem of designing a non-uniformly sparse array to produce a desired beam pattern is related to the calculation of the excitation coefficients and positions of the array sensors. In the last fifty years, many techniques have been proposed to address this problem. Among them the simulated annealing (SA) algorithm [4-5] and the genetic algorithm (GA) [6-7] have already been successfully used to synthesize the sparse linear array by removing some sensors from a fully populated half-wavelength array. However, these global optimization techniques find the solution with fewest array sensors from the

ISSN: 2005-4254 IJSIP

finite set of possible sensor locations, which are computationally expensive ways when synthesizing the arrays with many sensors. A non-iterative synthesis algorithm based on the matrix pencil method (MPM) is proposed in [8] to synthesize non-uniformly sparse arrays for achieving the prescribed pattern features and a reduced number of sensors. Nevertheless, this method performs poorly in terms of computational cost and array performances when dealing with shaped-beam patterns [9].

Recently, the compressive sensing (CS)-based optimization methods are exploited to solve the array synthesis problem. In [10-12], the reweighted ℓ_1 norm minimization ($IRWL_1$) [13] have lately been used to synthesize sparse arrays with the desired radiation pattern and the reduction in the number of the array sensors. The $IRWL_1$ that consists in solving a sequence of weighted convex optimization problems can produce fewer elements than the commonly used ℓ_1 norm. However, the maximally sparse solution is not always able to be achieved by this strategy. It has been shown in [14] that ℓ_p ($0<p<1$) norm minimization for sparse signal recovery outperforms $IRWL_1$ in many situations. That is, ℓ_p ($0<p<1$) norm minimization usually needs a smaller number of measurements to exactly reconstruct the sparse solution vector than $IRWL_1$. In this paper, we propose a synthesis method for sparse array design using the ℓ_p norm minimization to further reduce the number of sensors in a linear antenna array. By minimizing the peak sidelobe level (PSL) and the sparsity of an linear array denoted by the ℓ_p norm of excitations, the array excitation coefficients are optimized. Then, the sparse array are achieved by systematically removing the sensors whose excitations are approximately equal to zero. The ℓ_p norm minimization method performs better than $IRWL_1$ in the reduction of the number of the array sensors, despite requiring a little more iterations. Simulation results demonstrate the effectiveness of the proposed method.

This paper is organized as follows. In Section 2, the sparse array synthesis problem is formulated. In Section 3, the array synthesis methods based on $IRWL_1$ and ℓ_p norm minimization are presented, respectively. In Section 4 numerical simulation results are presented and discussed. Conclusions are drawn in Section 5.

2. Sparse Array Synthesis Problem

Consider a uniform spacing linear array with N-isotropic radiating elements located at $x_1, x_2, \ldots, x_N$ (with N large). For linear antenna arrays, the array factor $F(\theta)$ is given by [10]

$$F(\theta) = \sum_{n=1}^{N} w_n \exp^{jk x_n \sin\theta} \tag{1}$$

where $x_n = (n-1)d$ is the distance between the first and the nth sensors measured in wavelength, d is the inter-element spacing that is assumed to be very small with respect to the wavelength, $k = 2\pi/\lambda$ is the propagation constant, θ is the steering angle measured with respect to the x axis, and w_n is the complex coefficients of the nth sensor.

Let the steering vector

$$\mathbf{a}(\theta) = \left[\exp^{jkx_0 \sin\theta}, \exp^{jkx_1 \sin\theta} \cdots, \exp^{jkx_N \sin\theta}\right]^{\mathrm{T}} \tag{2}$$

and the excitation vector $\mathbf{w} = [w_1, w_2, \cdots, w_N]^{\mathrm{T}}$ respectively, where $[\cdot]^{\mathrm{T}}$ denotes the transpose operator. Substituting $\mathbf{a}(\theta)$ and $\mathbf{w}$ into (1) yields

$$F(\theta) = \mathbf{a}(\theta)^{\mathrm{T}} \mathbf{w} \tag{3}$$

The purpose of sparse array synthesis is to find optimal parameters of the sensors array to achieve the desired beam pattern subjected to the given far-field constraints. The constraints can be expressed as follows,

$$\begin{aligned} &\mathbf{a}(\theta_0)^{\mathrm{T}}\mathbf{w} = 1 \\ &\left\|\mathbf{a}(\theta_{\mathrm{SL}})^{\mathrm{T}}\mathbf{w}\right\|_\infty \le \varepsilon \end{aligned} \tag{4}$$

where θ_0 is the main beam radiation direction, θ_{SL} is the sidelobe region, ℓ_∞ denotes the infinity norm, and ε is a given upper bound of sidelobe levels. The constraint $\mathbf{a}(\theta_0)^{\mathrm{T}}\mathbf{w} = 1$ is used to guarantee the unit array response at the target direction. The addressed sparse array synthesis problem amounts to find a excitation vector $\mathbf{w}$ with as many zero or approaching zero components as possible for satisfying the given beam pattern constraints. Then the sparse array synthesis problem becomes

$$\begin{aligned} &\min_{\mathbf{w}} \|\mathbf{w}\|_0 \\ &s.t.\ \ \mathbf{a}(\theta_0)^{\mathrm{T}}\mathbf{w} = 1 \qquad \left\|\mathbf{a}(\theta_{\mathrm{SL}})^{\mathrm{T}}\mathbf{w}\right\|_\infty \le \varepsilon \end{aligned} \tag{5}$$

where $\|\mathbf{w}\|_0$ denotes the number of non-zero components of $\mathbf{w}$. Unfortunately, the objective function of the problem (5) is non-convex, and thus NP-hard to solve [10-12], since solving the problem (5) requires an intractable combinatorial search. By the use of the ℓ_1 norm, the optimization object function becomes convex and can be efficiently solved. The following problem is considered [12],

$$\begin{aligned} &\min_{\mathbf{w}} \|\mathbf{w}\|_1 \\ &s.t.\ \ \mathbf{a}(\theta_0)^{\mathrm{T}}\mathbf{w} = 1 \qquad \left\|\mathbf{a}(\theta_{\mathrm{SL}})^{\mathrm{T}}\mathbf{w}\right\|_\infty \le \varepsilon \end{aligned} \tag{6}$$

where $\|\mathbf{w}\|_1 = \sum_{i=1}^{N} |w_i|$ is the ℓ_1 norm of $\mathbf{w}$. This object function is convex, then it can typically be solved by the well-established interior point method, such as software toolbox CVX [15] .

3. Sparse Array Synthesis via IRWL$_1$ and ℓ_p Norm Minimization

3.1. Sparse Array Synthesis Method Based on IRWL$_1$

According to the theory in [13], IRWL$_1$ can provide more sparser solutions than the commonly used ℓ_1 norm because of the tighter approximation to ℓ_0 norm. Therefore, the IRWL$_1$ algorithm is applied to solving the array synthesis problem in [10-12]. At the *i*th iteration, the weighted ℓ_1 minimization of array synthesis problem to be solved is

$$\begin{aligned} &\min_{\mathbf{w}} \left\|\mathbf{Z}_1^{(i)}\mathbf{w}^{(i)}\right\|_1 \\ &s.t.\ \ \mathbf{a}(\theta_0)^{\mathrm{T}}\mathbf{w}^{(i)} = 1 \qquad \left\|\mathbf{a}(\theta_{\mathrm{SL}})^{\mathrm{T}}\mathbf{w}^{(i)}\right\|_\infty \le \varepsilon \end{aligned} \tag{7}$$

where the diagonal weight matrix $\mathbf{Z}_1^{(i)}$ used for the current iteration are computed from the value of the previous solution and started with $\mathbf{Z}^{(0)} = \mathbf{I}$. The *n*th diagonal element of $\mathbf{Z}_1^{(i)}$ is updated as follows

$$z_n^{(i)} = \left(\left|w_n^{(i-1)}\right| + \delta\right)^{-1} \tag{8}$$

It can be seen from (8) that the active elements with large magnitude excitations $w_n^{(i-1)}$ are assigned smaller weights at the current iteration for encouraging active elements to remain nonzero. Conversely, the small excitations lead to large weights so that the inactive elements are penalized to approach zero at the current iteration. The parameter $\delta > 0$ provides numerical stability and ensures that a nonzero estimate can be continue to work at the next step when a zero excitation occurs in the estimated $\mathbf{w}^{(i-1)}$ at the $(i-1)$ th iteration. The parameter δ should be set slightly smaller than the expected minimum nonzero magnitude of $\mathbf{w}$ [10-12]. Then the sensors that contribute the least to the array performance are removed systematically until the number of active elements no longer change.

3.2. Sparse Array Synthesis Method Based on ℓ_p Norm Minimization

It has shown [14] that by replacing the IRWL$_1$ with ℓ_p (0<p<1) norm, exact reconstruction is possible with substantially fewer measurements in many situations. In order to reduce more elements in a linear antenna array, an interesting alternative problem formulation for array synthesis can be expressed as

$$\begin{aligned} &\min_{\mathbf{w}} \|\mathbf{w}\|_p^p \\ &s.t. \;\; \mathbf{a}(\theta_0)^{\mathrm{T}}\mathbf{w} = 1 \qquad \left\|\mathbf{a}(\theta_{\mathrm{SL}})^{\mathrm{T}}\mathbf{w}\right\|_\infty \le \varepsilon \end{aligned} \tag{9}$$

where $\|\mathbf{w}\|_p^p = \left(\sum_{i=1}^{N}|w|^p\right)^{1/p}$ is the ℓ_p (0<p<1) quasi-norm. Although the ℓ_p norm minimization is a nonconvex, nonsmooth, and non-Lipschitz optimization problem, transforming the ℓ_p norm minimization into a series of weighted ℓ_1 norm minimization is a good way to solve this problem [16,17]. Then, the non-convex optimization problem (9) is transformed into iteratively reweighted ℓ_1 norm minimization problem, which attempts to find a convex penalty function more closely resembling the ℓ_p norm [16]. The ℓ_p norm minimization problem solved at the i th iteration is

$$\begin{aligned} &\min_{\mathbf{w}} \left\|\mathbf{Z}^{(i)}\mathbf{w}^{(i)}\right\|_1 \\ &s.t. \;\; \mathbf{a}(\theta_0)^{\mathrm{T}}\mathbf{w}^{(i)} = 1 \qquad \left\|\mathbf{a}(\theta_{\mathrm{SL}})^{\mathrm{T}}\mathbf{w}^{(i)}\right\|_\infty \le \varepsilon \end{aligned} \tag{10}$$

where $\mathbf{Z}^{(i)} = \mathrm{diag}(\mathbf{q}^{(i)})$ is a weighting diagonal matrix, and $\mathrm{diag}(\mathbf{q}^{(i)})$ represents a diagonal matrix constituted by the vector $\mathbf{q}^{(i)} = [q_1^{(i)}\; q_2^{(i)} \cdots q_N^{(i)}]$. In the first iteration $\mathbf{Z}^{(0)} = \mathbf{I}$, we update the weighting coefficients of the diagonal matrix $\mathbf{Z}^{(i)}$ as follows:

$$q_n^{(i)} = \left(\left|w_n^{(i-1)}\right| + \delta\right)^{p-1} \tag{11}$$

where the positive weights $q_n^{(i)}$ are computed from the values of the previous solution, and the function of the parameter $\delta > 0$ is same as that in (8), which provides numerical stability of algorithm even if a zero-valued component $w_n^{(i-1)}$ at the step i-1 appears. With the definitions of (10) and (11), the iteratively reweighted ℓ_1 norm minimizations are used to approach the ℓ_p norm asymptotically [16]. The sparse array is then obtained by removing those sensors whose excitations are approximately equal to zero when the number of magnitude coefficients in the solution $\mathbf{w}$ no longer change.

Compared with the sparse array synthesis methods based on IRWL$_1$ and ℓ_p norm minimization, the weights in the IRWL$_1$ algorithm used for sparse array synthesis are inversely proportional to the magnitude excitations at each iteration [10-12], while those in the ℓ_p norm minimization method are the exponential functions with respect to the magnitude excitations. Because of $0<p<1$, the $IRWL_1$ indeed penalizes nonzero coefficients more heavily than the ℓ_p norm minimization. Although the more heavy penalization induces a fast convergence, the $IRWL_1$ algorithm does perform worse in many situations than the ℓ_p norm minimization method which is more democratically penalize nonzero array excitations at each iteration [14]. The proposed sparse array synthesis method via ℓ_p norm minimization can construct the appropriate weights at each iteration and then achieve the maximally sparse array at cost of a little more iterations.

4. Simulation Result

Some numerical examples are presented to demonstrate the performance of the proposed method for synthesizing non-uniformly sparse linear arrays. It has been shown in [18] that $\ell_{1/2}$ regularization plays a representative role among all ℓ_p regularizations with p in (0,1). From this study, thus, the index p=1/2 is adopted in the following simulation experiments.

A linear array with the maximum length 21λ is synthesized, whose symmetric pattern is characterized by a side-lobe level SLL= -14.49 dB, while the main beam is confined in $|\sin\theta-\sin\theta_0|\leq 0.0436$. Prior to the array synthesis using the proposed method and the $IRWL_1$ method [10-12], a finer discretization of the array aperture with $d=0.1\lambda$ is required to resemble the continuous array. The distributions of the normalized magnitude excitations obtained by the proposed method and the $IRWL_1$ method are shown in Figure 1. We can clearly observe that the sparse nature of the solutions of the two synthesis methods, in which the magnitude excitations of only few sensors are not approach zero, are presented. The nonzero elements are the active sensors of the maximally sparse array, which contribute the most to the array performance, while the other sensors with the amplitude values approaching zero are then removed systematically. Thus, the optimized excitations and locations of sensors of the sparse array with the array aperture 21λ can be seen from Figure 1. The synthesized patterns of the optimized sparse array of the two synthesis methods are shown in Figure 2. The desired performance in terms of SLL and beamwidth have been realized by the $IRWL_1$ method using N=19 antennas and 5 iterations, while the same performances concerning final SLL and beamwidth of the proposed method have been achieved with N= 18 antennas and 7 iterations. Thus the synthesis method herein proposed can reduce more antennas, despite requiring a little more iterations.

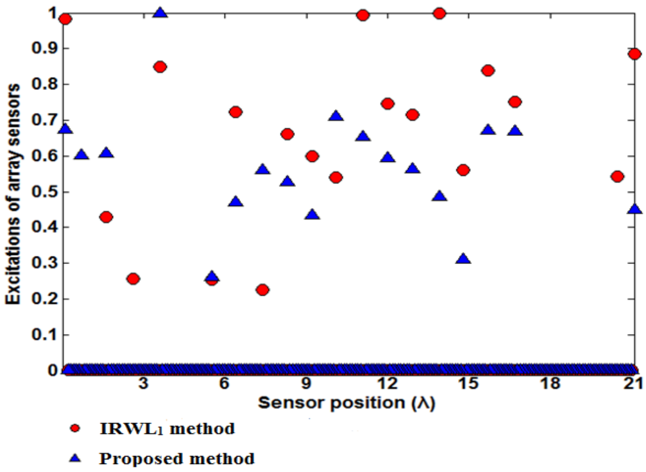

Figure 1. Synthesized Positions and Excitations of the Linear Arrays with the Maximum Length 21λ

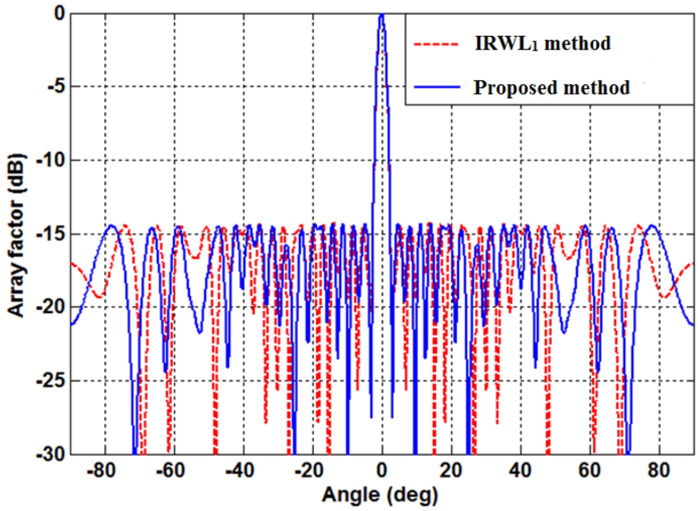

Figure 2. Synthesized Pattern by the Proposed Method Compared with the Synthesized Pattern by the IRWL$_1$ Method. The Array Aperture is 21λ, *SLL*= -14.49 dB, |sinθ-sinθ$_0$|≤0.0436

In order to verify the robustness of the proposed method in the reduction of the number of the elements in a linear antenna array, let us now consider the other problem of synthesizing a linear array with the maximum length 30λ, whose radiation pattern exhibits a side-lobe level *SLL*= -24 dB, with the main beam confined in $|\sin\theta - \sin\theta_0| \leq 0.0436$. By applying the IRWL$_1$ method, it has been possible to achieve the desired pattern by using *N*= 33 antennas and 5 iterations, while the same array performances concerning the SLL and beamwidth have been achieved with *N*= 29 antennas and 7 iterations by the synthesis method herein proposed, as shown in Figure 3 and Figure 4. Thus the proposed method achieves maximally sparse array at cost of a little more time-consuming.

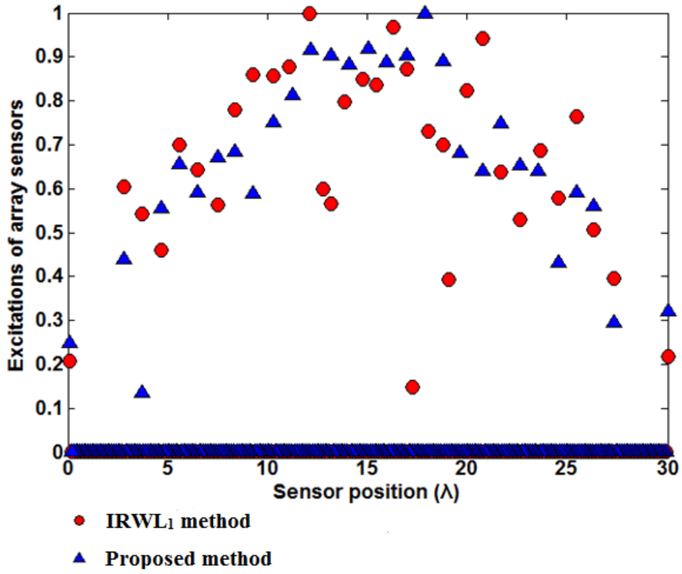

Figure 3. Synthesized Positions and Excitations of the Linear Arrays with the Maximum Length 30λ

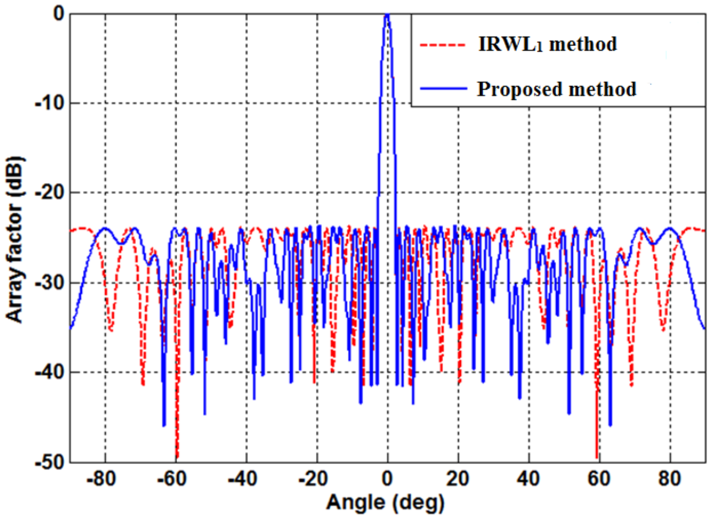

Figure 4. Synthesized Pattern by the Proposed Method Compared with the Synthesized Pattern by the IRWL$_1$Method. The Array Aperture is 30λ, *SLL*= -24dB, |sinθ-sinθ$_0$|≤0.0436

5. Conclusion

In this paper, a ℓ_p (0<p<1) norm minimization scheme is proposed for synthesizing a non-uniformly sparse array with the minimum number of sensors. Due to the non-convex optimization problem of the ℓ_p (0<p<1) norm minimization, a series of constrained weighted ℓ_1 minimization is used to approach the ℓ_p norm asymptotically, while the penalized weights at each iteration is different from the standard IRWL$_1$ method. The synthesis method based on ℓ_p(0<p<1) norm minimization, which more democratically penalizes nonzero array excitations at each iteration than the IRWL$_1$ method, has the ability to achieve minimally redundant sparse arrays under the given constraints on the array performances concerning the SLL and beamwidth, despite it converges after a little more iterations.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 61302188, and Grant 61372066, in part by the Natural Science Foundation of Jiangsu Province of China under Grant BK20131005, and in part by the project funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

References

[1] C. A. Balanis, “Antenna Theory: Analysis and Design (Second Edition)”, Wiley, New York, **(1997)**.
[2] R. J. Mailloux, “Phased Array Antenna Handbook (Second Edition)”, Artech House, Norwood, MA, **(2005)**.
[3] L. Cen, W. Ser, Z. L. Yu and R. Susanto, “An improved genetic algorithm for aperiodic array synthesis”, Proceedings of the IEEE International Conference Acoustics, Speech, Signal Processing, Las Vegas, Nevada, **(2008)** March 31-April 4.
[4] V. Murino, A. Trucco and C. S. Regazzoni, “Synthesis of unequally spaced arrays by simulated annealing”, IEEE Transactions on Signal Processing, vol. 44, **(1996)**, pp. 119–123.
[5] C. A. Meijer, “Simulated annealing in the design of thinned arrays having low sidelobe levels”, Proceedings of the IEEE South African Symposium on Communications and Signal Processing, Rondebosch, South Africa, **(1998)** September.
[6] A. Lommi, A. Massa, E. Storti and A. Trucco, “Sidelobe reduction in sparse linear arrays by genetic algorithms”, Microwave and Optical Technology Letters, vol. 31, **(2002)**, pp. 194-196.
[7] K. Chen, Z. S. He and C. L. Han, “A modified real GA for the sparse linear array synthesis with multiple constraints”, IEEE Transactions on Antennas and Propagation, vol. 54, **(2006)**, pp. 2169-2173.

[8] Y. Liu, Z. Nie and Q. Liu, "Reducing the number of elements in a linear antenna array by the matrix pencil method", IEEE Transactions on Antennas and Propagation, vol. 56, **(2008)**, pp. 2955-2962.
[9] G. Oliveri, F. Robol, M. Carlin and A. Massa, "Synthesis of large sparse linear arrays by bayesian compressive sensing", Proceedings of the the 5th European Conference on Antennas and Propagation, EUR Congressi, Rome, **(2011)** April 11.
[10] G. Priso and M. D'Urso, "Maximally sparse arrays via sequential convex optimization", IEEE Antennas Wireless Propagation Letter, vol. 11, **(2012)**, pp. 192-195.
[11] S. E. Nai, W. Ser, Z. L. Yu and H. Chen, "Beampattern synthesis for linear and planar arrays with antenna selection by convex optimization", IEEE Transactions on Antennas and Propagation, vol. 58, **(2010)**, pp. 3923–3930.
[12] B. Fuchs, "Synthesis of sparse arrays with focused or shaped beampattern via sequential convex optimizations", IEEE Transactions on Antennas and Propagation, vol. 60, **(2012)**, pp. 3499-3503.
[13] E. J. Candès, M. B. Wakin and S. P. Boyd, "Enhancing Sparsity by Reweighted ℓ_1 Minimization", Journal of Fourier Analysis and Applications, vol. 14, **(2008)**, pp. 877-905.
[14] S. Foucart and M. J. Lai, "Sparsest solutions of underdetermined linear systems via ℓ_p- minimization for $0<p\leq1$", Applied and Computational Harmonic Analysis, vol. 26, **(2009)**, pp. 395-407.
[15] M. Grant and S. Boyd, "CVX: MATLAB Software for Disciplined Convex Programming (version 1.22)", **(2012)**.
[16] X. J. Chen and W. J. Zhou, "Convergence of the reweighted ℓ_1 minimization algorithm for ℓ_2 - ℓ_p minimization", Computational Optimization and Applications, vol. 59, **(2014)**, pp. 47-61.
[17] D. D. Ge, X. Y. Jiang and Y. Y. Ye, "A note on the complexity of L_p minimization", Mathematical Programming, vol. 129, **(2011)**, pp. 285-299.
[18] Z. B. Xu, X. Y. Chang, F. M. Xu and H. Zhang, "$L_{1/2}$ Regularization: A thresholding representation theory and a fast solver", IEEE Transactions on neural Networks and Learning Systems, vol. 23, **(2012)**, pp. 1013-1027.

Authors

Jinli Chen, he received the Ph.D. degree from Nanjing University of Science and Technology, Nanjing, China, in 2010. Now, he is a Lecturer in College of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. His research is focused on array signal processing, communication signal processing, and MIMO radar array signal processing.

Huasong Cao, he received his bachelor's degrees from Nanjing University of Information Science and Technology, in 2013. Now, he is a master of electronics and communication engineering at Nanjing University of Information Science and Technology, his research interest is sparse array synthesize and compressed sensing.

Jiaqiang Li, he received the Ph.D. degree from Shanghai Jiao tong University, Shanghai, China, in 2007. Now, he is a Associate Professor in College of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. His research is focused on communication signal processing, and radar array signal processing.

Junxiang Ge, he received the Ph.D. degree from Southeast University, Nanjing, China, in 1991. Now, he is a Professor in College of Electronic and Information Engineering, Nanjing University of Information Science and Technology, Nanjing, China. His research is focused on microwave and millimeter wave, antenna design, and microwave remote sensing.

