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Spatially Constrained Mixture Model and Image Segmentation: A Review

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Abstract

The mixture model is a commonly used approach for image segmentation. However, it doesn't consider the spatial information. In order to overcome this disadvantage, several spatially constrained mixture models have been proposed. In this paper, these spatially constrained mixture models and their experimental results on synthetic and real world images are presented. These experimental results demonstrate that the spatially constrained mixture models can achieve competitive performance compared to the standard mixture model.

Keywords: *Mixture model, spatial information, image segmentation, expectation-maximization algorithm*

1. Introduction

Image segmentation plays an important role in image processing and computer vision. Its task is to classify image pixels based on the coherence of certain features such as intensity, color, texture, motion, location. Many methods have been previously proposed for image segmentation.

The mixture model [1-2] is one of the most commonly used model for clustering and image segmentation. In this approach, the pixels are viewed as coming from a mixture of probability distributions, each representing a different component. The parameters of the probability distributions can be estimated very efficiently through maximum likelihood (ML) using the expectation maximization (EM) algorithm [3]. When the parameters have been estimated, each pixel can be assigned using the maximum a posteriori (MAP) rule.

The main advantage of the mixture model is that it has a simple mathematical form and it is easy to implement. It has been used successfully in a number of applications [4-5]. A drawback of the mixture model is that it doesn't use the spatial information about the data. In order to overcome this drawback, several approaches with spatial constraint have been proposed.

Markov random field (MRF) [6-8] provides a convenient and consistent way to account for spatial dependencies between image pixels. The spatially variant finite mixture model (SVFMM) [9-10] assumes that the prior distribution form a Markov random field. The prior distribution of the SVFMM depends on their neighboring pixels and their corresponding parameters. Thus, these models work well in medical images for segmentation [5]. However, the drawback is that they are computationally expensive to estimate parameters. Various approximations have been introduced to tackle this problem. The gradient projection algorithm9 was proposed to implement the M-step of the EM algorithm. A closed-form updates equation followed by an efficient projection method [10] is used to estimate the parameters.

In Ref. [11], a new family prior distribution has been presented. The prior probability is based on Gauss-Markov random field, which controls the degree of smoothness for each cluster. Parameters can be estimated in closed form via the MAP estimation using the EM

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algorithm. In Ref. [12], the Dirichlet compound multinomial-based spatially variant finite mixture model (DCM-SVFMM) has been proposed. This model exploits the Dirichlet compound multinomial probability to model the probabilities of class labels and the Gauss-Markov random field to model the spatial prior.

Another approach to take the commonality of location into account is an extension of mixture model [13]. This approach is quite similar to the mixture model and easy to implement. It introduces a novel prior distribution which acts like a mean filter to incorporate the spatial relationships between neighboring pixels. The gradient approach has been successfully used for adjusting the parameters. In Ref. [14], a spatially constrained generative model was proposed for segmentation. It assumes the prior distributions share similar parameters for neighboring pixels and introduces a pseudo-likelihood quantity to measure the similarity between neighboring pixels priors. These spatially constrained mixture models are very commonly used for image segmentation [15] [16].

In the mixture model-based methods, we use the EM algorithm [3] to estimate the parameters. When all the parameters have been estimated, we can compute the posterior probability. Finally, each pixel can be assigned to a class using the maximum a posterior (MAP) decision rule. Numerical experiments are presented to assess the performance of the mixture model-based methods both with simulated data and real natural images. We compare the results of these mixture model-based methods both visually and quantitatively.

2. A Review of Mixture Model-Based Methods for Image Segmentation

In this section, we discuss three groups of mixture model-based approaches for image segmentation. The first group is standard Gaussian mixture model [2]. The second group is the spatially variant finite mixture model (SVFMM) [9-10], which assumes the prior distributions that generate the pixel labels form a Markov random field. The third one is a class-adaptive spatially variant mixture model [11], which assumes the prior probability is based on Gauss-Markov random field.

Let $X = \{x_i, i \in I\}$ denote an observed image, where x_i is the observation of pixel i and I is the set of pixels in the scene. We also denote a label set $Y = \{1,2,\ldots K\}$, K is the total number of the classes. The goal of the image segmentation problem is to find a labeling y that assigns each pixel i a label $y_i \in Y$, where y consider both the observed data and the spatial smooth. For each pixel i , given the posterior probability $P(y_i = k \mid x_i)$ for all classes, it is assigned to the class with the largest posterior probability

$$x_i \in class\ k : IF\ k = \arg\max P(y_i = t \mid x_i) \tag{1}$$

2.1. Standard Mixture Model

The mixture model [2] is one of the Bayesian-based methods. It is a flexible and powerful technique for image segmentation. In the standard mixture model framework, it assumes a common prior distribution π which is a discrete distribution with K states, whose parameters π_k are unknown, and holds:

$$P(y_i = k) = \pi_k \tag{2}$$

where the prior distribution satisfies the constraints

$$0 \le \pi_k \le 1 \; and \; \sum_{k=1}^{K} \pi_k = 1 \tag{3}$$

The mixture model also assumes that the density function at a pixel observation is given by

$$\begin{aligned} P(x_i) &= \sum_{k=1}^{K} P(y_i = k) P(x_i \mid y_i = k) \\ &= \sum_{k=1}^{K} P(y_i = k) P(x_i \mid \theta_k) \end{aligned} \tag{4}$$

where $P(x_i | y_i = k)$ is a density conditional on the class label k , θ_k is the parameter of the kth component distribution. In general, we assume that the probability distribution of the mixture model is Gaussian distribution, which is given by:

$$P(x_i \mid \theta_k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{(x_i - \mu_k)^2}{2\sigma_k^2} \right) \tag{5}$$

The log-likelihood function of standard Gaussian mixture model is given by

$$\begin{aligned} \mathrm{L}(\pi, \theta) &= \sum_{p=1}^{N} \log \sum_{k=1}^{K} P(y_i = k) P(x_i \mid y_i = k) \\ &= \sum_{i=1}^{N} \log \sum_{k=1}^{K} \pi_k P(x_i \mid \theta_k) \end{aligned} \tag{6}$$

The log-likelihood function is considered as a function of the parameters π and θ . As can be seen, the advantage of the standard mixture model is that it has a simpler form, and requires a small number of parameters. However, the main drawback is that the pixels are considered independent in the mixture model.

2.2. Spatially Variant Finite Mixture Model

In order to take into account the spatial dependence between image pixels, a set of spatially constrained mixture model have been proposed for image segmentation [9-13]. The classical and famous one is spatially variant finite mixture model (SVFMM) [9-10] .

In the SVFMM framework, the prior distribution $P(y_i = k)$ is defined in the following form:

$$P(y_i = k) = \pi_{ik} \tag{7}$$

The prior distribution π_{ik} of pixel i belonging to the class k should satisfies the constraints

$$0 \le \pi_{ik} \le 1 \; and \; \sum_{k=1}^{K} \pi_{ik} = 1$$

The SVFMM assumes that the prior distribution π form a Markov random field. The Markov random field of the prior distribution is defined as:

$$P(\pi) = \frac{1}{Z} \exp(-U(\pi)) \tag{8}$$

where Z is a normalizing constant called the partition function. The energy function $U(\pi)$ denotes the clique potential function on π which considers the spatial interaction information and can be computed as follows:

$$U(\pi) = \beta \sum_{i=1}^{N} \sum_{j \in N(i)} \sum_{k=1}^{K} \left(\pi_{ik} - \pi_{jk}\right)^2 \tag{9}$$

where $N(i)$ is the neighborhood of i, β is a free parameter.

The log-likelihood function for the spatially variant finite mixture model is given by

$$L(\theta, \pi) = \sum_{i=1}^{N} \log \sum_{k=1}^{K} \pi_{ik} P(x_i \mid \theta_k) - \beta \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j \in N(i)} \left(\pi_{ik} - \pi_{jk}\right)^2 \tag{10}$$

Compared to the log-likelihood function of standard mixture model (6), the above log-likelihood function is quite complex. Therefore, the limitation of SVFMM method is that it requires a large amount of computational power to solve the constrained optimization problem of the prior distribution π. Another limitation of the SVFMM is that it requires a greater number of parameters compared to the standard mixture model.

2.3. Class-Adaptive Spatially Finite Mixture Model

The main drawback of the SVFMM [9-10] is that only one parameter in (10) is used to capture the smoothness of all the clusters and in all directions. In the class-adaptive spatially finite mixture model [11] the prior probability is considered to be based on Gauss-Markov random field. The prior of this model can capture the smoothness of each class in different degrees and adapt better to the data. The Gauss-Markov random field prior probability for π in (8) is given by

$$P(\pi) \propto \prod_{k=1}^{K} \beta_k^{-N} \exp\left(-\frac{1}{2} \frac{\sum_{i=1}^{N} \sum_{j \in N(i)} \left(\pi_{ik} - \pi_{jk}\right)^2}{\beta_k^2} \right) \tag{11}$$

where the parameter k captures the spatial smoothness of class k.

The log-likelihood function for the class-adaptive spatially finite mixture model is given by

$$L(\theta, \pi) = \sum_{i=1}^{N} \log \sum_{k=1}^{K} \pi_{ik} P(x_i \mid \theta_k) - \frac{1}{2} \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j \in N(i)} \left(\pi_{ik} - \pi_{jk}\right)^2 - \sum_{k=1}^{K} \log \beta_k^N \tag{12}$$

Note that the parameter k captures the spatial smoothness of class k. Thus, this prior can enforce smoothness of different degree in each class. Moreover, the prior can be further refined by allowing smoothness that varies both within class and along different spatial directions. In this case, the form of the prior probability becomes

$$P(\pi) \propto \prod_{d=1}^{D} \prod_{k=1}^{K} \beta_{kd}^{-N} \exp\left(-\frac{1}{2} \frac{\sum_{i=1}^{N} \sum_{j \in N(i)} \left(\pi_{ik} - \pi_{jk}\right)^2}{\beta_{kd}^2} \right) \tag{13}$$

where D is the total number of the considered directions, the parameter kd captures not only the class variance for class k but also the variance within class k at a certain spatial direction d.

And then the log-likelihood function for this class-adaptive spatially finite mixture model becomes

$$L(\theta,\pi)=\sum_{i=1}^{N}\log\sum_{k=1}^{K}\pi_{ik}P(x_i\mid\theta_k)-\frac{1}{2}\sum_{d=1}^{D}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j\in N(i)}\frac{\left(\pi_{ik}-\pi_{jk}\right)^2}{\beta_{kd}^2}-\sum_{d=1}^{D}\sum_{k=1}^{K}\log\beta_{kd}^{N} \quad (14)$$

In this Gauss-Markov random field-based model, the EM algorithm is utilized for estimating the parameters. The details of the algorithm are given in Ref.[11].

3. Experiments

In this section, we provide experimental results on synthetic and real-world images for evaluating these mixture model-based methods. We analysis the noise robustness and compare the segmentation result of the standard Gaussian mixture model (termed MM), the Spatially Variant Finite Mixture Model (termed SVFMM), the Class-Adaptive Spatially Variant Finite Mixture Model (termed CA-SVFMM). All the methods are implemented in the MATLAB environment. The source code for SVFMM can be downloaded from http://www.cs.uoi.gr/˜kblekas/. In all the experiments, we set the parameter = 0:1. The code for CA-SVFMM is based on Sfikas's Library, which can be downloaded from http://www.cs.uoi.gr/˜sfikas/.

3.1. Synthetic Images

In this subsection, we illustrate the performance of the proposed model with references for synthetic image segmentation. We compare the accuracy of MM, SVFMM and CA-SVFMM. The misclassification ratio (MCR) [6] is used to measure the segmentation accuracy, which is computed by the ratio between the number of misclassified pixels and the total number of pixels.

In the experiment, we use a synthetic image similar to the one used in [6-10] (see Figure 1(a)). The simulated image has three classes (K=3) sampled from an MRF model using the Gibbs sampler [17-18]. The gray levels for the three classes are 55, 115 and 225 respectively. Figure 1(b) shows the same image with added Gaussian noise (mean=0, variance=0.02). We show a comparison of the MM, SVFMM, CA-SVFMM for segmenting this three-class synthetic image. Figure.1(c)-Figure 1(e) show the segmentation results obtained by the MM, SVFMM, CA-SVFMM and the proposed method, respectively. Compared to these three methods, it is easy to view that the proposed method obtains the best result in noisy environment. To further examine its robustness to noise, the evaluation of the proposed method within noisy environment is presented. The results obtained with varying levels of Gaussian noise are presented in Figure 2. As can be seen, the spatially constrained mixture models have lower MCR compared with the standard mixture model.

Form the above example we can conclude here that the segmentation results of the spatially constrained mixture models are significant quantitative and with the higher degree of robustness with respect to noise.

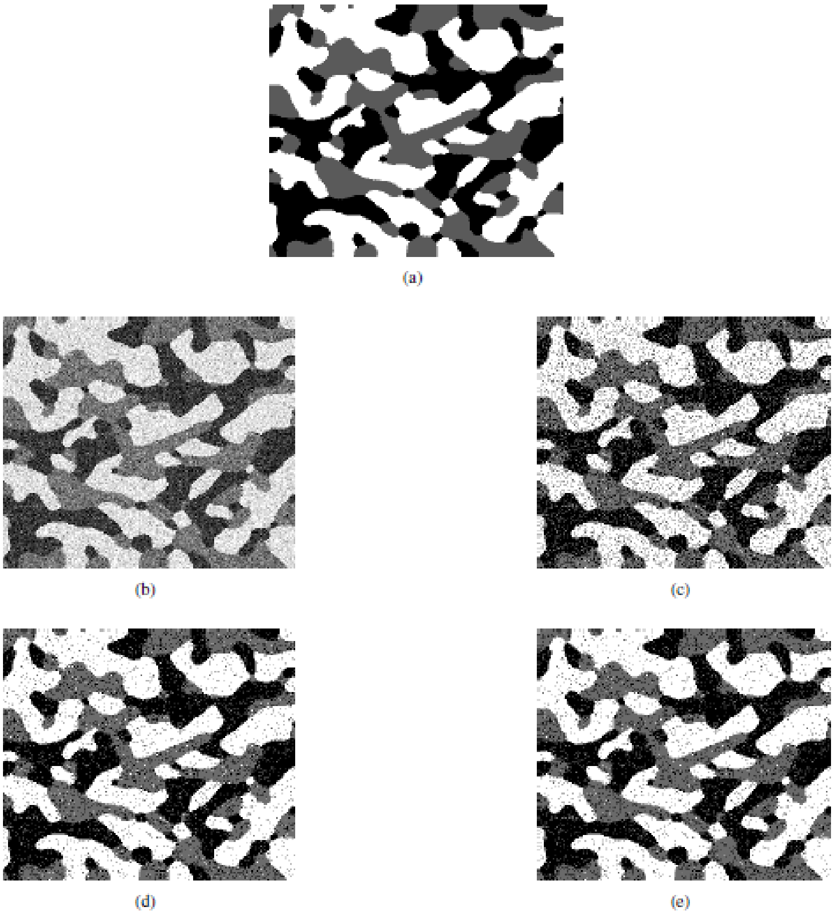

Figure 1. Experiment on Synthetic Image. (a) Original Image. (b) Corrupted Original Image with Gaussian Noise (0 mean, 0.02 Variance). (c) MM (MCR=19.24%). (d) SVFMM (MCR=10.02%). (e) CASVFMM (MCR=8.98%)

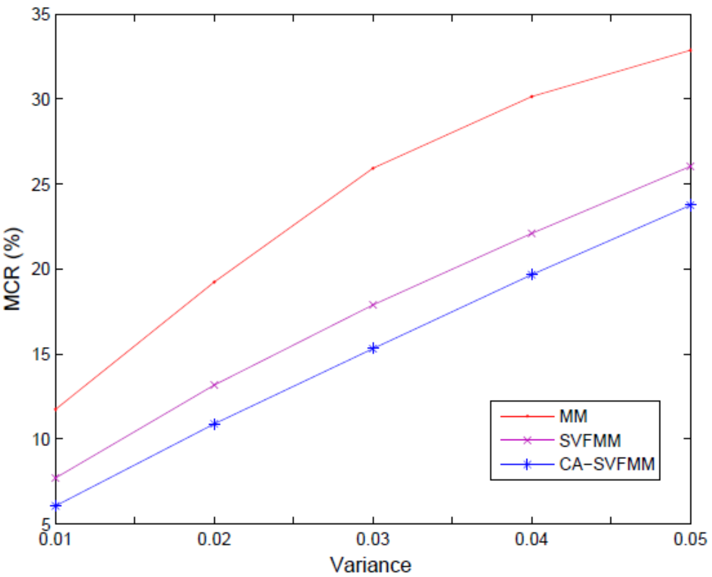

Figure 2. MCR at Different Variance Values (Gaussian Noise Mean =0) using Different Methods

3.2. Real World Images

We have also evaluated and compared these methods for the segmentation of RGB natural images. The real world images are obtained from the Berkeley's image segmentation database [19]. This database consists a set of natural images and their ground truth segmentation results provided by different individuals. In this paper, we use the Probabilistic Rand index (PRI) [20] to evaluate the performance of the proposed algorithm. It contains values in the range [0-1], with values closer to 1 indicating a good result.

In the first experiment, we tried to segment a color image shown Figure 3(a) into five classes. For illustrative purpose, images in Figure 3(b), Figure 3(c) and Figure.3(d) show the segmentation results obtained by using MM, SVFMM and CA-SVFMM, respectively. It can be observed that theses spatially constrained mixture models obtain good results when compared to the standard mixture model. The PRI values with their associated methods are also displayed in the Figure 3(a). As expected, theses spatially constrained mixture models outperforms the standard mixture model with a higher PRI value.

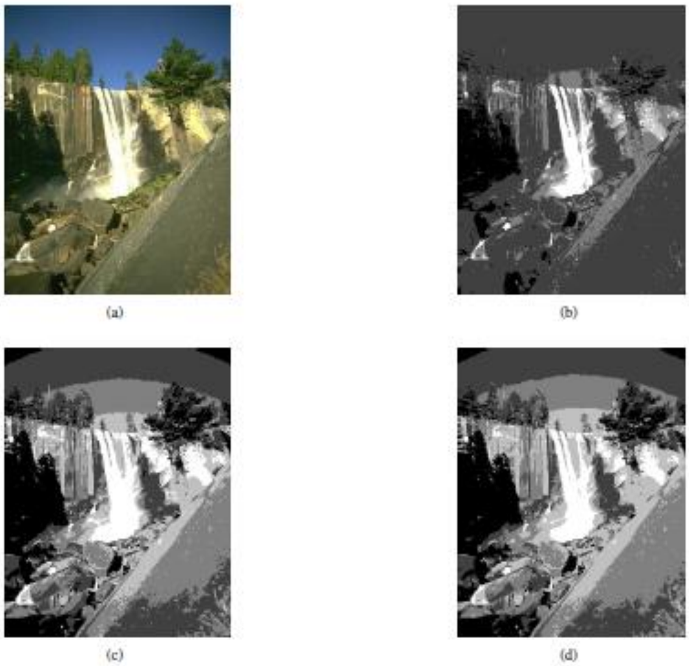

Figure 3. Experiment on Color Image. (a) Original Image 27059. The Segmentation Results Obtained by (b) MM (PRI=0.532), (c) SVFMM ((PRI=0.733), (d) CA-SVFMM (PRI=0.736)

To further examine the accuracy and the efficiency of theses spatially constrained mixture models, they are applied to the case of different class numbers. The image shown in Figure 4(a) is the original color image. The number of the classes could be three, four, and five. Figure 4(b)–Figure 4(d) present the three-class segmentation results obtained by employing the standard MM, SVFMM and CA-SVFMM, respectively. Figure 4(e)–Figure 4(g) present the four class segmentation results obtained by employing the standard MM, SVFMM and CA-SVFMM, respectively. From visual inspection of the results, theses spatially constrained mixture models demonstrate better performances compared to the standard mixture model. Moreover, the results of theses spatially constrained mixture models are with higher PRI values.

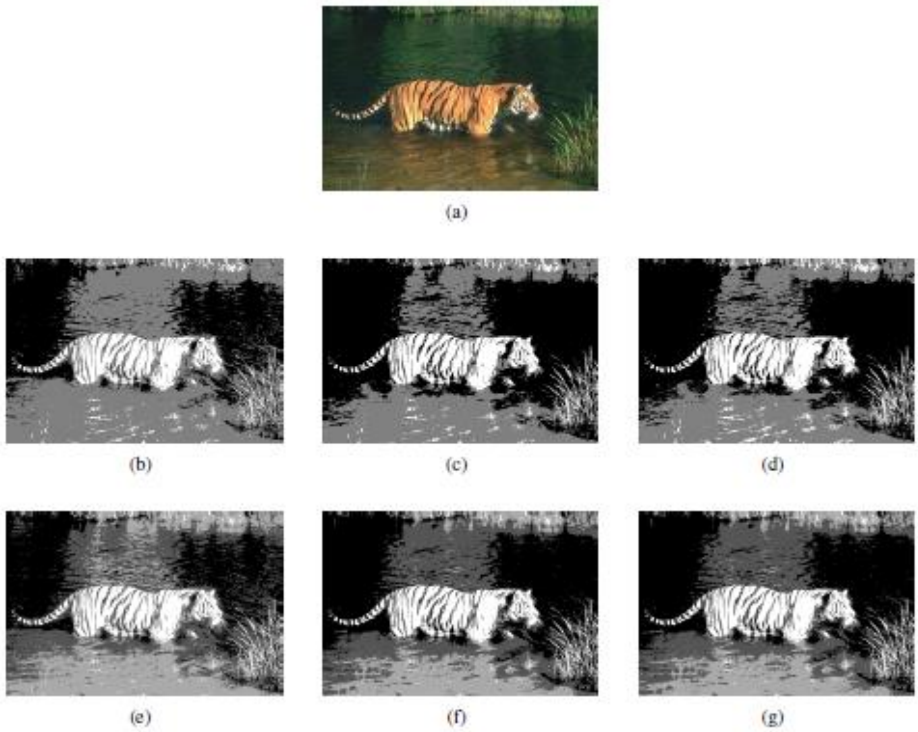

Figure 4. Experiment on Color Image. (a) Original Image 108073, Three-Class Segmentation Results Obtained by (b) MM (PRI=0.580), (c) SVFMM (PRI=0.597), (d) CA-SVFMM (PRI=0.600). The Four-Class Segmentation Results Obtained by (e) MM (PRI=0.556), (f) SVFMM (PRI=0.575), (g) CA-SVFMM (PRI=0.578)

4. Conclusion

In this paper, we have presented a review of mixture model-based methods. The mixture model is a commonly used approach for image segmentation. In order to improve the segmentation accuracy, the spatial information is taken into the mixture model. We present two classical spatially constrained mixture models. We use the EM algorithm to estimate the parameters of the model. When all the parameters have been estimated, we can compute the posterior probability. Finally, each pixel can be assigned to a class using the maximum a posterior (MAP) decision rule. These methods have been tested with many synthetic and real world images, the experimental results demonstrate the excellent performance of the spatially constrained mixture models in segmenting the images compared to the standard mixture model.

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