

Unambiguous Bearing Estimation of Coherent Signals Using Acoustic Vector Sensor

Yan Jing^{1,2}, Yi Shen¹, Naizhang Feng² and Mingjian Sun^{2*}

¹. School of Astronautics, Harbin Institute of Technology; Harbin, 150001, P. R. China

². School of Information and Electrical Engineering, Harbin Institute of Technology at Weihai; Weihai 264209, P. R. China

Abstract

This paper makes full use of the characteristics of acoustic vector sensor (AVS), which can simultaneously measure both acoustic pressure scalar and particle velocity vector at single point in acoustic field. A new method is introduced by extending modified multiple signal classification (MMUSIC) to AVS, named as V-MMUSIC, which can be used to bearing estimation for coherent signals. Simulation results show that this method can achieve bearing estimation of coherent or unrelated signals and remove the port/starboard indistinct phenomenon. The proposed method can accomplish full spatial, unambiguous and small error bearing estimation in the low SNR.

Keywords: *acoustic vector sensor; modified multiple signal classification algorithm; bearing estimation; coherent signals*

1. Introduction

Acoustic signal processing of sensor array is an important part, which has been utilized in many applications, such as sonar, electromagnetic, navigation and positioning of underwater device, distributed sensor networks, and so on. Bearing estimation is one of the primary problems in acoustic signal processing by using scalar acoustic pressure sensors (APSs) or acoustic vector sensors (AVSs). The output of each APS is a scalar relating to the acoustic pressure. Each AVS is composed of velocity sensors and pressure sensor, which can simultaneously measure both the acoustic pressure and two or three orthogonal components of the acoustic particle velocity. Hence, AVSs completely characterize the acoustic field at a point in space, which should outperform the scalar APSs in accuracy of bearing estimation [1].

Many traditional bearing estimation algorithm, such as minimum variance distortion-less response (MVDR), Capon, multiple signal classification (MUSIC), root-MUSIC and estimation signal parameter via rotational invariance techniques (ESPRIT) have been extensively investigated for AVS arrays [2-8]. However, these algorithms fail to estimate the bearing of coherent or highly correlated signals. Based on AVS, abundant research have been developed to solve this issue, such as polarization smoothing algorithm (PSA) [9], signal subspace processing [10], particle velocity field smoothing (PVFS) [11-13], propagator method [14] and sparsely spatial smoothing [15]. The PVFS method eliminates relevance of coherent signals by putting the different elements covariance matrices in together. In [15], One-dimensional sparsely spatial smoothing is applied to estimate the two-dimensional bearing of the coherent sources based on a sparsely-distributed AVS array. The weighed Toeplitz matrix (WTM) and improved WTM methods have been implemented as a preprocessing stage of MUSIC to

*Corresponding Author

This research is supported by the National Natural Science Foundation of China (Grant No.61201307 and Grant No.61371045), the Fundamental Research Funds for the Central Universities (Grant No. HIT. NSRIF.2013132).

locate coherent signals without port/starboard indistinct phenomenon by using AVS array [16].

In this paper, a new method, named as V-MMUSIC, is proposed to estimate the bearing of coherent signals by using AVS array. Simulation results demonstrate that the proposed approach has good bearing estimation performance without the port/starboard ambiguous phenomenon by comparing to MMUSIC [17] based on APS array (P-MMUSIC). The root mean square error (RMSE) and bias of V-MMUSIC are similar to that of MUSIC based on AVS array (V-MUSIC) [8] for unrelated signals in the low SNR. V-MMUSIC can resolve the bearing estimation of the coherent signals but V-MUSIC cannot.

The organization of the paper is as follow. Section 2 illustrates the bearing estimation theory of P-MMUSIC method. Section 3 proposes the V-MMUSIC approach for bearing estimation. Simulation results are displayed in section 4, and conclusions are drawn in section 5.

2. P-MMUSIC Method

Suppose that K narrow-band far-field acoustic signals of wavelength λ with azimuth angle $\theta_1, \theta_2, \dots, \theta_K$ ($\theta_k \in (0, \pi]$) are impinging on an uniform linear APS array of D ($D > K$) sensors whose distance d of adjacent two sensors is half wavelength. The array output $\mathbf{X}(t)$ can be written as

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t) \quad (1)$$

where $\mathbf{X}(t) = [x_1(t), x_2(t), \dots, x_D(t)]^T$ is the $D \times 1$ received signals matrix, $\mathbf{S}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$ represents the $K \times 1$ acoustic signal matrix, $\mathbf{N}(t) = [n_1(t), n_2(t), \dots, n_D(t)]^T$ is the $D \times 1$ additive white Gaussian noise signal with zero means and variance $\sigma^2 \mathbf{I}$. $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_K)]$ is the $D \times K$ steering matrix. The k -th column of \mathbf{A} is denoted by

$$\mathbf{a}(\theta_k) = [1, \exp(-j\tau), \dots, \exp(-j(D-1)\tau)]^T \quad (2)$$

$$\tau = 2\pi d \sin(\theta_k) / \lambda \quad (3)$$

The covariance matrix of $\mathbf{X}(t)$ can be described as $\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)] \in D \times D$, where superscript H represents conjugate transpose.

In order to estimate bearing of coherent signals, \mathbf{R} can be reconstructed as

$$\mathbf{R}' = \mathbf{R} + \mathbf{J}\mathbf{R}^* \mathbf{J} \quad (4)$$

where \mathbf{J} is the $D \times D$ exchange matrix whose entries are all zero except the one on the vice diagonal, superscript $*$ represents conjugate.

By carrying out eigenvalue decomposition, \mathbf{R}' can be written as

$$\mathbf{R}' = \mathbf{U}_s \mathbf{A}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{A}_n \mathbf{U}_n^H \quad (5)$$

where \mathbf{U}_s is the signal subspace spanned by the eigenvector \mathbf{A}_s corresponding to the K larger eigenvalue. \mathbf{U}_n is the noise subspace spanned by the eigenvector \mathbf{A}_n corresponding to the $D - K$ smaller eigenvalue.

According to the column vector of steering matrix \mathbf{A} and noise subspace \mathbf{U}_n are orthogonal in the ideal condition, the P-MMUSIC spatial spectrum is given by

$$P_{\text{P-MMUSIC}}(\theta) = [\mathbf{a}^H(\theta) \mathbf{U}_n \mathbf{U}_n^H \mathbf{a}(\theta)]^{-1} \quad (6)$$

The bearing estimation of the coherent or uncorrelated signals is obtained by searching for the maximum $P_{\text{P-MMUSIC}}(\theta)$ in potential angle space.

3. V-MMUSIC Method

Consider that K narrow-band far-field acoustic signals with center frequency f_0 and bearing angle θ_k , $k=1, \dots, K$ ($\theta_k \in (0, 2\pi]$) are impinging on a uniform linear AVS array of D ($D > K$) sensors. The distance d of adjacent two sensors is half wavelength. Here, an AVS is composed of one pressure sensor and two collocated orthogonal velocity sensors. Thus, each AVS outputs acoustic pressure $p(t)$ and two orthogonal components of particle velocity ($\mathbf{v}_x, \mathbf{v}_y$). It can be shown that

$$\mathbf{v}(t) = \frac{p(t)}{\rho_0 c} \mathbf{h}(\theta_k) \quad (7)$$

where ρ_0 is the ambient density and c is the sound velocity in the medium. The constant $\rho_0 c$ is defined as acoustic impedance, which do not affect the relationship between particle velocity and acoustic pressure. In order to simplify representation, let $\rho_0 c = 1$. Bearing vector of two-dimensional AVS is denoted as $\mathbf{h}(\theta_k) = [1, \cos \theta_k, \sin \theta_k]^T$.

The output of AVS array, which is associated with one vector and one signal, can be denoted by

$$\begin{bmatrix} y_p(t) \\ \mathbf{y}_{vx}(t) \\ \mathbf{y}_{vy}(t) \end{bmatrix} = \mathbf{a}(\theta) \otimes \mathbf{h}(\theta) \cdot p(t) + \begin{bmatrix} e_p(t) \\ \mathbf{e}_{vx}(t) \\ \mathbf{e}_{vy}(t) \end{bmatrix} \quad (8)$$

where $y_p(t)$, $\mathbf{y}_{vx}(t)$ and $\mathbf{y}_{vy}(t)$ are the array output vector of acoustic pressure and particle velocity of x and y direction, respectively. $e_p(t)$, $\mathbf{e}_{vx}(t)$ and $\mathbf{e}_{vy}(t)$ are corresponding components to noise. $\mathbf{a}(\theta)$ is steering vector and $p(t)$ is measured acoustic pressure.

Then, the output of AVS array, which is connected with D vector sensors and K signals, can be written as

$$\mathbf{y}(t) = \mathbf{B}(\theta) \mathbf{p}(t) + \mathbf{e}(t) \quad (9)$$

where $\mathbf{y}(t) \square [y_p^{(1)}(t), \mathbf{y}_{vx}^{(1)}(t), \mathbf{y}_{vy}^{(1)}(t), \dots, y_p^{(D)}(t), \mathbf{y}_{vx}^{(D)}(t), \mathbf{y}_{vy}^{(D)}(t)]^T \in 3D \times 1$ is the observed sensor outputs vector, $\mathbf{B}(\theta) = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \dots, \mathbf{b}(\theta_K)] \in 3D \times K$ is steering matrix, $\mathbf{b}(\theta_k) = \mathbf{a}(\theta_k) \otimes \mathbf{h}(\theta_k) \in 3D \times 1$, \otimes denotes Kronecker product, $\mathbf{a}(\theta_k)$ is equal to equation (2), $\mathbf{p}(t)$ includes the acoustic pressure of K signals, $\mathbf{e}(t) \square [e_p^{(1)}(t), \mathbf{e}_{vx}^{(1)}(t), \mathbf{e}_{vy}^{(1)}(t), \dots, e_p^{(D)}(t), \mathbf{e}_{vx}^{(D)}(t), \mathbf{e}_{vy}^{(D)}(t)]^T \in 3D \times 1$ is the additive white Gaussian noise.

The covariance matrix of $\mathbf{y}(t)$ can be described as $\mathbf{R}_{3D} = E[\mathbf{y}(t) \mathbf{y}^H(t)] \in 3D \times 3D$. In practical, for the received data is finite, \mathbf{R}_{3D} can be replaced by sampling covariance matrix $\hat{\mathbf{R}}_{3D}$, that is

$$\hat{\mathbf{R}}_{3D} = \frac{1}{L} \sum_{t=1}^L \mathbf{y}(t) \mathbf{y}^H(t) \quad (10)$$

where L is the number of snapshots.

$\hat{\mathbf{R}}_{3D}$ can be reconstructed as

$$\hat{\mathbf{R}}'_{3D} = \hat{\mathbf{R}}_{3D} + \mathbf{J}_{3D} \hat{\mathbf{R}}_{3D}^* \mathbf{J}_{3D} \quad (11)$$

where $\hat{\mathbf{R}}_{3D}^*$ is the conjugate of $\hat{\mathbf{R}}_{3D}$, \mathbf{J}_{3D} is the $3D \times 3D$ exchange matrix.

$3D$ eigenvalues are obtained by making eigenvalue decomposition of $\hat{\mathbf{R}}'_{3D}$, which are arranged in order

$$\gamma_1 \geq \gamma_2 \geq \dots \geq \gamma_K \geq \gamma_{K+1} \geq \dots \geq \gamma_{3D} \geq 0 \quad (12)$$

Let \mathbf{V}_s denotes the signal subspace composed of the eigenvectors $\mathbf{\Gamma}_s$ relating to the K larger eigenvalues, and let \mathbf{V}_n symbolizes the noise subspace composed of the eigenvectors $\mathbf{\Gamma}_n$ relating to the remaining $3D - K$ eigenvalues:

$$\hat{\mathbf{R}}'_{3D} = \mathbf{V}_s \mathbf{\Gamma}_s \mathbf{V}_s^H + \mathbf{V}_n \mathbf{\Gamma}_n \mathbf{V}_n^H \quad (13)$$

The proposed method, *i.e.* V-MMUSIC spatial spectrum is given by

$$P_{\text{V-MMUSIC}}(\theta) = [\mathbf{b}^H(\theta) \mathbf{V}_n \mathbf{V}_n^H \mathbf{b}(\theta)]^{-1} \quad (14)$$

The bearing estimation of the coherent or uncorrelated signals is acquired by searching for potential angle of the maximum response in the $P_{\text{V-MMUSIC}}(\theta)$.

4. Simulation Results

4.1. Acoustic Signals

In the simulations, acoustic signals are two simulated ship-radiated noise, which are nonstationary random process. Ship-radiated noise spectrum is comprised of a continuous wideband spectrum and a narrowband line spectrum.

The mathematical model of ship-radiated noise can be described as

$$s(t) = [1 + a(t)]s_c(t) + s_l(t) \quad (15)$$

where $a(t)$ is the periodic modulation waveform, $s_c(t)$ is the time-domain waveform of continuous spectrum, $s_l(t)$ is the time-domain waveform of line spectrum.

The amplitude spectrum of $s(t)$ is given by

$$P_s = 20 \log_{10} |a(t)| \quad (16)$$

According to modeling and simulation of ship-radiated noise in [18], the amplitude spectrum of acoustic signal s1 and s2 are obtained and presented in Figure 1.

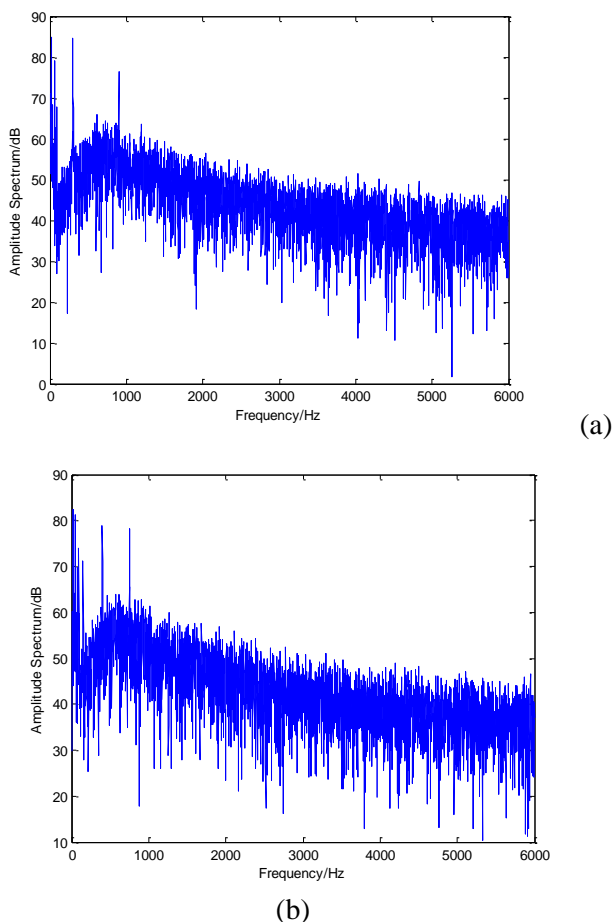


Figure 1. Amplitude Spectrum of Ship-Radiated Noise.

(a) acoustic signal s1. (b) acoustic signal s2.

The propeller speed of signal s1 and s2 are 180r/min and 360r/min, respectively. The propeller leaf number of signal s1 and s2 are five and four, respectively. Source s1 and s2 are regarded as the uncorrelated signal whose correlation coefficient is 0.0085. Ocean ambient noise is assumed to be independent from each other.

4.2. Simulation of Bearing Estimation

The elements number of APSs array and AVSs array are all 8, and 100 the number of snapshots is 100.

Firstly, the performance of V-MMUSIC is demonstrated by comparing with P-MMUSIC and V-MUSIC in resolving the bearing estimation of unrelated signals.

We consider acoustic signal s1 and s2 at the azimuth $[-20^\circ, 50^\circ]$ with -10dB signal noise ratio (SNR). The angular spectrum of P-MMUSIC, V-MMUSIC and V-MUSIC method are shown in the Figure 2. In this figure, it is observed that: (1) V-MMUSIC can efficiently estimate the bearing of two unrelated signals corresponding to two peaks of angular spectrum; (2) V-MMUSIC and V-MUSIC have similar estimation ability, maintain sharp spectrum peak without the port/starboard indistinct phenomenon in low SNR; (3) P-MMUSIC produces two wrong angles except for two right angles, whose sidelobe is far greater than V-MMUSIC's. Thus, P-MMUSIC cannot provide bearing estimation in total space, which has port/starboard indistinct occurrence.

Secondly, the effectiveness of V-MMUSIC is verified by comparing with P-MMUSIC and V-MUSIC in resolving the bearing estimation of coherent signals.

We consider acoustic signal s_1 , s_1 , and s_2 at the azimuth $[-20^\circ, 10^\circ, 40^\circ]$ with -10dB signal noise ratio (SNR). Two signals arriving at -20° and 10° are coherent. The angular spectrum of three methods are presented in the Figure 3. Figure 3 shows that: (1) V-MMUSIC has ability to estimate bearing of coherent sources in low SNR without the port/starboard ambiguous phenomenon; (2) V-MUSIC fails to estimate the azimuth of coherent signals; (3) P-MMUSIC cannot accomplish a good bearing estimation, which produces three wrong angles except for three right angles.

Thirdly, the estimated bias and RMSE of V-MMUSIC and V-MUSIC for two unrelated signals are investigated in different SNR.

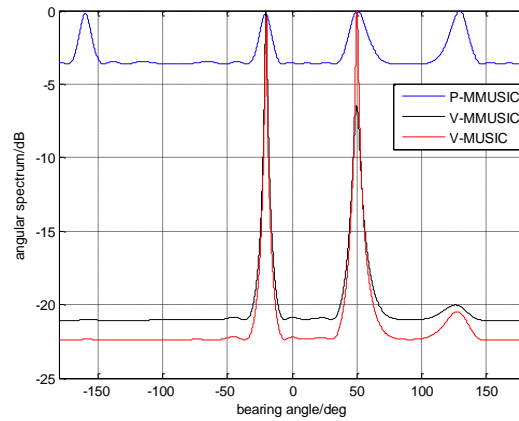


Figure 2. Bearing Estimation of Unrelated Signals by Using P-MMUSIC, V-MMUSIC and V-MUSIC Method

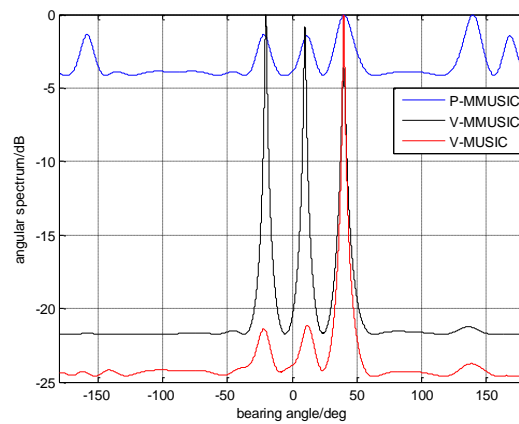


Figure 3. Bearing Estimation of Coherent Signals by Using P-MMUSIC, V-MMUSIC and V-MUSIC Method

The bias is defined as

$$Bias = \frac{1}{NK} \sum_{i=1}^K \sum_{n=1}^N (\hat{\theta}_{ni} - \theta_i) \quad (17)$$

The RMSE is defined as

$$RMSE = \sqrt{\frac{1}{NK} \sum_{i=1}^K \sum_{n=1}^N (\hat{\theta}_{ni} - \theta_i)^2} \quad (18)$$

where K is the number of acoustic signals, N is number of Monte Carlo simulations, $\hat{\theta}_{ni}$ is estimated bearing angle and θ_i is reality bearing angle.

Here, $K=2$, $N=100$, $\theta = [-20^\circ, 50^\circ]$, bias vs. SNR and RMSE vs. SNR of two methods are displayed in Figure 4 and Figure 5, respectively.

It is concluded from Figure 4 and Figure 5 that the bias and RMSE of bearing estimation of V-MMUSIC and V-MUSIC are approaching equality when the SNR is less than -15dB. The difference between two methods for bias and RMSE is not exceeding 0.1° when SNR increases from -15dB to 10dB. Therefore, the proposed V-MMUSIC method has small bias and RMSE when SNR is greater than -15dB.

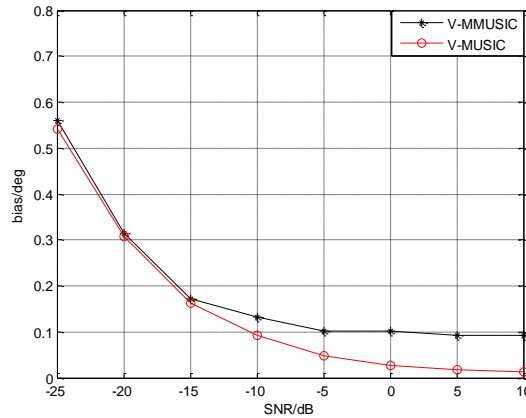


Figure 4. Estimation Bias of Two Methods in Different SNR

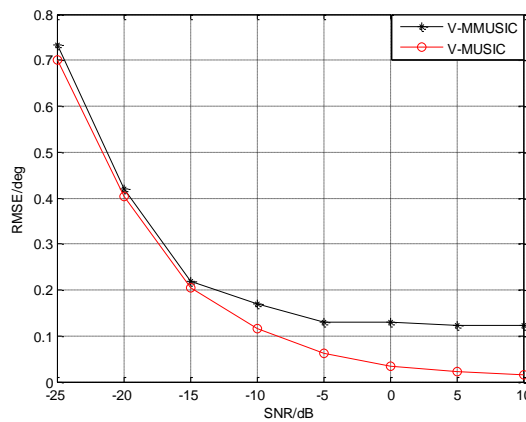


Figure 5. Estimation RMSE of Two Methods in Different SNR

5. Conclusions

In this paper, a modified MUSIC method based on AVS, named as V-MMUSIC, is proposed and applied to bearing estimation. According to the corresponding simulations, the following conclusions are drawn that: (1) the V-MMUSIC method can achieve bearing estimation of coherent or unrelated signals; (2) V-MMUSIC has not the port/starboard ambiguous phenomenon, and achieves bearing estimation in whole space; (3) V-MMUSIC has small the estimation bias and RMSE for unrelated signals when SNR is greater than -15dB.

References

- [1] A. Nehorai and E. Paldi, "Acoustic vector-sensor array processing", *IEEE Transactions on Signal Processing*, vol. 42, (1994), pp. 2481-2491.
- [2] M. Hawkes and A. Nehorai, "Acoustic vector-sensor beamforming and Capon direction estimation", *IEEE Transactions on Signal Processing*, vol. 46, no. 9, (1998), pp. 2291-2304.
- [3] M. D. Zoltowski and K. T. Wong, "Closed-Form eigenstructure based direction finding using arbitrary but identical subarrays on a sparse uniform Cartesian array grid", *IEEE Transactions on Signal Processing*, vol. 48, no. 8, (2000), pp. 2205-2210.
- [4] K. T. Wong and M. D. Zoltowski, "Self-initiating MUSIC-based direction finding in underwater acoustic particle velocity-field beamspace", *IEEE Journal of Oceanic Engineering*, vol. 25, no. 2, (2000), pp. 262-273.
- [5] K. T. Wong and M. D. Zoltowski, "Root-MUSIC based azimuth-elevation angle of arrival estimation with uniformly spaced but arbitrarily oriented velocity hydrophones", *IEEE Transactions on Signal Processing*, vol. 47, no. 12, (1999), pp. 3250-3260.
- [6] K. T. Wong, "Beam patterns of an underwater acoustic vector hydrophone located away from any reflecting boundary", *IEEE Journal of Oceanic Engineering*, vol. 27, no. 3, (2002), pp. 628-637.
- [7] K. T. Wong and M. D. Zoltowski, "Closed-form underwater acoustic direction-finding with arbitrarily spaced vector hydrophones at unknown locations", *IEEE Journal of Oceanic Engineering*, vol. 2, no. 3, (1997), pp. 566-575.
- [8] Z. Yao, J. Hu and D. Yao, "A bearing estimation algorithm using an acoustic vector sensor array based on MUSIC. *Acta Acustica*", (in Chinese), vol. 33, no. 4, (2008), pp. 305-309.
- [9] D. Rahamim, J. Tabrikian and R. Shavit, "Source localization using vector sensor array in a multipath environment", *IEEE Transactions on Signal Processing*, vol. 52, no. 11, (2004), pp. 3096-3103.
- [10] H. Chen and J. Zhao, "Coherent signal-subspace processing of acoustic vector sensor array for DOA estimation of wideband sources", *Signal Processing*, vol. 85, no. 4, (2005), pp. 837-847.
- [11] J. Tao, W. Chang and W. Cui, "Vector field smoothing for DOA estimation of coherent underwater acoustic signals in presence of a reflecting boundary", *IEEE Sensors Journal*, vol. 7, no. 8, (2007), pp. 1152-1158.
- [12] J. Tao, W. Chang and Y. Shi, "Direction-finding of coherent sources via particle-velocity-field smoothing", *IET Radar Sonar and Navigation*, vol. 2, no. 2, (2008), pp. 127-134.
- [13] J. He, S. Jiang, J. Wang and Z. Liu, "Particle-velocity-field difference smoothing for coherent source localization in spatially nonuniform Noise", *IEEE Journal of Oceanic Engineering*, vol. 35, no. 1, (2010), pp. 113-119.
- [14] J. He and Z. Liu, "Two-dimensional direction finding of acoustic sources by a vector sensor array using the propagator method", *Signal Processing*, vol. 88, no. 10, (2008), pp. 2492-2499.
- [15] X. Yuan, "Coherent source direction finding using a sparsely distributed acoustic vector sensor array", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 3, (2012), pp. 2710-2715.
- [16] Y. Wu, G. Li, Z. Hu and Y. Hu, "Unambiguous directions of arrival estimation of coherent sources using acoustic vector sensor linear arrays", *IET Radar, Sonar and Navigation*, vol. 9, no. 3, (2015), pp. 318-323.
- [17] D. Kundu, "Modified MUSIC algorithm for estimating DOA of signals", *Signal Processing Magazine*, vol. 48, no. 1, (1996), pp. 85-90.
- [18] Z. He and Y. Zhang, "Modeling and simulation research of ship-radiated noise", *Audio Engineering*, (in Chinese), no. 12, (2005), pp. 52-55