

## Research on Two Kinds of Adaptive Synchronization Method for Uncertain Fourth Order Chaotic Systems based on Sigmoid Function and Soft Function Method

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### Abstract

*Synchronization of chaotic systems can be used in the secure communication so it greatly attracted interests of many researchers. The bounded characteristics of chaotic system was used and an uniformed adaptive law was designed to solve system uncertainties which make the whole design very simple. And two kinds of functions was used to take place of traditional sign function, then the chattering problem was greatly improved. A Lyapunov function was chosen to guarantee the stability of the whole design. At last, detailed simulation was done to show the rightness and effectiveness of the proposed method.*

**Keywords:** Chaotic systems; nonlinear; robust control; adaptive synchronization; uncertainty; desired trajectory

### 1. Introduction

Chaotic system has many advantages such as it has random characteristics and rich dynamic response [1-9], especially it can be used in secure communication with synchronization of two chaotic systems. So it aroused interests of researchers in many countries because of its military application [10-13].

But the system uncertainties will always exist in every real engineering systems, so how to solve uncertainties is the biggest problem that affect synchronization effect [14-17]. And many researchers designed sliding mode controller since it can provide a strong robustness for controllers. But the sliding mode strategy will has a disadvantage that it will cause chattering problems because of the use of sign function. So in this paper, we introduced two kinds of functions to take place of the sign function, one is the soft function and another is the sigmoid function. Simulation result showed that it can improve the synchronization effect and reduce chattering problem obviously.

Also many researchers designed many kinds of adaptive controllers [18-21] by using adaptive strategy and robust design method. But most of the design were based on different structure and different function of systems, so adaptive law was very complex in some situation that subsystems are different from each other. In this paper, a novel design was proposed by using the bounded characteristics of chaotic system that a uniformed formation of adaptive law design method was applied in every subsystem. So the whole

controller design of very simple although the control object is a fourth order system. So the main advantage of the proposed method is that it is very simple and easy to be applied in control engineering, also it can reduce chattering problem by introducing soft function and sigmoid function.

## 2. Problem Description

Considering the following driver system and response system, where parameters of response system is known and there exists unknown parameters and uncertain nonlinear function in driven system.

The driven system can be written as

$$\dot{x} = f_x(x) + F_x(x)\theta_x + \Delta(x,t) \quad (1)$$

The response system can be written as

$$\dot{y} = f_y(y) + bu \quad (2)$$

Take a four dimension system as a example, the main driven system can be written as

$$\dot{x}_1 = f_{x_1}(x_1, \dots, x_4) + \sum_{j=1}^{p_1} F_{x_1j}(x_1, \dots, x_4)\theta_{x_1j} + \sum_{j=1}^{q_1} \Delta_{x_1j}(x,t) \quad (3)$$

$$\dot{x}_2 = f_{x_2}(x_1, \dots, x_4) + \sum_{j=1}^{p_2} F_{x_2j}(x_1, \dots, x_4)\theta_{x_2j} + \sum_{j=1}^{q_2} \Delta_{x_2j}(x,t) \quad (4)$$

$$\dot{x}_3 = f_{x_3}(x_1, \dots, x_4) + \sum_{j=1}^{p_3} F_{x_3j}(x_1, \dots, x_4)\theta_{x_3j} + \sum_{j=1}^{q_3} \Delta_{x_3j}(x,t) \quad (5)$$

$$\dot{x}_4 = f_{x_4}(x_1, \dots, x_4) + \sum_{j=1}^{p_4} F_{x_4j}(x_1, \dots, x_4)\theta_{x_4j} + \sum_{j=1}^{q_4} \Delta_{x_4j}(x,t) \quad (6)$$

And the slave response system can be written as

$$\dot{y}_1 = f_{y_1}(y_1, \dots, y_4) + b_1u_1 \quad (7)$$

$$\dot{y}_2 = f_{y_2}(y_1, \dots, y_4) + b_2u_2 \quad (8)$$

$$\dot{y}_3 = f_{y_3}(y_1, \dots, y_4) + b_3u_3 \quad (9)$$

$$\dot{y}_4 = f_{y_4}(y_1, \dots, y_4) + b_4u_4 \quad (10)$$

When  $\theta_x$  is unknown parameter, and the number of unknown parameter is  $\sum_{i=1}^n p_i$ ,

and the number of uncertain nonlinear function is  $\sum_{i=1}^n q_i$ ,  $b_i$  is a known constant [8-10].

So the robust adaptive control target for chaotic system with unknown parameter and uncertain nonlinear function is to design the control  $u_i = u_i(x, y, \hat{r}_{ij})$ ,  $\hat{r}_{ij} = f_{ij}(z_1, z_2, z_3)$  such that the state of slave system can trace state of master system, such as  $y \rightarrow x$ .

Considering the above adaptive PID control is too complex, so the design of robust adaptive synchronization need benefit the boundedness of chaotic system. And use a bounded function to describe the uncertain region of the unknown information of driven system and response system, then design a robust adaptive controller to synchronize two systems [11-14].

### 3. Assumption

Two assumptions are built for the above system to simplify the analysis.

Assumption 1: the driven system and response system have the same structure, it means that it has the same dimension.

Assumption 2: the nonlinear function satisfies the below conditions, for  $1 \leq i \leq n$ ,  $1 \leq j \leq p_2$ , there exists a unknown positive constant  $r_{ij} \leq d_{ij}$  such that

$$\begin{aligned} & f_{y_i}(y_1, \dots, y_4) - f_{x_i}(x_1, \dots, x_4) - \sum_{j=1}^{p_1} F_{x_{ij}}(x_1, \dots, x_4) \theta_{x_{ij}} - \sum_{j=1}^{q_1} \Delta_{x_{ij}}(x, t) \\ & \leq r_{i1} |z_1| + r_{i2} |z_2| + r_{i3} |z_3| \end{aligned} \quad (11)$$

where  $d_{ij}$  is a known constant. Because the chaotic system is bounded, so it is easy to be satisfied for many chaotic systems [15-17].

### 4. Robust Adaptive Synchronization Law Design based on Soft Function Method

Define the error variable as  $z_i = y_i - x_i$ , where the error system can be written as

$$\begin{aligned} \dot{z}_i &= f_{y_i}(y_1, \dots, y_4) - f_{x_i}(x_1, \dots, x_4) \\ & - \sum_{j=1}^{p_1} F_{x_{ij}}(x_1, \dots, x_4) \theta_{x_{ij}} - \sum_{j=1}^{p_2} \Delta_{x_{ij}}(x, t) + b_i u_i \end{aligned} \quad (12)$$

Then it also has

$$z_i \dot{z}_i \leq r_{i1} |z_1| |z_i| + r_{i2} |z_2| |z_i| + r_{i3} |z_3| |z_i| + z_i b_i u_i \quad (13)$$

Use the adaptive method to design the control  $u_i$  as

$$u_i = b_i^{-1} [\hat{r}_{i1} |z_1| + \hat{r}_{i2} |z_2| + \hat{r}_{i3} |z_3|] \frac{z_i}{|z_i| + \varepsilon_i} \quad (14)$$

Where  $\frac{z_i}{|z_i| + \varepsilon_i}$  is a soft function and has a characteristic similar to sign function.

The error of assumption is defined as

$$\tilde{r}_{ij} = r_{ij} - \hat{r}_{ij} \quad (15)$$

Solve its derivative as

$$\dot{\tilde{r}}_{ij} = -\dot{\hat{r}}_{ij} \quad (16)$$

Considering that if  $\varepsilon_i$  is a small positive constant, then

$$\frac{z_i}{|z_i| + \varepsilon_i} \approx \text{sign}(z_i) \quad (17)$$

Where the parameter adaptive law is designed as

$$\dot{\hat{r}}_{ij} = -|z_i z_j| \quad (18)$$

Choose the Lyapunov function as

$$V_1 = \sum_{i=1}^n z_i^2 + \sum_{j=1}^{j=n} \sum_{i=1}^{i=n} \tilde{r}_{ij} \quad (19)$$

It is easy to get

$$\dot{V}_1 \leq 0 \quad (20)$$

So the system is stable and synchronization is realized [18-21].

## 5. Robust Adaptive Synchronization Law Design based on Sigmoid Function Method

Define the error variable as  $z_i = y_i - x_i$ , where the error system can be written as

$$\begin{aligned} \dot{z}_i = & f_{y_i}(y_1, \dots, y_4) - f_{x_i}(x_1, \dots, x_4) \\ & - \sum_{j=1}^{p_1} F_{x_{ij}}(x_1, \dots, x_4) \theta_{x_{ij}} - \sum_{j=1}^{p_2} \Delta_{x_{ij}}(x, t) + b_i u_i \end{aligned} \quad (21)$$

Then it also has

$$z_i \dot{z}_i \leq r_{i1} |z_1| |z_i| + r_{i2} |z_2| |z_i| + r_{i3} |z_3| |z_i| + z_i b_i u_i \quad (22)$$

Use the adaptive method to design the control  $u_i$  as

$$u_i = b_i^{-1} [\hat{r}_{i1} |z_1| + \hat{r}_{i2} |z_2| + \hat{r}_{i3} |z_3|] h_i(z_i) \quad (23)$$

Where  $h_i(z_i)$  is a sigmoid function and it has a characteristic similar to sign function and it can be written as

$$h_i(z_i) = \frac{1 - e^{-\tau_i z_i}}{1 + e^{-\tau_i z_i}} \quad (24)$$

The error of assumption is defined as

$$\tilde{r}_{ij} = r_{ij} - \hat{r}_{ij} \quad (25)$$

Solve its derivative as

$$\dot{\tilde{r}}_{ij} = -\dot{\hat{r}}_{ij} \quad (26)$$

Considering that if  $\tau_i$  is a big positive constant, then

$$\frac{1 - e^{-\tau_i z_i}}{1 + e^{-\tau_i z_i}} \approx \text{sign}(z_i) \quad (27)$$

We can also design adaptive law as

$$\dot{\hat{r}}_{ij} = -|z_i z_j| \quad (28)$$

Choose the Lyapunov function as

$$V_1 = \sum_{i=1}^n z_i^2 + \sum_{j=1}^{j=n} \sum_{i=1}^{i=n} \tilde{r}_{ij} \quad (29)$$

It is easy to get

$$\dot{V}_1 \leq 0 \quad (30)$$

So the system is stable and the synchronization is realized according to Lyapunov stability theorem.

## 6. Numerical Simulation for Synchronization with Soft Function Method

We choose a four dimension chaotic system as an example to do the numerical simulation to test the rightness of the proposed method and its mode can be written as

$$\dot{x}_1 = a(x_2 - x_1) + k_{1b} \sin x_4 \quad (31)$$

$$\dot{x}_2 = bx_1 - x_2 - x_1x_3 + k_{2b} \cos x_4 \quad (32)$$

$$\dot{x}_3 = x_1x_2 - cx_3 + k_{3b} \sin(x_4x_3) \quad (33)$$

$$\dot{x}_4 = \sin(x_1x_2) + k_{4b} \cos(x_3) \quad (34)$$

Where  $a = 10$  ,  $b = 28$  ,  $c = \frac{8}{3}$  ,  $k_{1b} = 0.1$  ,  $k_{2b} = 0.1$  ,  $k_{3b} = 0.2$  ,  $k_{4b} = -1$  ,the above system is a chaotic system and it has an attractor. We set initial value as  $x_1 = -0.2, x_2 = 0.7, x_3 = 5, x_4 = 0.2$  , then its free movement can be shown as following Figure 1 to Figure 4.

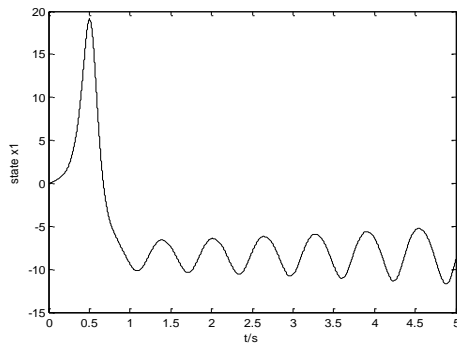


Figure 1. The Curve of State x1

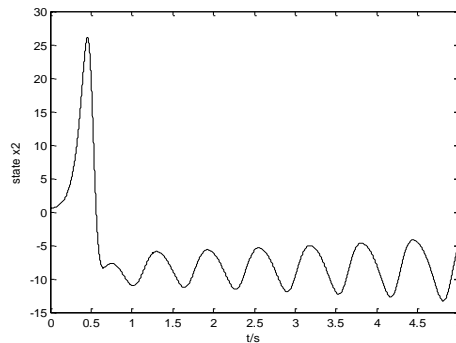


Figure 2. The Curve of State x2

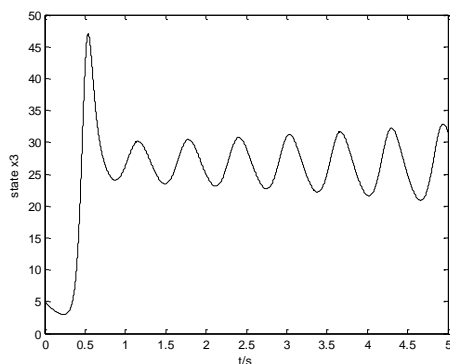


Figure 3. The Curve of State x3

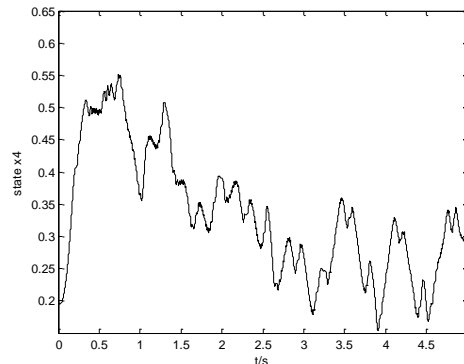


Figure 4. The Curve of State x4

We set  $a, b, c$  as unknown parameters and set  $k_{lb}$  as coefficient of nonlinear function. The structure of response system is known and it can be described as follows:

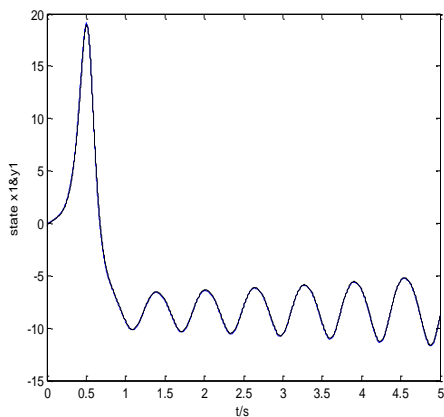
$$\dot{y}_1 = a_y (y_2 - y_1) + u_1 \quad (35)$$

$$\dot{y}_2 = b_y y_1 - y_2 - y_1 y_3 + u_2 \quad (36)$$

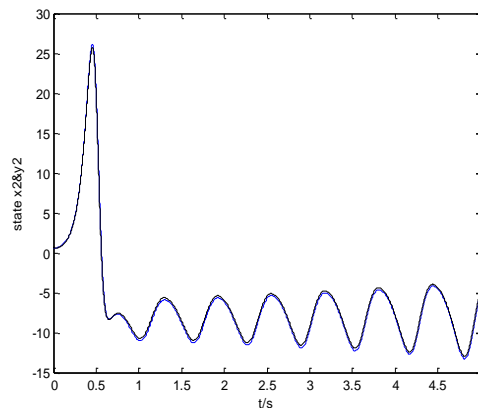
$$\dot{y}_3 = y_1 y_2 - c_y y_3 + u_3 \quad (37)$$

$$\dot{y}_4 = \sin(y_1 y_2) + u_4 \quad (38)$$

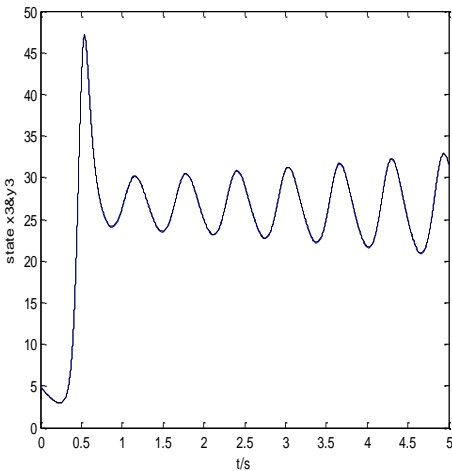
Choose parameter as  $a_y = 9$ ,  $b_y = 25$ ,  $c_y = 2$ , and the initial state of response system is  $y_1 = -0.3, y_2 = 0.5, y_3 = 3, y_4 = 0.7$ , use above robust adaptive strategy with soft function method, the simulation result is as follows:



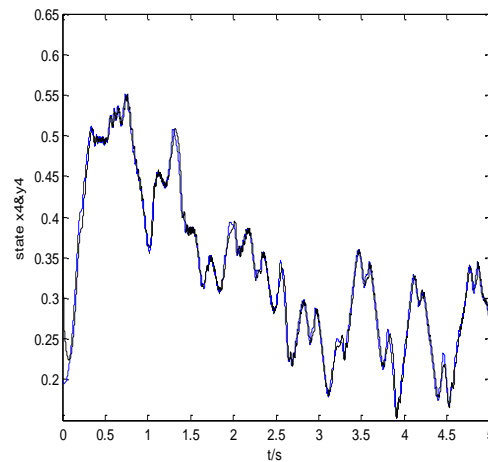
**Figure 5. The Curve of State x1 and y1**



**Figure 6. The Curve of State x2 and y2**

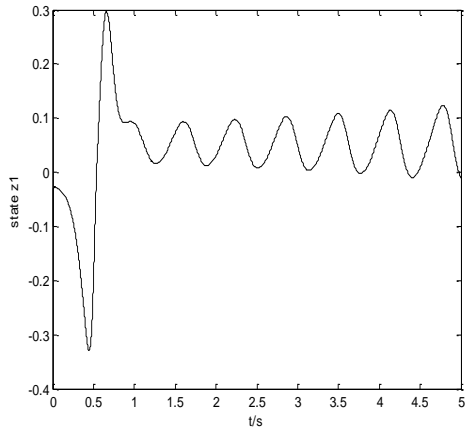


**Figure 7. The Curve of State x3 and y3**

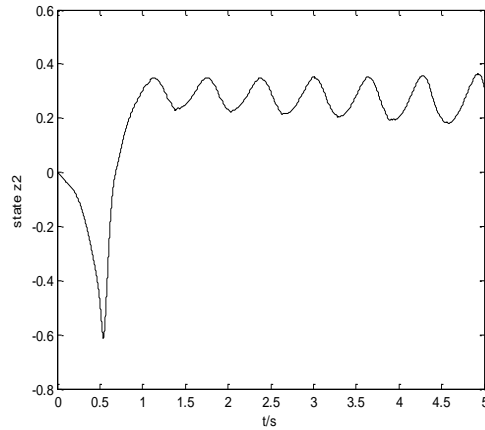


**Figure 8. The Curve of State x4 and y4**

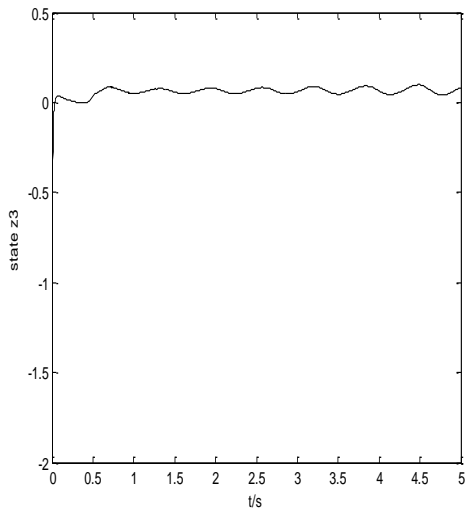
And according to above simulation figures, we can make a conclusion that the response chaotic system can trace the driven chaotic system very well. So the synchronization effect is very good. And the synchronization errors can see following figures.



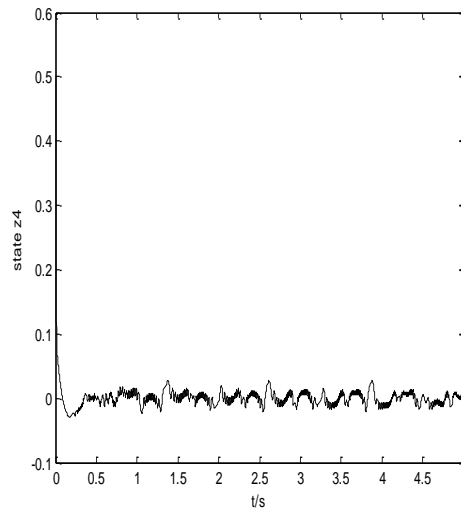
**Figure 9. The Curve of State z1**



**Figure 10. The Curve of State z2**



**Figure 11. The Curve of State z3**

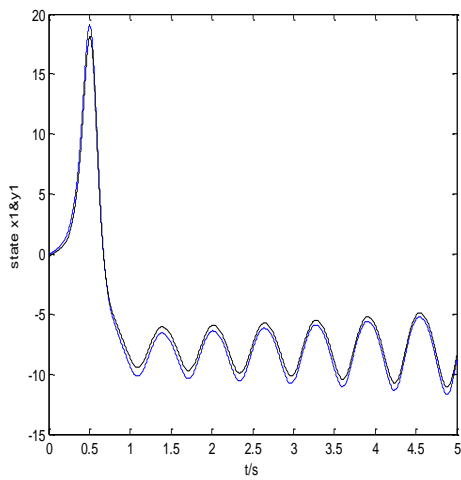


**Figure 12. The Curve of State z4**

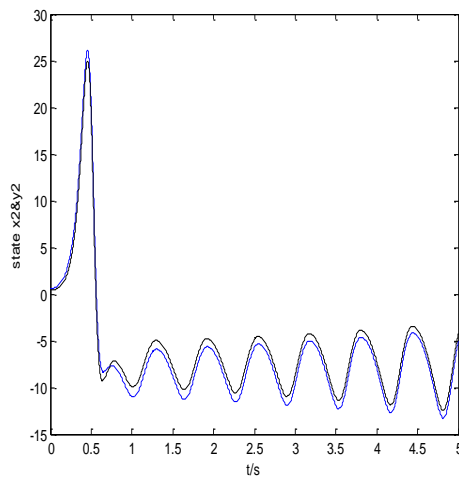
So we can make a conclusion that the synchronization error is converged to a small interval near zero. And if we increase the gain, then the errors can be further reduced.

## 7. Numerical Simulation for Synchronization with Sigmoid Function Method

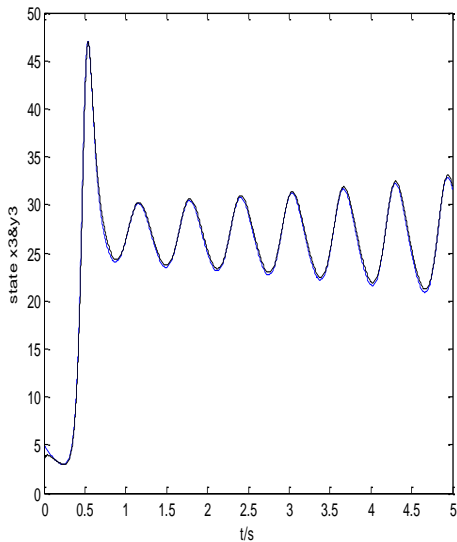
And we tried the sigmoid function method to do the simulation, and the setting of initial value of systems and adaptive law weights are the same as above soft function method. And we choose  $\tau_i = 0.2$ , then simulation results can see following figures.



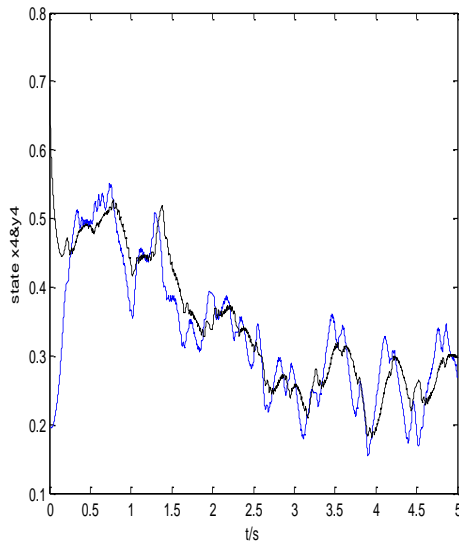
**Figure 13. The Curve of State x1 and y1**



**Figure 14. The Curve of State x2 and y2**



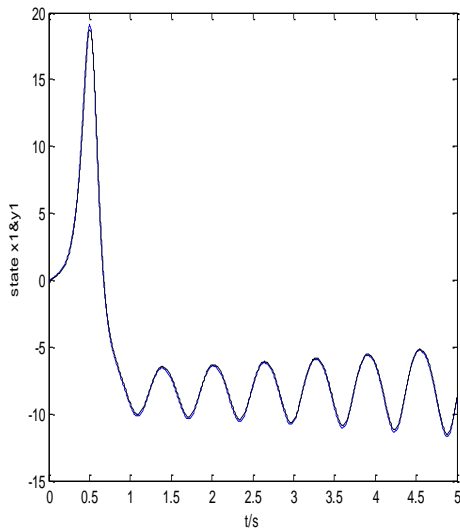
**Figure 15. The Curve of State x3 and y3**



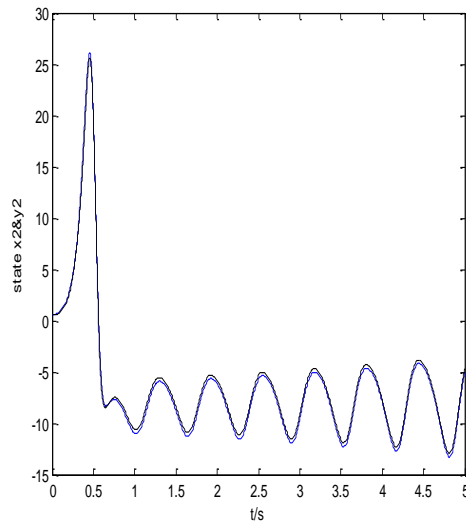
**Figure 16. The Curve of State x4 and y4**

And according to above simulation result and figures, it is obvious that the synchronization effect is not as good as soft function method with the same setting of controller gains. And we increase  $\tau_i = 2$ , then simulation result can see following figures.

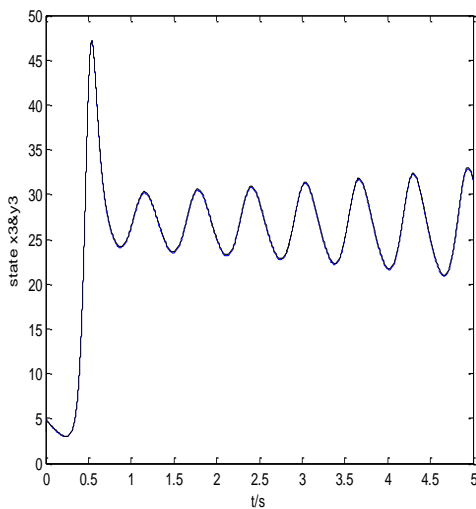




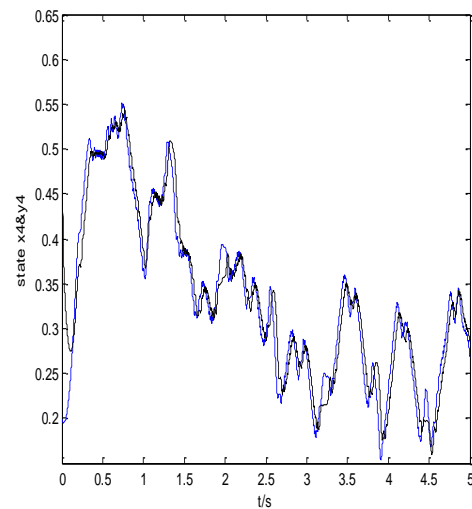
**Figure 13. The Curve of State x1 and y1**



**Figure 14. The Curve of State x2 and y2**



**Figure 15. The Curve of State x3 and y3**



**Figure 16. The Curve of State x4 and y4**

Then the synchronization effect was improved greatly. So the sigmoid function method was also very useful in some situation of synchronization of chaotic systems.

## 8. Conclusion

In order to realize synchronization of two chaotic systems with uncertain parameters and nonlinearities, two kinds of adaptive and robust method based on soft function and sigmoid function were proposed. The biggest advantage of the proposed two methods is that it do not need the detailed information of the uncertain part of systems and a uniformed adaptive law was designed for every subsystem by using the bounded characteristics of chaotic system. Also by using sigmoid function and soft function method, the chattering problem was greatly suppressed compared with traditional sign

function method in sliding mode control design. And at last, detailed simulation result shows the rightness and effectiveness of the proposed method.

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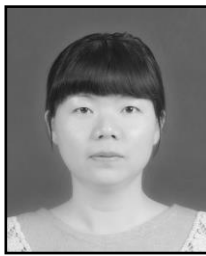
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