Segmentation for Range Image Based on Snake Active Contour Model

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Abstract

Range image segmentation is one of the most common problems in the field of computer vision. In view of the defect about traditional range image segmentation methods, this paper introduces a new range image segmentation method based on snake active contour model (SCAM). First, this paper expresses the snake active contour model in the form of parameters, and illustrates the contour line data points inside and outside of the movement and pseudo code description of the motion of a point algorithm; then constructs energy function, pushing the Euler equation and discretization; finally gets the results by Cholesky decomposition. Numerical experiment results show that the method is accurate and efficient, of good effect and the segmentation results are consistent with human subjective visual perception.

Keywords: Range image segmentation; Snake active contour model; Energy function; Cholesky decomposition

1. Introduction

Segmentation is one of the old complex problems that has not yet been solved in computer vision field. The segmentation of point cloud algorithms in the field of 3D research is still very immature, by transforming the 3D laser point cloud into the 2D range images, this paper generally adopt the dimension reduction approach. Then use point cloud based on picture processing of mesh method for image segmentation to get the segmented plane domain. According to one-to-one mapping relations between 2D image points and 3D spatial point and corresponding laser point cloud can be partitioned into finite surface.

So far, the number of the image segmentation method is vast. More mature traditional range image segmentation methods have range image segmentation based on differential invariants and region growing [1], range image segmentation based on binary morphology [2] and range image segmentation based on morphology watershed region [3]. Among them, range image segmentation method based on differential operation need complex operation and is sensitive to noise, the common defect of which the boundary curve is not smooth and clear; the boundary curve is made up of the segmentation of the object of free shape. Snake active contour model is also known as parameter active contour model [4]. Due to its computational efficiency and simplify, it is particularly suitable for modeling and extracting complex shape curved objects’ Boundary contour. Therefore, snake active contour model in edge detection, target segmentation, medical image processing and video tracking in [5-8] have gained a wide application and development of far-reaching for more than 20 years. It is still one of the most active research topics in computer vision fields and a major breakthrough [9] in this field.

The advantages of the proposed method has the following several aspects: (1) In the difficult circumstances of target area for feature selection, the method can
directly use the gray or texture data which is extracted, and therefore has a broader scope of application; (2) The method has overcome the deficiency of the initial contour interactive selection, the algorithm has higher robustness and flexibility. The comparative experiments prove that the proposed algorithm is feasible.

2. Basic Knowledge

2.1. Region Segmentation Description

The Segmentation of a dictation image is related to the outline extract. Suppose that $R$ is used to express the whole region of the distant image, then the so-called Regional Segmentation is to divide $R$ into $n$ parts as $R_i, R_2, \ldots, R_n$, these parts mentioned above are up to the five conditions that is following:

1. $\bigcup_{i=1}^{n} R_i = R$;
2. $R_i$ is continuous region, $i = 1, 2, \ldots, n$;
3. $R_i \cap R_j = \emptyset$, for any form of $i \neq j$;
4. $P(R_i) = \text{TRUE}$, on the condition of all forms of $i = 1, 2, \ldots, n$;
5. $P(R_i \cup R_j) = \text{FALSE}$, for each adjoin region, on condition that $R_i$ is the logic.

2.2. Curve Evolution Mechanism

Curves can be of three types: simple curves, common curves and complication curves. There being curvature in curves, and in curvature there being positive and negative, on the promotion of the normal force, the direction that the curves more varied: some more outward, others inwards.

Simple curves, when under the force of curvature, its evolution possess particular mathematized characteristic, that is: For all simple curves, seriously warped or not, provided it is simple curves it would be evaluated to be a circle on the force of curvature, then lost.

There being two important parameters in the description of the geometry characters of curves curvature and the unit normal vector, the former describes the direction, the latter states level, thus curves evolution just take advantage of these two elements to study the change of the curve over time. Its evolution procedure could be regards as on the force of $F$, the evolution its direction $N$ and the speed $V$.

When it comes to the procedure of the curve evolution, it can be treat as the process of different force level to the curve, and force symbol energy. All things in the world tend to be in the form of minimum energy, hence they are in a balance stale for the time being.

In the Segmentation of image, we figure out the outline at last, which means the minimum energy on the control of normal force what it evaluates to the outline, it energy is the in the minimum state, it is harmonious, its speed is zero and keep step, the part is segmented at this time.

3. Fundamental Principle

Snake active contour model was first promoted by its inventor Kass [10], the basic idea is to put some data points can be fitted to a certain shape deformable closed contour curves, by defining the energy function to change the shape of a closed curve to close to the actual contour in the image target. Then the image segmentation process is transformed into the process of solving the minimum value
of energy function. The closed curve with the minimum energy is the actual contour of the target. In the process of approximation of the boundary of the object, the closed curve constantly changing shape, like a snake crawling.

Snake active contour model is a set of ordered points, these points’ head and the tail are connected by straight lines constituting a contour line. Formula (1) for the parameter vector available:

\[ C(s,t) = \{[x_i(s,t), y_i(s,t)]|s \in [0,1], i = \{1,2,\ldots,L\} \} \] (1)

In Formula (1), \(x_i(s,t)\) and \(y_i(s,t)\) stand for each control point coordinates in the image, \(t\) is the time parameter, \(s\) is the independent variable, which is described the boundary in the form of Fourier transform. It’s normalized arc length parameters along a closed curve \(C(s,t)\).

### 3.1. Data Point Movement

Snake active contour model on the data points \(P_{i,t} = [x_i(s,t), y_i(s,t)]\) shown in Figure 1, the algorithm shown in Figure 2 in Algorithm 1. Data point has been movement, except when hard constraints (moving point across data points, edges).

![Figure 1. a. Data Points Inward Movement](image)

**Figure 1. a. Data Points Inward Movement**

![Figure 1. b. Data Points Outward Movement](image)

**Figure 1. b. Data Points Outward Movement**

**Algorithm 1.** Current point \(P_{i,t}\) on the contour move to a lower energy position \(P_{i,t+1}\).

**Input:** \(t\) times \(L\) data points; each \(P_{i,t}\) with velocity \(V_{i,t}\) and acceleration \(A_{i,t}\).

**Output:** \(t+1\) times \(L\) data points; each \(P_{i,t+1}\) has velocity \(V_{i,t+1}\) and acceleration \(A_{i,t+1}\).

\(P_{i,t}\) was calculated for each point which was not stopped because of hard constraints at every intervals \(\Delta t\):

1. Calculate the force \(A_{i,[t]}\) suffered by the neighboring points \(P_{i,t}\);
2. Acceleration vector \(A_{i,[t]}\) is calculated by force;
3. Computing speed \(V_{i,t+1} = V_{i,t} + A_{i,t} \Delta t\);
4. Calculate the new position \(P_{i,t+1} = P_{i,t} + V_{i,t} \Delta t\);
5. If \(P_{i,t+1}\) is within the control of the data points, it is locked in this position.

**Figure 2. Contour Data Point Movement Algorithm**
3.2. Construct Energy Function

The snake active contour is used to fit the data points is an optimization problem, namely in the contour data points can be iterative approximation of the actual target energy minimization problem by solving the boundary, active contour curve move in picture airspace. Its energy function [11] \( E_{\text{snake}} \) using the Formula (2) is defined:

\[
E_{\text{snake}} = E_{\text{internal}} + E_{\text{out}}
\]  

(2)

Where \( E_{\text{internal}} \) is the outline of the internal energy, it encourages active contour deformation stretching and bending deformation, and include the elastic energy and bending energy. The internal energy only relates to the shape of snake, whereas with the image data is independent; \( E_{\text{out}} \) is the outline of external energy, it attracts contour points by characteristics of interest in an image and guide the active contour curve towards the direction of movement of the object boundary, the external energy is only concerned with the image data.

Internal energy function \( E_{\text{internal}} \) is defined in Formula (3):

\[
E_{\text{internal}} = \int_0^1 (\alpha(s)\|C_s(s)\|^{2} + \beta(s)\|C_{ss}(s)\|^{2})ds
\]  

(3)

Differential expressed by the subscript, first-order differential express the rate of change of the active contour curve length in Formula (3), used to limit the length of a large segment of small changes in contour, elasticity function coefficients \( \alpha(s) \) can control the speed of the active contour curve stretching; Second order differential represents the rate of change of curvature of activity profile curve. Rigid function coefficient \( \beta(s) \) used for controlling activities along the normal direction of the profile curve toward the target contour speed. Weighting function coefficients \( \alpha(s) \) and \( \beta(s) \) play a reconcile role. Proper adjustments of two coefficients make active contour curve continuity, smoothness and the degree of bending during deformation.

External energy function \( E_{\text{out}} \) is defined in Formula (4):

\[
E_{\text{out}} = E_{\text{image}} + E_{\text{constraint}}
\]  

(4)

Where \( E_{\text{image}} \) is the image energy, indicating fitting degree between contours and image brightness, image gradient and line endpoints; where \( E_{\text{constraint}} \) is bound energy, constraint information is provided by the user in an interactive manner, and almost not used. Image energy function \( E_{\text{image}} \) can be expressed as the sum of three:

\[
E_{\text{image}} = \int_0^1 (w_{\text{line}}E_{\text{line}} + w_{\text{edge}}E_{\text{edge}} + w_{\text{term}}E_{\text{term}})ds
\]  

(5)

Where lines \( w_{\text{line}}E_{\text{line}} \) guide the snake contour to dark ridge, it is attracted toward (the edge of) the position of the image intensity gradient by the edge \( w_{\text{edge}}E_{\text{edge}} \), and attracted toward the snake contour endpoint by the endpoint \( w_{\text{term}}E_{\text{term}} \). In practice, the vast majority of the models only use the edge of items. Suppose \( I(x, y) \) is a range image and contains a continuous area, defined as the edge of the item:

\[
E_{\text{edge}} = -|\nabla(G_\sigma(x, y)*I(x, y))|^2 \sum_{i=1}^L |C(i)|^2
\]  

(6)
Where \((G_\sigma(x, y))\) whose standard deviation is \(\sigma\) is a two-dimensional Gaussian function, in order to reduce the noise calculation of the gradient, convolution operation range image \(I(x, y)\) using it. \(\nabla\) is the gradient operator.

So far, the energy functional \(E_{\text{snake}}\) of the snake active contour models can be more completely expressed as:

\[
\int_0^1 \left( \alpha(s) \left\| C_y(s) \right\|^2 + \beta(s) \left\| C_x(s) \right\|^2 + \gamma(s) E_{\text{edge}} \right) ds
\]

### 3.3. Numerical Solution

According to the variation principle, in order to achieve the minimum energy \(E_{\text{snake}}\), active contour curve \(C(s, t)\) must meet the following two independent Euler equations:

\[
\frac{\partial}{\partial s} \left( \alpha(s) \frac{\partial C}{\partial s} \right) + \frac{\partial^2}{\partial s^2} \left( \beta(s) \frac{\partial^2 C}{\partial s^2} \right) + \frac{1}{2} \frac{\partial E_{\text{edge}}}{\partial x} = 0
\]

\[
\frac{\partial}{\partial s} \left( \alpha(s) \frac{\partial C}{\partial s} \right) + \frac{\partial^2}{\partial s^2} \left( \beta(s) \frac{\partial^2 C}{\partial s^2} \right) + \frac{1}{2} \frac{\partial E_{\text{edge}}}{\partial y} = 0
\]

In order to solve the problem with numerical method, we need to use finite difference method discretizes the Euler equations into the following form:

\[
\sum_{i=1}^{L} \left[ \alpha(i) \left\| C(i+1) - C(i) \right\|^2 / h^2 + \beta(i) \left\| C(i+1) - 2C(i) + C(i-1) \right\|^2 / h^4 \right] + \frac{1}{2} \frac{\partial E_{\text{edge}}}{\partial s} = 0
\]

\[
\frac{1}{h} \left\{ \alpha(i+1) \frac{x_{i+1} - x_i}{h} - \alpha(i) \frac{x_i - x_{i-1}}{h} \right\} +
\frac{1}{h^2} \left\{ \beta(i+1) \frac{x_{i+2} - 2x_{i+1} + x_i}{h^2} - 2\beta(i) \frac{x_{i+1} - 2x_i + x_{i-1}}{h^2} + \beta(i-1) \right\} +
\frac{1}{2} \frac{\partial E_{\text{edge}}}{\partial x} \bigg|_{x_{i-1}, x_i} = 0
\]

By merging the different coefficients of orderly data points, the formula (10) can be expressed as:

\[
f_i(x) = a_i x_{i-2} + b_i x_{i-1} + c_i x_i + d_i x_{i+1} + e_i x_{i+2}
\]

Forum (10) will be rewritten into a linear matrix form about the \(x\) by

\[
f_i(x) = -\frac{1}{2} \frac{\partial E_{\text{edge}}}{\partial x} \bigg|_{x_{i-1}, x_i} \cdot a_i = \frac{\beta(i-1)}{h^4} \cdot x_{i-2}, b_i = \frac{2[\beta(i) + \beta(i-1)]}{h^4} - \frac{\alpha(i)}{h^2},
\]

\[
c_i = \frac{\beta(i+1) + 4\beta(i) + \beta(i-1)}{h^4} + \frac{\alpha(i+1) + \alpha(i)}{h^2}, d_i = \frac{2[\beta(i+1) + \beta(i)]}{h^4} - \frac{\alpha(i+1)}{h^2}, e_i = \frac{\beta(i+1)}{h^4}
\]

as follows:

\[
A \mathbf{x} = \mathbf{f}_i(x, y)
\]

Where \(f_i(x, y)\) is the edge strength \(E_{\text{edge}}\) along the X-axis of a first-order differential, \(A\) is five diagonal banded matrix, which is determined by the number of discrete points \(L\).
Similarly, we can get:

\[ \mathbf{Ay} = f_r(x, y) \quad (13) \]

Then, with Cholesky factorization for solving the Formula (12) and (13), and obtain the coordinate expression of the position vector:

\[
\mathbf{A} = \mathbf{R}^T \mathbf{R} \\
\mathbf{x} = \mathbf{R}^{-1} \left( (\mathbf{R}^T)^{-1} \mathbf{f}_x(x, y) \right) \\
\mathbf{y} = \mathbf{R}^{-1} \left( (\mathbf{R}^T)^{-1} \mathbf{f}_y(x, y) \right) \quad (14)
\]

In the Formula (14), \( \mathbf{R} \) is an upper triangular matrix and it can be solved by the Cholesky factorization.

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4. Experimental Results

Combine the programming environment of VC++6.0 with MATLAB7.0 realize the segmentation of range image of Mickey in Figure 3:

\[ \text{Figure 3. a. Range Image  b. Initial Contour  c. Snake Model (1/3)} \]
\[ \text{d. Snake Model (2/3)  e. Final Segmentation  f. Result of Literature [3]} \]
In order to explain the universality of the method in the paper, continue to segment the depth image of the workpiece, Results as shown in Figure 4.

![Figure 4. a. Range Image   b. Initial Contour     c. Snake Model (1/3)
  d. Snake Model (2/3)    e. Final Segmentation    f. Result of Literature [3]](image)

5. Conclusion

Parametric active contour model has the incomparable advantages of image segmentation based on data-driven methods, such as Laplace operator, threshold and region growing. Advantages are as follows: ① Image data, the initial estimate, the target boundary and constraint-based knowledge in a unified parametric active contour model energy function; ② After properly initialized, parametric active contour model can automatically converge to the minimum energy state; ③ Due to the integration of advanced information, you can focus on the extraction of the desired goal at the beginning of the extraction process, and the minimization of energy from coarse to fine in the scale space can greatly expand the capture area and reduce the computational complexity, and therefore can greatly reduce the amount of calculation; ④ Due to the use of this model can eliminate texture or noise caused the discontinuities on the target extracted by a desired; ⑤ Traditional template method is different is that this model is not due to the constraints of the target shape, we can extract the target of arbitrary shape, and can adapt to changes in the target shape occur in a timely manner, and therefore has a great value in the target trail.

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