International Journal of Signal Processing, Image Processing and Pattern Recognition
Vol.9, No.9, (2016), pp.147-154
http://dx.doi.org/10.14257/ijsip.2016.9.9.14

Multiple Signal Estimation Using Weighting Music Algorithm

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Abstract

Subspace partition is a common method in normal MUSIC algorithm that divides the signal covariance matrix into signal subspace and noise subspace by eigenvalue decomposition. By this method, the effect of environmental noise is curbed. However, when the signal angle interval becomes small and the signal-noise ratio reduces, some certain limitations in multiple signal estimation such as loss and confusion will be presented, which means the normal method of estimation is unable to distinguish those signals we need actually. A modified MUSIC algorithm is proposed in this paper to solve the problem. A modified part in the spatial spectrum called weighting function is introduced. Some weighted operation are given to the steering vectors when the spatial spectrum is formed, making the most of subspaces and there eigenvalues. Some simulations followed are taken to discuss the performace of the modified method. Through the analysis we can see that, under the condition of a small signal angle interval and a low signal-noise ratio, the improved algorithm could achieve satisfactory result for the DOA estimation.

Keywords: *MUSIC algorithm, weighting function, eigenvalue decomposition*

1. Introduction

Multiple signal classification algorithm [1] (MUSIC) is one of the methods about array signal processing [2-4]. This algorithm has a good performance of signal estimation, for instance, a significant estimation variance which is close to the cramer-rao bound and a moderate computational work. Taking eigenvalue decomposition [5] (EVD) with the covariance matrix which is made up of array signals, the two subspaces called signal subspace and noise subspace are divided and the orthogonality between them is utilized to build the spatial spectrum which include the signal parameters, such as Direction of Arrival [6-7] (DOA) and location [8-9].

The classical MUSIC algorithm has certain limitations in DOA estimation especially in multiple signals estimation. When the several incident angles of signals are close, the division of subspaces appears blurred which causes a certain loss and confusion of signal parameters.

To get more accurate information about target azimuth, some improvements are put forward. A novel weighting function W is introduced to the MUSIC spatial spectrum. By some weighted operation to the steering vectors in spatial spectrum, the improved algorithm could provide a better noise suppression and more accurate estimated results.

2. Research Method

Consider a uniform linear array (ULA) for the convenience purpose. Some array parameters are given. The number of array elements is M. The spacing between two

ISSN: 2005-4254 IJSIP

adjacent elements is $d = 0.5\lambda$, where λ is the carrier wavelength. Suppose that there are N $(N \le M)$ far-field narrowband signals arrive at the ULA with an angle θ, as shown in Figure 1.

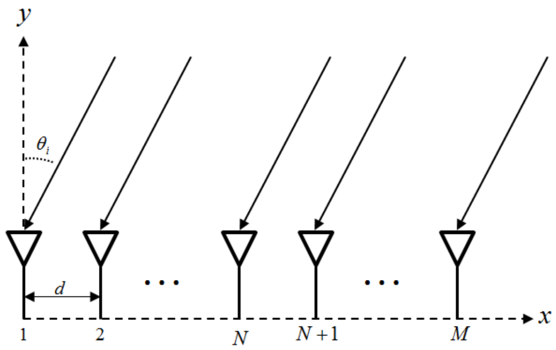

Figure 1. Uniform Linear Array

The environmental parameters are also given. Suppose that there is a zero mean and the variance σ_n^2 Gaussian white noise (GWN) in the environment, having no correlation with the signals monitored. The data received by array elements can be expressed as,

$$X(t) = AS(t) + N(t) \tag{1}$$

Where $A = [\alpha(\theta_1), \alpha(\theta_2), \cdots, \alpha(\theta_N)]$ denotes the array manifold matrix, $\alpha(\theta_i) = [1, e^{j2\pi \sin(\theta_i)/\lambda}, \cdots, e^{j2\pi(M-1)\sin(\theta_i)/\lambda}]$ denotes the steering vector. $S(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^T$ denotes the vector of source waveforms, $N(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$ denotes the vector of noise received by array elements.

Some conditions should be satisfied before we use MUSIC algorithm to estimate the signal parameters. First, the number of array elements should be more than the signal number. Then, the steering vectors made up by signals with different incident angles are independent. Also, a non-singular matrix for covariance matrix is necessary. The last, the environmental noise should satisfy the condition as follow,

$$\begin{cases} E\{n_i(t)\} = 0 \\ E\{n_i(t) n_i^H(t)\} = \sigma^2 \\ E\{n_i(t) n_i^T(t)\} = 0 \end{cases} \tag{2}$$

Consider a covariance matrix of array signals R_X , take eigenvalue decomposition and arrange the eigenvalues from large to small,

$$\lambda_1 \ge \lambda_2 \ge \ldots \ge \lambda_N \ge \lambda_{N+1} \ge \ldots \ge \lambda_M \tag{3}$$

Then we get

$$R_X = U_s \Lambda_s U_s^H + U_n \Lambda_n U_n^H \tag{4}$$

Where $U_s = [u_1, u_2 \ldots u_N]$ denotes the signal subspace, corresponding to the N large eigenvalues of $\Lambda_s = diag\{\lambda_1, \lambda_2, \ldots \lambda_N\}$. $U_n = [u_{N+1,} u_{N+2,} \ldots u_M]$ denotes the noise subspace, corresponding to the $M - N$ small eigenvalues of $\Lambda_n = diag\{\lambda_{N+1}, \lambda_{N+2}, \ldots \lambda_M\}$. When $k = N+1, \ldots, M$,

$$\begin{cases} R_X \lambda_k = \lambda_k u_k = \sigma^2 u_k \\ R_X \lambda_k = (AR_S A^H + \sigma^2 I_M) u_k \end{cases} \tag{5}$$

To come to this case, we get,

$$AR_S A^H u_k = \mathbf{0}_{M\times 1} \tag{6}$$

$$A^H u_k = \mathbf{0}_{N\times 1} \tag{7}$$

Further,

$$A^H U_n = \mathbf{0}_{N\times(M\text{-}N)} \tag{8}$$

From the formula above we know that the array manifold and noise subspace have orthogonality. It is worth noting that the signal subspace is spanned by the steering vector of array manifold. We can consider that the signal subspace and noise subspace satisfy the orthogonal condition, marked $spanU_s \perp spanU_n$. Then the spatial spectrum of MUSIC is constructed as follow,

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) U_n U_n^H a(\theta)} \tag{9}$$

Then scanning the spatial spectrum within the searching scope, the N incident angles will be found. The steps of algorithm are as follows,

1. According to the data received by array, calculate the covariance matrix R .

2. Do eigenvalue decomposition with R , get the eigenvalues and corresponding eigenvectors

3. Determine the signal numbers and divide the signal subspace U_s and nocie subspace U_n .

4. Construct the spatial spectrum of MUSIC algorithm P_{MUSIC} , search the spectral peaks and find the incident angles of signals.

To get more accurate information about target azimuth, some improvements are put forward. In classical MUSIC algorithm, if the number of signals we estimated is error, the noise subspace divided is not in conformity with the actual, leading to a big error even a failed DOA estimation, especially in the case of a small SNR. An advisable method is that all the characteristic vectors of the signal covariance matrix are considered and a novel weighting should be taken to the characteristic vectors. After this, the proper weight coefficient will reduce the effect of signal eigenvectors and the noise eigenvectors are entirely used.

The novel weighting function W is introduced to the MUSIC spatial spectrum, then the spatial spectrum changes as,

$$W = \begin{bmatrix} \lambda_1^q & 0 & 0 & 0 \\ 0 & \lambda_2^q & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_M^q \end{bmatrix} \tag{10}$$

$$P_{MUSIC} = \frac{1}{a^H(\theta)\hat{U}_N\hat{U}_N^H W\hat{U}_N\hat{U}_N^H a(\theta)} \tag{11}$$

In the formula above, $\lambda_i, i = 1, 2, \cdots M$ is the eigenvalues of the array covariance matrix. $q \le 0$ is a customized parameter. Different W causes different estimation performance. When W is a unit matrix, that means a uniform weighting is selected to the eigenvectors, which evolve into a classical algorithm. When $W = e_1 e_1^H$, where $e_1 = [1, 0, \text{L}\ , 0]^T$ belongs to a vector of noise subspaces, the minimum inner product algorithm correspond.

3. Results and Discussions

To verify the performance of the improved algorithm, named weighting MUSIC algorithm (W-MUSIC), some simulations and discussions are taken in this section. Consider a uniform linear array which is consisted of 8 non-direction array elements. The incident angles of two independent signals are 30 degree and 60 degree. The Signal to Noise Ratio is 10 dB. The surrounding noise is Gaussian white noise. The estimated results of two algorithms are as Figure 2.

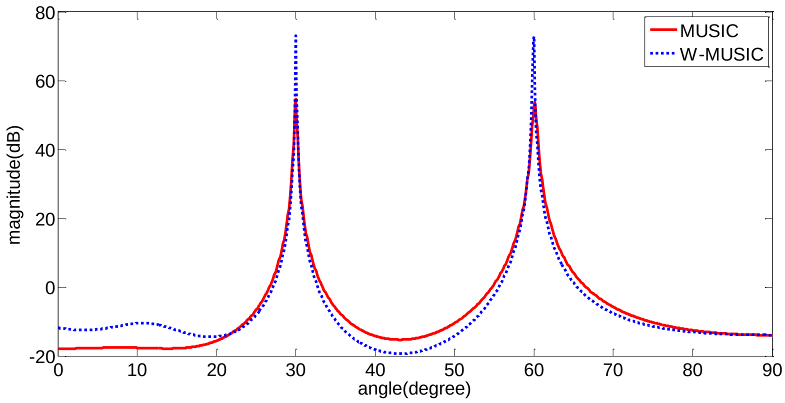

Figure 2. The Estimated Results of Two Algorithms, Classical MUSIC Algorithm and Weighting MUSIC Algorithm, 8 Array Elements, SNR =10dB, Incident Angles are 30 Degree and 60 Degree

As Figure 2, shows, the red solid line is the spatial spectrum of classical MUSIC algorithm, the blue dotted line is the spatial spectrum of weighting MUSIC algorithm. The searching range of angle is from 0 degree to 90 degree. We can see that the peaks in spatial spectrum of weighting MUSIC are higher and sharper than the classical MUSIC. That means the weighting MUSIC algorithm can provide a better estimation performance.

Then we discuss the angle resolution of two algorithms. Consider the same array configuration as mentioned above. Three far field narrowband signals exist, which is no correlation among them. The incident angles of three are -20 degree, 0 degree and 4 degree. SNR is 10 dB. The surrounding noise is GWN. The estimated results of two algorithms are as Figure 3.

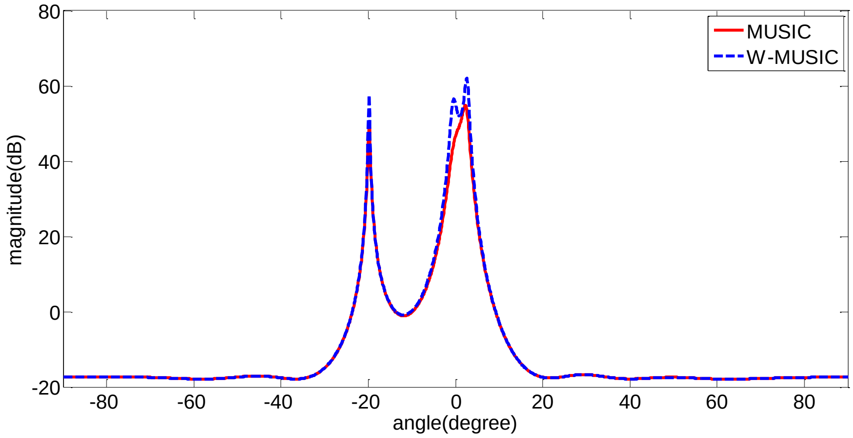

Figure 3. The Estimated Results of Two Algorithms, Classical MUSIC Algorithm and Weighting MUSIC Algorithm, 8 Array Elements, SNR =10dB, Incident Angles are -20 Degree, 0 Degree and 4 Degree

As Figure 3, shows, the red solid line and the blue dotted line respectively represent the classical and weighting MUSIC algorithm. We can see that there are only two peaks in spatial spectrum of classical MUSIC and the two peaks corresponding to 0 degree and 4 degree are mixed. However there are three obvious peaks in the spatial spectrum of weighting MUSIC. That means the improved algorithm can offer a better angle resolution especially the angle difference is small.

The performances of signal estimation about the two algorithms are discussed next. Consider a ULA with 8 independent Omni-directional array sensors, array element interval $d = 0.5\lambda$, λ is the signal wavelength. 2 far-field narrowband signals arrive with the angle $\theta = (\theta_1, \theta_2)$, which the estimation value is $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$. To facilitate the calculation and description, a successful DOA estimation is defined as,

$$\begin{cases} Success, if \ \left|\hat{\theta}_1 - \theta_1\right| + \left|\hat{\theta}_2 - \theta_2\right| < \left|\hat{\theta}_1 - \hat{\theta}_2\right| \\ fail, other \end{cases} \tag{12}$$

Then the probability of success estimation (PSE) η is defined as,

$$\eta = \frac{N_{success}}{N_{all}} \times 100\% \tag{13}$$

Where, $N_{success}$ denotes the times of success estimation, N_{all} denotes the total times of DOA estimation. The root mean square error (RMSE) of DOA estimation is as follow,

$$RMSE = \sqrt{\frac{1}{100}\left(\sum_{j=1}^{100}\left(\hat{\theta}_{1,j} - \theta_1\right)^2 + \sum_{j=1}^{100}\left(\hat{\theta}_{2,j} - \theta_2\right)^2\right)} \tag{14}$$

Study the performances about the classical and modified algorithms in the case that different angle offset of two signals. The range of angle offset $\Delta\theta = \left|\theta_1 - \theta_2\right|$ is 2 degree to 20 degree, SNR is 10 dB, as Figure 4 shows.

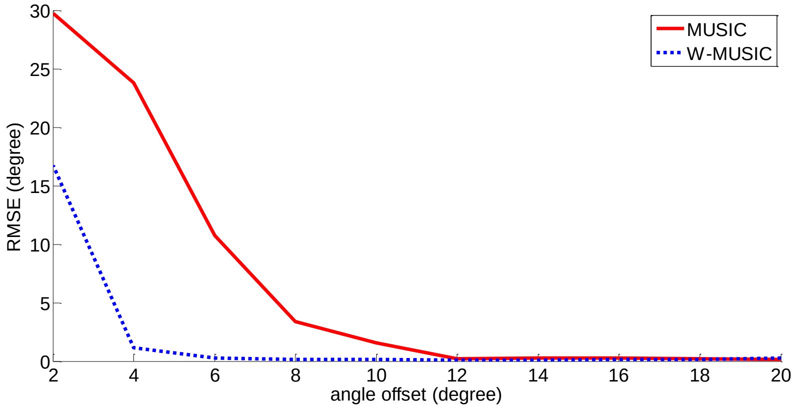

Figure 4. DOA Estimation RMSE Against $\Delta\theta$

One can see in Figure 4, the x axis is the angle offset $\Delta\theta$, while the y axis is the RMSE of DOA estimation. As $\Delta\theta$ is small, for instance, $\Delta\theta$=2° , the [RMSE(MUSIC), RMSE(W-MUSIC)] are [29.1°, 16.9°], large deviation, which means the results are unable to reflect the DOA of signals correctly. When $\Delta\theta$ is an proper value, for instance, $\Delta\theta$=6° , the [RMSE(MUSIC), RMSE(W-MUSIC)] are [24.3°, 2.1°]. One can see that the modified algorithm has higher estimation precision than the standard.

Figure 5 shows the RMSE of DOA estimation about two methods against the SNR of signals, the two incident angles of signals are $\theta_1 = \pi/3$, $\theta_2 = \pi/6$. Using 100 Monte Carlo simulations to calculate the RMSE, the results are as follows,

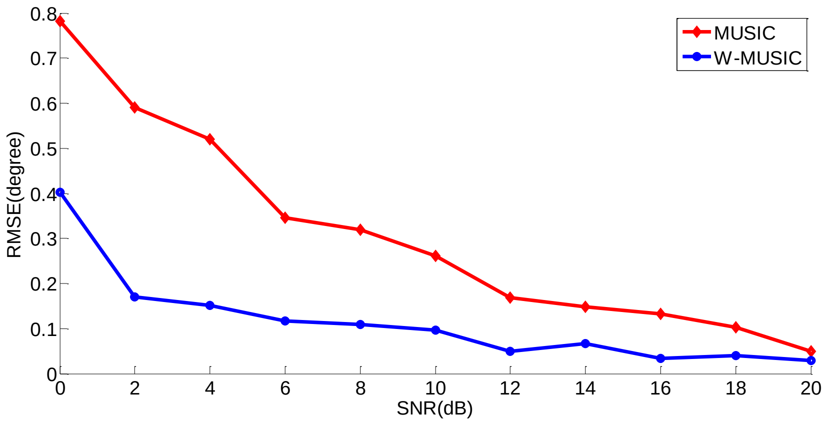

Figure 5. DOA Estimation RMSE against SNR

One can see that under the condition of higher SNR, for instance, 20dB, the two methods both have good performances of DOA estimation, the RMSE of each is 0.07 degree and 0.05 degree. With the SNR decreases, the RMSE of both two algorithm increases, however, the W-MUSIC (blue line with circular marks) has lower RMSE than the classic method (red line with diamond marks).

Study the probability of success estimation of the two methods mentioned above against the SNR with the range is 0dB to 20dB. The simulation conditions are given as the same mentioned above, as Figure 6 shows,

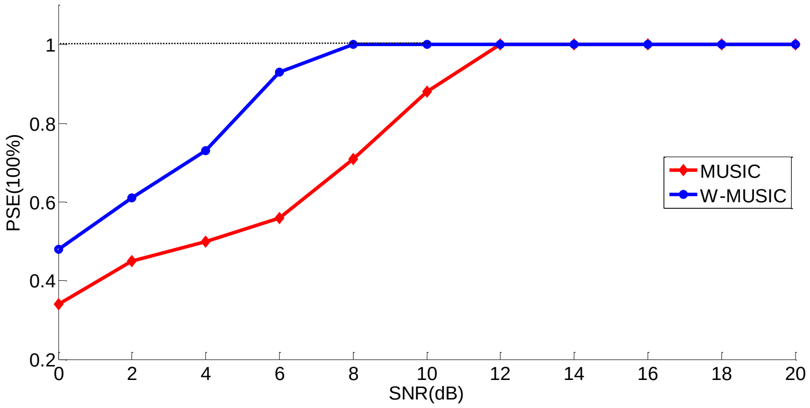

Figure 6. PSE Against SNR

One can see that in Figure 6, the x axis is the SNR, range [0dB, 20dB], 2dB step. The y axis is the PSE of DOA estimation. With the increase of SNR, the PSE of the two algorithms are both gradually increased, while the PSE of W-MUSIC (blue line with circular marks) is higher than the classic method (red line with diamond marks), for instance, when SNR is 6dB, the PSE of two methods are 0.56 and 0.93 respectively. To reach the condition of one hundred percent PSE, the W-MUSIC need a smaller SNR than the classic, 8dB and 12dB respectively.

One can see from these simulations above that, under the condition of a small signal angle interval and a low signal-noise ratio, the improved algorithm could provide a better signal resolution and offer more accurate estimated results than the classical.

4. Conclusion

This article proposes the cyclic cross correlation MUSIC algorithm to estimate multiple signals with less array elements. Simulation results show that the cyclic cross correlation MUSIC algorithm can provide a better estimation performance especially when the signals number is more than the array elements. This algorithm has a better noise suppression and a larger array aperture than the standard, having a certain reference value.

Acknowledgments

The author acknowledges the funding of following science foundation: National Natural Science Foundation of China (Grant Nos. 60908028, 60971068, 10979065, and 61275201).

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