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The Improved Wavelet Threshold Function and Its Application

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Abstract

Images will produce noise in the process of storage and collection. Wavelet threshold de-noising is a simple and effective de-noising method, but the choice of threshold function is a key. The hard-threshold function is discontinuous and there is the deviation between the signal processed by the soft-threshold function and the real signal, so this paper constructs a new threshold function at the origin sufficiently smooth to deal with above problems. A parameter is added to the new threshold function, which is between the soft-threshold and hard-threshold function by adjusting the parameter. The new threshold function can remove the noise effectively, and the image information is well preserved. Hence it plays an important role in follow-up edge detection. The de-noising method with improved wavelet threshold is presented, and then uses morphological edge detection on the new image in this paper. The result shows that the method can detect the complete edge effectively, and the visual effect and objective evaluation are good.

***Keywords**: wnavelet, de-noising, threshold function, edge detection*

1. Introduction

Edge, as important basis of image segmentation and image analysis, it contains most of the information of the image. Edge is the most basic feature of the image. It has the extensive application prospect [1]. Edge detection has wide application in image segmentation, image matching, feature extraction, and other fields, is also the vital research content of computer vision [2]. There are many methods to do edge detection, such as Sobel operator, Roberts operator, Laplace operator, Canny operator and so on, which is the traditional algorithms of edge detection [3-4]. The morphological edge detection with the unique advantages in image edge detection, is becoming image processing tool, and increasingly shows good application prospect.

Digital image is interfered with noise in the process of gathering and transmission. Noise has a great influence on the subsequent processing of digital image-edge detection. So image preprocessing should be firstly done to remove the noise of the image when image edge detection is adopted [5]. Nowadays, many kinds of de-noising methods have been studied, such as average filtering, median filtering, wiener filtering and so on. Although these methods can remove part of noise, the effect is not good [6]. For instance, de-noising ability of some of those is too weak enough to filter out noise completely. Others remove excess noise, which leads that real edge was disposed. Wavelet threshold de-noising method is a new method for de-noising field, and the de-noising effect is better. This method can be easily implemented and operated, which also has more extensive adaptability. The wavelet threshold de-noising generally includes hard threshold de-noising method and soft threshold de-noising method. Hard and soft threshold method

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inherit the good properties of wavelet transform, such as easy implementation and simple operation, so it gains widespread application.

However, there also exist defects. For example, some points of hard threshold function are discontinuous. Soft threshold function has some deviations with real images in wavelet coefficients. Above these faults have affected the de-noising effect. The literatures [7-9] have had made corresponding improvement based on the above consideration. The functions of two thresholds are continuous in literature [7-8], which in most cases is not smoothness and inconvenient for mathematical processing. The method proposed in literature [9] not only overcomes faults of soft and hard threshold method, also has a certain smoothness, nevertheless the constructed threshold function has more complexity. In order to solve these problems, this paper constructs a new threshold function, which is sufficiently smooth at the origin, more continuous, almost smooth everywhere and more simple. The experimental results confirmed that de-noising effect of the new method is better than that proposed by the literatures [7-9].

2. Improved Threshold De-Noising Function

2.1. Wavelet Threshold De-Noising

The model of noise signal is

$$h(i) = f(i) + n(i), \ i = 0,1,\cdots,n-1 \tag{1}$$

where $f(i)$ is the original signal, $h(i)$ is the signal with noise, $n(i)$ is noise signal. We

make discrete wavelet transform on formula (1). We have

$$\omega_h(j,i) = \omega_f(j,i) + \omega_n(j,i) \tag{2}$$

Here $j = 0,1,\cdots,J$; $i = 0,1,\cdots,n-1$, $\omega_f(j,i)$, $\omega_n(j,i)$, $\omega_h(j,i)$ are respectively corresponding wavelet coefficients with Original signal, noise signal and signal with noise.

After the image using wavelet transforms, the absolute value is less than the threshold of wavelet coefficient is removed as the noise signal, the most important aspect of threshold de-noising is to set the threshold value. Too higher threshold will remove a useful high-frequency information as a noise signal and the resulting image blurring, such as edge information. Too small threshold will retain excessive noise and less obvious de-noising effect.

Steps of wavelet threshold de-noising processing signals can be divided as follows.

Step1. Choose appropriate wavelet and wavelet decomposition layers, calculate wavelet coefficients of noisy signals.

Step2. Select the appropriate threshold and process on high frequency coefficient.

Step3. Reconstruct signal according to the processed wavelet coefficients.

2.2. Hard and Soft Threshold

The traditional threshold de-noising method contains the hard and the soft threshold de-noising method. The expressions show following.

The expression of hard threshold function is

$$\hat{\omega}_{j,i} = \begin{cases} \omega_{j,i}, & |\omega_{j,i}| \ge T \\ 0, & |\omega_{j,i}| < T \end{cases} \tag{3}$$

The image is shown as follows.

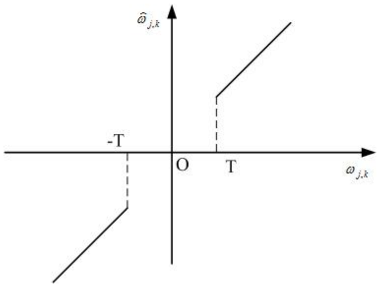


Figure 1. The Hard Threshold Function

The expression of soft threshold function is

$$\hat{\omega}_{j,i} = \begin{cases} \mathrm{sgn}(\omega_{j,i})\left(\left|\omega_{j,i}\right| - T\right), & \left|\omega_{j,i}\right| \geq T \\ 0, & \left|\omega_{j,i}\right| < T \end{cases}. \tag{4}$$

Where, $\omega_{j,i}$ is wavelet coefficients. $\hat{\omega}_{j,i}$ is the new wavelet coefficients. T is the threshold value. The image is shown as follows.

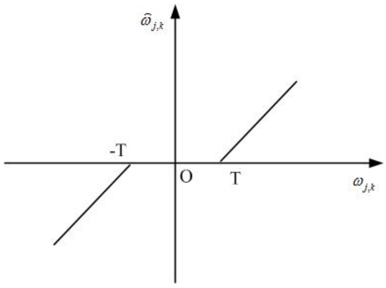


Figure 2. The Soft Threshold Function

In practice, although two kinds of de-noising methods that soft threshold and hard threshold obtain wide spread application and the good effect in image processing, they still exist a few problems in itself. The discontinuity of the hard threshold function, results in poor continuity of wavelet coefficients of hard threshold processing. The continuity of the wavelet coefficient value processed by soft threshold is good, which is also easy to handle. But when the wavelet coefficient is large, the estimated wavelet coefficients value compared with the original wavelet coefficient values have a fixed bias, which will bring inevitable errors to reconstruct the signal. It is the most important fact to select the appropriate threshold function.

2.3. The Improved-Threshold Method

Disadvantage of the above hard and soft threshold function influenced the de-noising effect of the image. The literatures [7-9] have made corresponding improvement in different degrees. The threshold function of literature [7-9] were

$$\hat{\omega}_{j,i}=\begin{cases}\mathrm{sign}(\omega_{j,i})(|\omega_{j,i}|-\dfrac{\lambda}{\exp(\dfrac{|\omega_{j,i}|-\lambda}{\lambda})}), & |\omega_{j,i}|\geq\lambda\\ 0\ , & |\omega_{j,i}|<\lambda\end{cases} \tag{5}$$

$$\hat{\omega}_{j,i}=\begin{cases}\mathrm{sign}(\omega_{j,i})\cdot\sqrt{\dfrac{\omega_{j,i}^2+(|\omega_{j,i}|-\lambda)^2}{2}}\ , & |\omega_{j,i}|\geq\lambda\\ 0, & |\omega_{j,i}|<\lambda\end{cases} \tag{6}$$

$$\hat{\omega}_{j,i}=\begin{cases}\omega_{j,i}-\lambda\mathrm{sign}(\omega_{j,i})(1-\dfrac{2}{\exp(\left|\dfrac{\lambda}{\omega_{j,i}}\right|^n-1)+1+n}), & |\omega_{j,i}|\geq\lambda\\ \dfrac{2\mathrm{sign}(\omega_{j,i})|\omega_{j,i}|^{n+1}}{[\exp(\left|\dfrac{\omega_{j,i}}{\lambda}\right|^n-1)+1+n]\lambda^n}, & |\omega_{j,i}|<\lambda\end{cases} \tag{7}$$

Where λ is the threshold value. $\omega_{j,i}$ is wavelet coefficient before processing. $\hat{\omega}_{j,i}$ is new wavelet coefficient, n is the adjustable factor.

The smoothness of threshold function is poorer in literatures [7-8]. The proposed literature [9] methods can not only overcome the deficiencies in the hard threshold and soft threshold method, also has a certain smoothness, but its more complex.

We construct a threshold function by a smooth function.

$$\hat{\omega}_{j,i}=\begin{cases}\gamma\omega_{j,i}-\dfrac{1}{2}\gamma\lambda\mathrm{sign}\left(\omega_{j,i}\right), & |\omega_{j,i}|\geq\lambda\\ \dfrac{1}{2}\gamma\lambda\mathrm{sign}\left(\omega_{j,i}\right)g(\omega_{j,i})\exp(\lambda^{-2}), & |\omega_{j,i}|<\lambda\end{cases} \tag{8}$$

Where γ is adjustable factor. The expression of smooth function is

$$g\left(x\right)=\begin{cases}\exp(-x^{-2}), & x\neq 0\\ 0, & x=0\end{cases} \tag{9}$$

For the threshold value, this paper will choose the traditional threshold. The expression is

$$\lambda=\sigma\sqrt{2\ln N} \tag{10}$$

Where σ is the noise standard deviation. N is length of signal. The image of improved threshold function is Figure3.

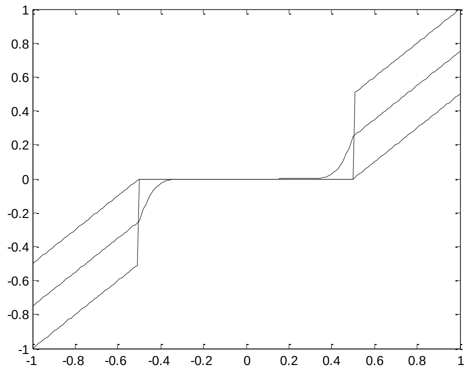
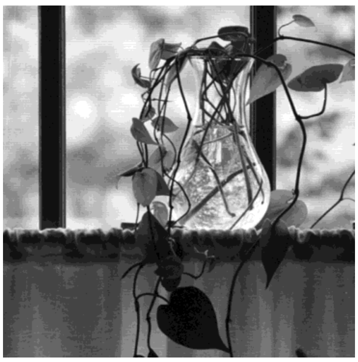


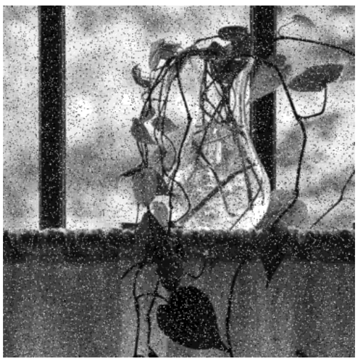
Figure 3. The Improved Threshold Function in This Paper

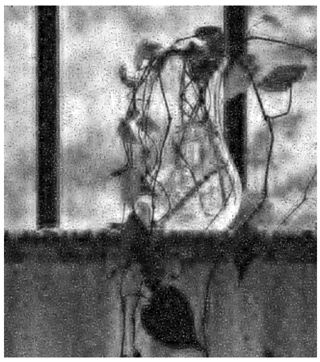
This paper constructs the new threshold function, which is sufficiently smooth at the origin, more continuous, almost smooth everywhere and more simple. The advantage of the improved threshold function is adjustable. By adjusting the size of the experimental parameters simply, it is possible to get better de-noising effect. Meanwhile, the new threshold function can also adapt to different needs about de-noising.

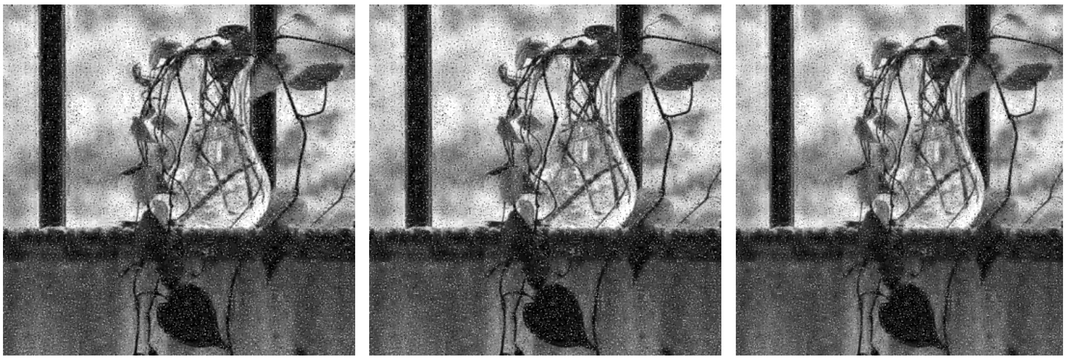
2.4. The Simulation Experiments

Salt and pepper' noise with density of 0.1 is separately added into the images of Flower, Man and House, and three layers of wavelet decomposition are adopted on the original image with the sym4 wavelet, to detect the performance of the algorithm. Then we use the soft threshold method, the hard threshold method, the algorithms in literatures [7-9] and in this paper the algorithm to remove the noise of the noise images. Look from the vision, the Figure 4 to 6, the method can effectively remove noise and reduce the image fuzzy in this paper. The de-noising method of this paper is better than the hard threshold, soft threshold, literatures [7- 9].

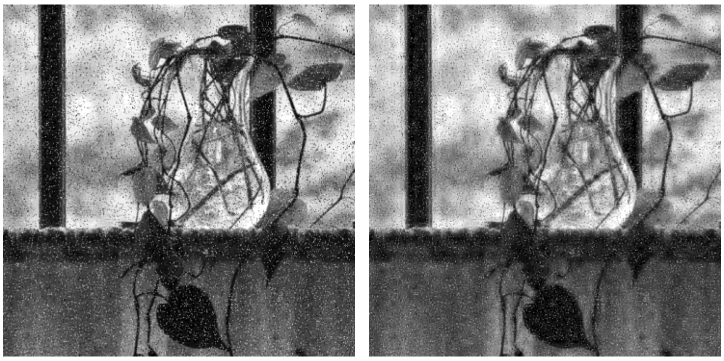

(a) The original image


(b) The image with noise


(c) The hard threshold



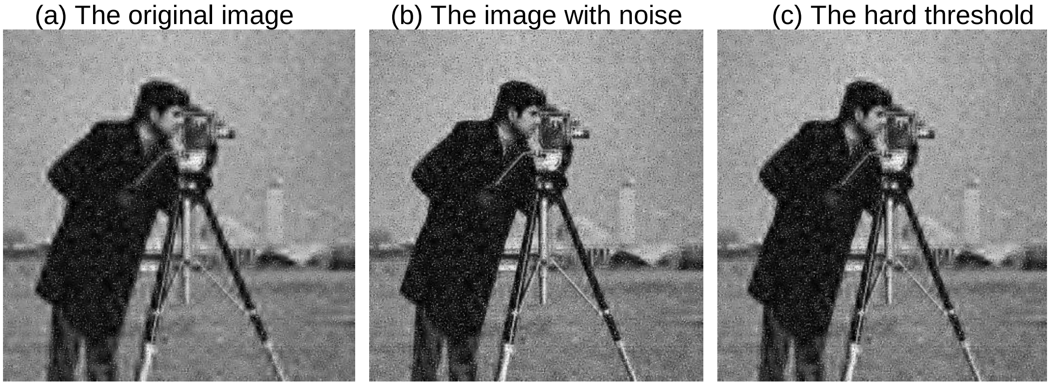
(d) The soft threshold (e) The literature [7] (f) The literature [8]



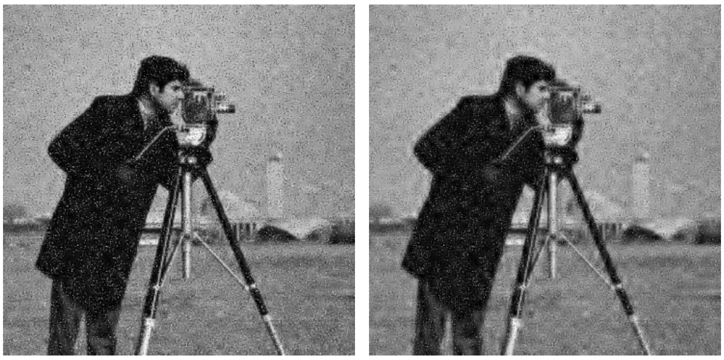
(g) The literature [9] (h) This Paper

Figure 4. The Results of Flower after De-noising by Six Methods

(a) The original image (b) The image with noise (c) The hard threshold

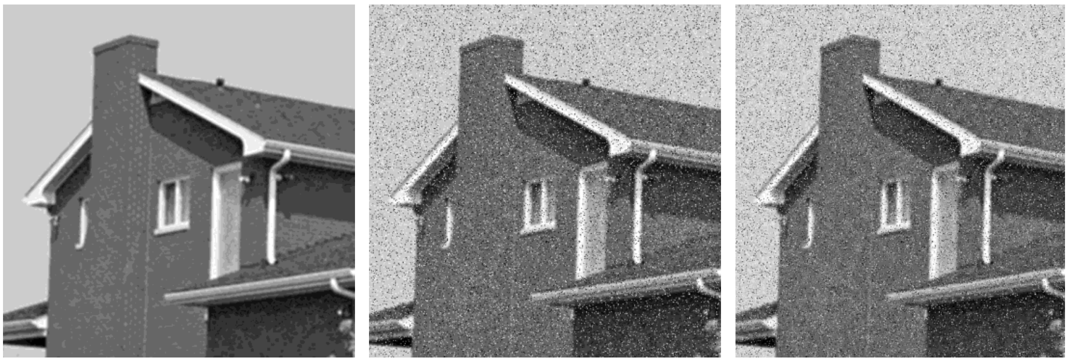


(d) The soft threshold (e) The literature [7] (f) The literature [8]

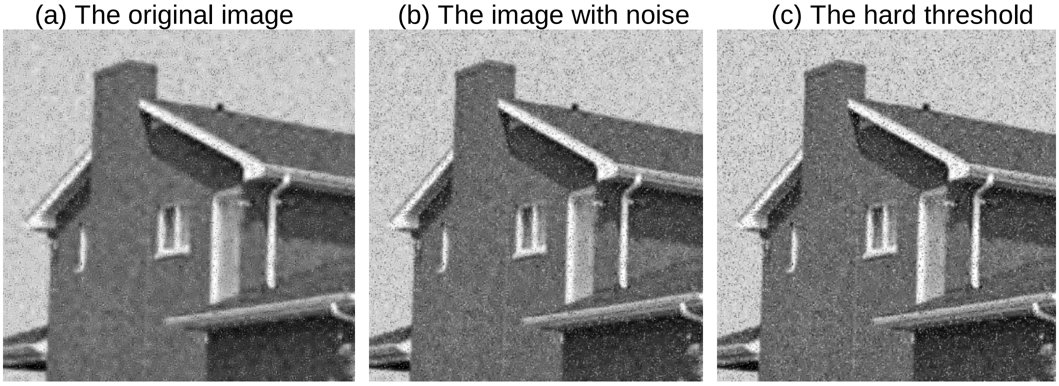


(g) The literature [9] (h) This Paper

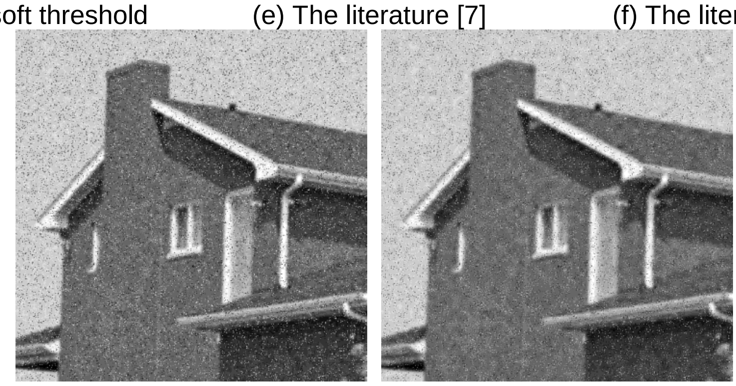
Figure 5. The Results of Man after De-noising by Six Methods



(a) The original image (b) The image with noise (c) The hard threshold



(d) The soft threshold (e) The literature [7] (f) The literature [8]



(g) The literature [9] (h) This Paper

Figure 6. The Results of House after De-noising by Six Methods

MSE (Mean Square Error), PSNR (Peak Signal to Noise Ratio) and SSIM (Structural Similarity), from image edge detection, is adopted as evaluation standards to evaluate the effect of improved methods more objectively and effectively in this paper.

Table 1. The Comparison of Objective Evaluation

Flower	MSE	PSNR	SSIM
The hard threshold	2038.834	15.0370	0.8102
The soft threshold	1841.832	15.4783	0.8214
The Literature [7]	2022.791	15.0713	0.8118
The Literature [8]	1930.266	15.2746	0.8169
The Literature [9]	1890.344	15.3654	0.8167
This Paper	1432.469	16.5699	0.8560

Table 2. The Comparison of Objective Evaluation

Man	MSE	PSNR	SSIM
The hard threshold	1930.899	15.2699	0.7764
The soft threshold	1383.887	16.7218	0.8259
The Literature [7]	1837.861	15.4770	0.7848
The Literature [8]	1658.927	15.8910	0.8000
The Literature[9]	1559.126	16.2310	0.8141
This Paper	1192.521	17.3776	0.8475

Table 3. The Comparison of Objective Evaluation

House	MSE	PSNR	SSIM
The hard threshold	1829.529	15.5074	0.7683
The soft threshold	1417.089	16.6168	0.8088
The Literature [7]	1629.529	16.0102	0.7875
The Literature [8]	1761.991	15.6708	0.7749
The Literature [9]	1608.833	16.0657	0.7902
This Paper	1144.963	17.5429	0.8401

3. Edge Detection

3.1. Classical Edge Detection Based on Mathematical Morphology

Mathematical morphology filtering is a nonlinear filtering emerging based on set theory, including four basic operations: dilation, corrosion, opening and closing

[10]. $G(x,y)$ is the image edge. $f(x,y)$ is the input image. $e(x,y)$ is a structural element. D_f and D_e are respectively the domain of function $f(x,y)$ and $e(x,y)$.

For the grayscale morphology, dilation, erosion, opening and closing can be defined as follows.

The dilation is

$$(f \oplus e)(x,y) = \max\left\{ f(x-x_1,\ y-y_1) + e(x_1,y_1) \mid x-x_1, y-y_1 \in D_f, x_1, y_1 \in D_e \right\}$$

The erosion is

$$(f \Theta e)(x,y) = \min\left\{ f(x+x_1, y+y_1) - e(x_1,y_1) \mid x+x_1, y+y_1 \in D_f, x_1, y_1 \in D_e \right\}$$

The opening is

$$f \circ e = (f \Theta e) \oplus e$$

The closing is

$$f \bullet e = (f \oplus e) \Theta e$$

According to the basic operation of morphology, three classical edge detection operators are listed here.

The dilation operator is

$$G_d^1 = f(x,y) \oplus e(x,y) - f(x,y) \tag{11}$$

The erosion operator is

$$G_e^1 = f(x,y) - f(x,y) \Theta e(x,y) \tag{12}$$

The dilation erosion operator is

$$G_{d\varepsilon}^1 = f(x,y) \oplus e(x,y) - f(x,y) \Theta e(x,y) \tag{13}$$

Obviously, the dilation operator of edge detection (G_d^1) can obtain the outside edge of the image. The erosion operator (G_ε^1) can get the inside edge of the image. The dilation erosion operator ($G_{d\varepsilon}^1$) is equivalent to the sum of dilation operator and erosion operator. The edge detector positions accuracy, the disadvantage is that the edge of the positioning width is twice the results of G_d^1 and G_ε^1. At the edge of a lower demand for the position of the cases, the operator is a good choice.

3.2. Improved Operator of Morphology in Edge Detection

Advantage of morphology operation is not only flexible and varied morphological transformation mode, the selection of structural elements also affects the effect of morphological edge detection [11].

Take the structural elements

$$B_1 = [0,1,0;1,1,1;0,1,0]\ ; \quad B_2 = [1,1,1;1,1,1;1,1,1] \tag{14}$$

The main steps of the edge detection are shown:

Step1. Use the structural elements (B_1 and B_2) for edge detection and get the image edge.

$$y_d = (((f \circ B_1) \bullet B_2) \oplus B_2) \circ B_2 - (f \circ B_1) \bullet B_2 \tag{15}$$

$$y_e = (f \circ B_1) \bullet B_2 - (((f \circ B_1) \bullet B_2) \Theta B_2) \bullet B_2 \tag{16}$$

Step2. Making the image edges is obtained in Step 1 do minimum operation to get details edge.

$$E_{\min} = \min\{y_d, y_e\} \tag{17}$$

Step3. Making the image edges is obtained in Step 1 do summation operation.

$$y_{de} = y_d + y_e \tag{18}$$

Step4. Making the image edges is obtained in Step 2 and Step 3 does summation operation to get the image edge finally.

$$y = y_{de} + E_{\min} \tag{19}$$

It can enhance the edge information, to better detect the image edge.

This paper uses the above threshold de-noising method for reducing noise of image, then use the improved morphological edge detection method for image edge detection.

The detailed flow chart is as follows.

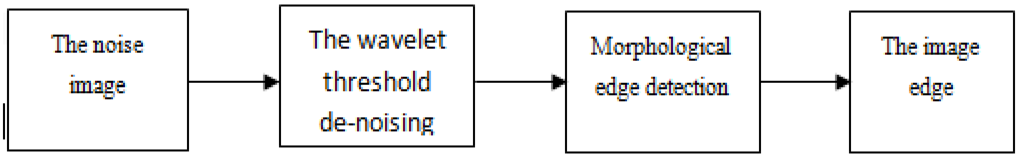
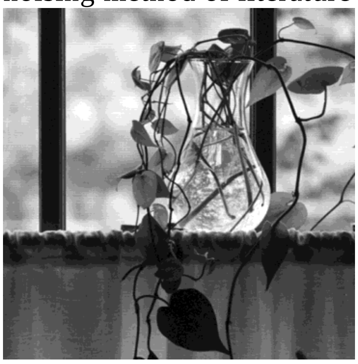


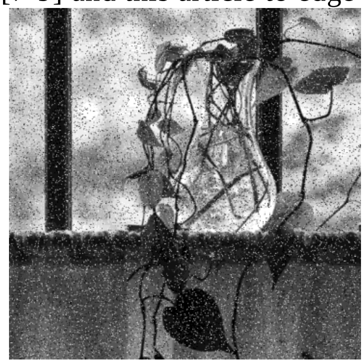
Figure 7. Edge Detection Flow Diagram

3.3. The Simulation Experiments

In order to verify the effectiveness of the de-noising method for edge detection in this paper, we select the processed image of Flower, Man and House images by de-noising method of literature [7-9] and this article to edge detection.

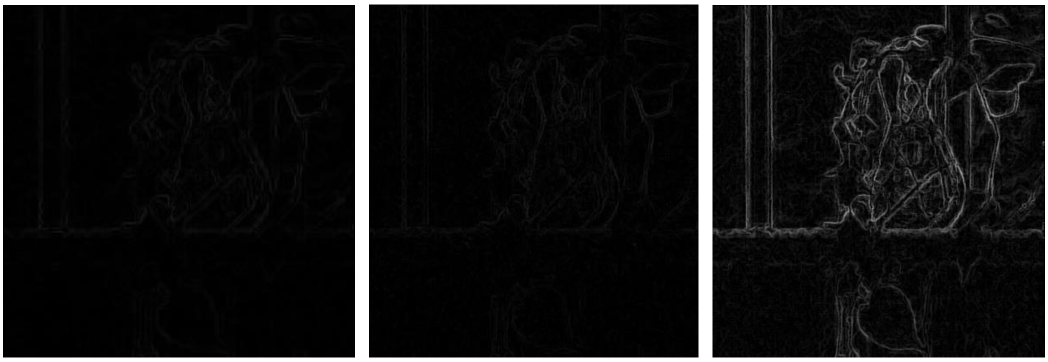


(a) The original image



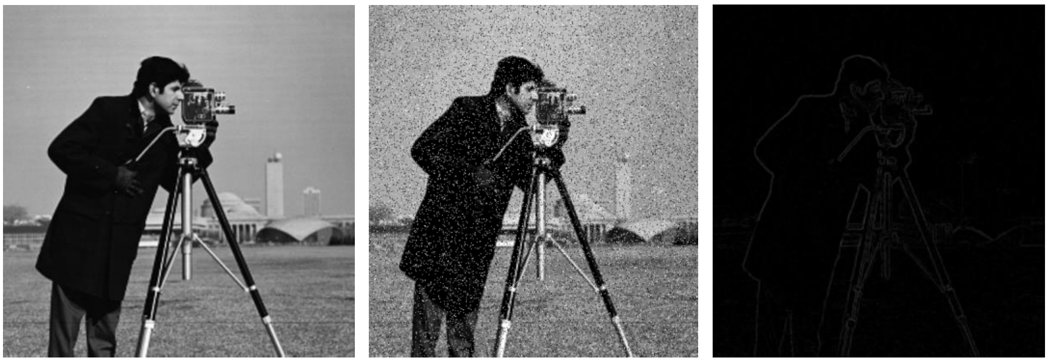
(b) The noise image

(c) The Literature [7]

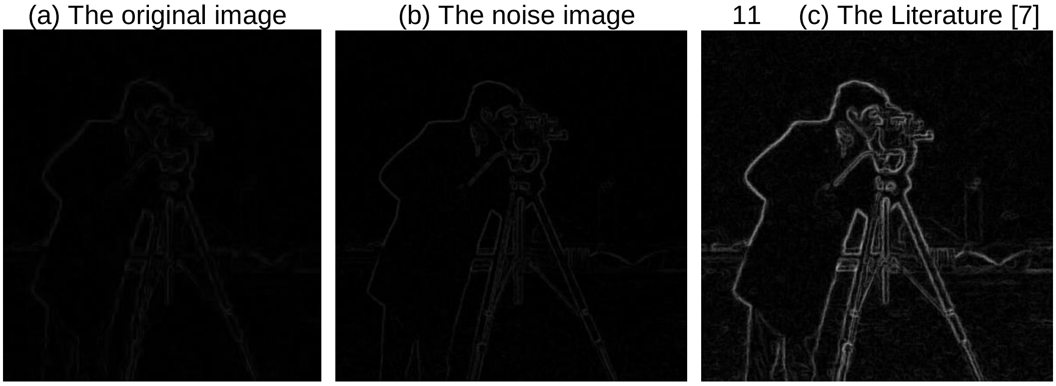


(d) The Literature [8] (e) The Literature [9] (f) This Paper

Figure 8. The Results of Flower after Edge Detection

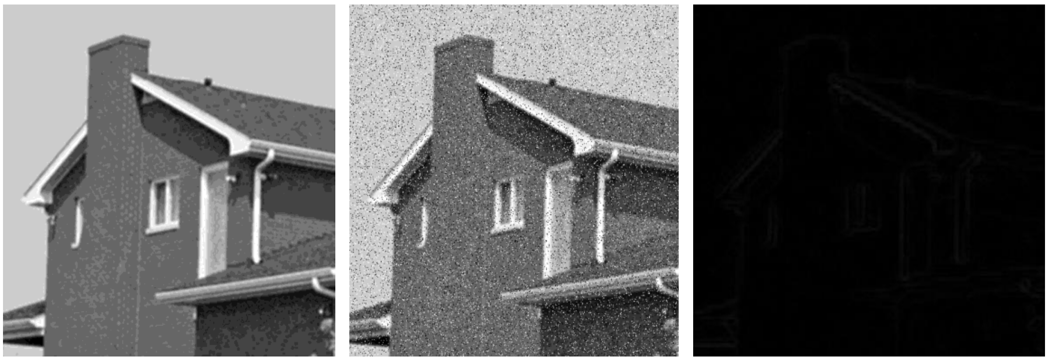


(a) The original image (b) The noise image 11 (c) The Literature [7]

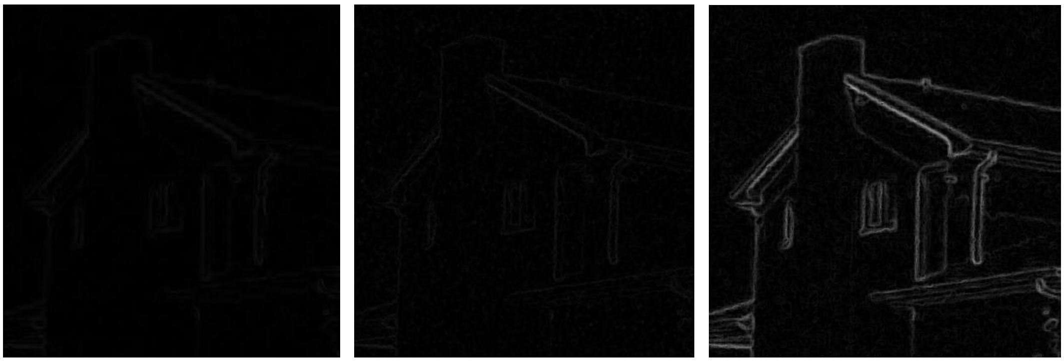


(d) The Literature [8] (e) The Literature [9] (f) This Paper

Figure 9. The Results of Man after Edge Detection



(a) The original image (b) The noise image (c) The Literature [7]



(d) The Literature [8]   (e) The Literature [9]   (f) This Paper

Figure 10. The Results of House after Edge Detection

Table 4.The Comparison of MSE

MSE	Flower	Man	House
The Literature [7]	18150	17371	21438
The Literature [8]	18099	17367	21420
The Literature [9]	17964	17323	21310
This Paper	17803	17267	20856

Table 5. The Comparison of PSNR

PSNR	Flower	Man	House
The Literature [7]	5.5422	5.7326	4.8191
The Literature [8]	5.5543	5.7336	4.8225
The Literature [9]	5.5866	5.7444	4.8458
This Paper	5.6012	5.8388	4.9023

The images above and the data showed that the method in this paper is better. Threshold functions constructed by Literatures [7-8] are discontinuous and blurred image severely in de-noising process, so the edges can not be detected properly. Threshold function constructed by Literature [9] is continuous, to a certain extent, reduce the fuzzy of image, but it is more complex. This paper constructs the new threshold function, which is sufficiently smooth at the origin, more continuous almost smooth everywhere and more simple. As a result, greatly reduces the fuzzy of image. So it is important to detect the intact image edge.

4. Conclusions

Digital image is interfered with noise in the process of gathering and transmission. When the image edge is detected, the noise is detected in edges inevitably. In order to be able to detect the true and exact image edge, must do image de-noising. In this paper, combined with the advantages and disadvantages of hard and soft threshold function obtain the new threshold function. This paper uses the improved wavelet threshold method for image de-noising, and then uses the method of mathematical morphology for image edge extraction. The experimental results show the effectiveness of this method to the edge detection.

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