## An Improved Image Registration Based on Nonsubsampled Contourlet Transform and Zernike Moments

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#### Abstract

Feature-based registration is an effective and most widely used image registration method currently. It includes three critical steps, feature extraction, feature matching and transformation parameters estimation. This paper mainly explores the first two steps. In one of Chahira Serief's paper about image registration, feature points extraction based on nonsubsampled contourlet transform (NSCT) was proposed and feature points matching based on Zernike moments was adopted. The registration accuracy and robustness of his algorithm are acceptable, but it can still be improved. In this paper, an improved scheme of this registration algorithm is proposed. The rotation invariance of NSCT-based feature points extraction is improved, which is beneficial to extract homologous feature points. And the reliability and effectiveness of Zernike moments-based feature points matching are improved, which can improve the matching accuracy. The improved registration algorithm can realize registration of images related by larger scaling, rotation and translation transformation. The simulation results show that the registration robustness is further improved, and the registration accuracy is still high.

**Keywords**: Image registration, Feature extraction, Nonsubsampled contourlet transform, Zernike moments

### 1. Introduction

Image registration is a geometric alignment process of two or more images of the same scene taken at different times, by different sensors or from different viewpoints. Featurebased registration is an effective and most widely used image registration method currently[1]. It utilizes only certain feature of image instead of all whole image pixels to obtain the transformation parameters. So feature-based registration can greatly reduce computational costs and possesses good invariance to intensity change and occlusion in image. Feature-based registration include three critical elements, feature extraction, feature matching and transformation parameters estimation. In the proposed algorithm, we improved the rotation invariance of NSCT-based feature points extraction and the reliability and effectiveness of Zernike moments-based feature points matching. We extract significant image feature points across spatial and directional resolutions exploiting the improved NSCT-based feature points extraction. And feature points matching is realized using the improved Zernike moments-based feature points matching that is performed by computing the correlation coefficients of the Zernike moments descriptor vectors of two concentric circular neighborhoods respectively centered on each feature point and each obtained first-matching point. These obtained matching points pairs are then used to make transformation parameters estimation. The experimental

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results demonstrate better robustness and acceptable accuracy of the improved registration algorithm.

The rest of the paper is structured as follows. In Section 2, we describe image registration algorithm based on NSCT and Zernike moments and discuss the existing problems of it. In Section 3, the proposed improved image registration algorithm based on NSCT and Zernike moments is introduced in detail. Transformation model selection and parameters estimation are discussed in section 4. Experimental results of the improved registration algorithm are presented and analyzed in section 5. Finally, in section 6, the conclusion is drawn.

# 2. Image Registration Based on NSCT and Zernike Moments and the Existing Problems of It

The process of Chahira Serief 's image registration algorithm based on NSCT and Zernike moments was carried out in three main steps, extracting significant image feature points exploiting NSCT-based feature points extraction, realizing feature points matching using Zernike moments-based feature points matching, making transformation parameters estimation using the obtained matching result. This can be shown as the flowchart in Figure 1.

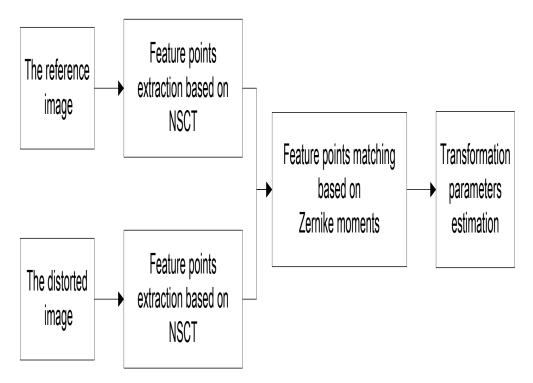


Figure 1. The Flowchart of the Image Registration Based on NSCT and Zernike Moments

## 2.1. Feature Points Extraction Method Based on NSCT

Contourlet transform [2] approximates an image using radical structure similar to contour segment. The support interval of radical is "long bar" structure whose aspect ratio varying with scale, and of directionality and anisotropy. So contourlet transform can better represent a natural image than Wavelets transform. However, due to downsampling for image, contourlet transform is not of shift- invariance, while shift-invariance is desirable for many image analysis applications. NSCT [3] is shift-invariant version of the

contourlet transform as it eliminates the downsamplers in the two-stage implementation of contourlet transform. its structure is shown in Figure 2.

For per stage decomposition, image is decomposed into low frequency part and high frequency part by nonsubsampled pyramid filter (NSP) first, and then high-frequency part is divided into several directions by the nonsubsampled directional filter bank (NSDFB).

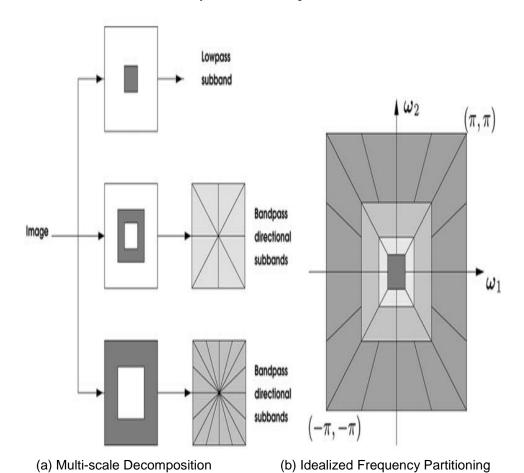


Figure 2. Nonsubsampled Contourlet Transform

The NSCT provides not only multiresolution analysis, but also geometric and directional representation and it is shift invariant so that each pixel of the transform subbands corresponds to that of the original image in the same location, so the geometric information can be gathered pixel by pixel from NSCT coefficients. The process of the NSCT-based feature points extraction method is shown in Figure 3.

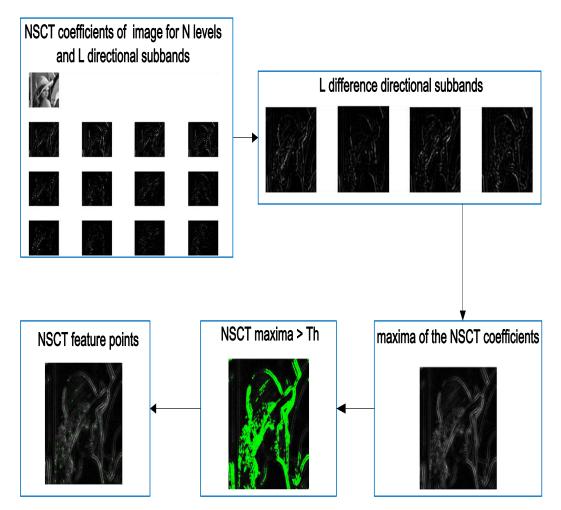


Figure 3. The Flowchart of NSCT-Based Feature Points Extraction Method

In the first step, we compute the NSCT coefficients of the image for N levels and L directional subbands. In the second step, compute the difference of corresponding directional subbands between two certain levels to obtain L difference directional subbands. In the third step, compute the maximum magnitude of the obtained L difference directional subbands at each pixel location to obtain "maxima of the NSCT coefficients". In the fourth step, record points that NSCT maxima > Th. Where the thresholding Th= $c(\delta + \mu)$ , the value of parameter c is defined by the user, and  $\delta$  and  $\mu$  are the standard deviation and mean of the NSCT maxima image. In the final step, find a local maximum point in each neighbourhood of size w×w centred on each recorded maxima point to eliminate maxima point that are very close to each other. The locations of the obtained threshold NSCT maxima are taken as the extracted feature points.

### 2.2. Feature Points Matching Based on Zernike Moments

After feature points are extracted from both reference image and distorted image, a corresponding mechanism is established using matching similarity measure based on Zernike moments between the two feature points sets. The correspondence is evaluated using a circular neighborhood of radius R centered on each feature point [4]. The reason for selecting the complex Zernike moments as feature descriptors is that the magnitudes of Zernike moments possess rotation invariance property [5]. To achieve translation and scale invariance, the image should be first made translation and scale normalization

utilizing its regular moments. Then, rotation invariant Zernike moments features are extract from the translation and scale normalized image.

Regular moments of digital image f(x, y) are defined as formula (1):

$$m_{pq} = \sum_{x} \sum_{y} x^{p} y^{q} f(x, y) \tag{1}$$

Where  $m_{pq}$  is the (p + q)th order moment of image f(x, y).

Image f(x, y) can be translation and scale normalized by transforming it into image g(x, y), and this is defined as formula (2):

$$g(x,y) = f\left(\frac{x}{\alpha} + \bar{x}, \frac{y}{\alpha} + \bar{y}\right) \tag{2}$$

Where 
$$\bar{x} = \frac{m_{10}}{m_{00}}$$
,  $\bar{y} = \frac{m_{01}}{m_{00}}$ ,  $\alpha = \sqrt{\beta/m_{00}}$ , with  $m_{10}$ ,  $m_{01}$ ,  $m_{00}$  regular moments of

image f (x, y) and  $\beta$  a predetermined value.

Zernike moments of digital image g(x, y) are defined as formula (3):

$$Z_{pq} = \frac{q+1}{\pi} \sum_{x} \sum_{y} g(x, y) V_{pq}^{*}(\rho, \theta) , \qquad x^{2} + y^{2} \le 1$$
 (3)

Where

$$R_{pq}(\rho) = \sum_{s=0}^{(p-|q|)/2} (-1)^s \cdot \frac{(p-s)!}{s! \left(\frac{p+|q|}{2}-s\right)! \left(\frac{p-|q|}{2}-s\right)!} \rho^{p-2s}$$

with p a non-negative integer, p-|q| even,  $p\ge |q|$ , and  $\rho$  length of vector from origin to (x, y) pixel,  $\theta$  angle between the vector and x axis in counterclockwise direction.

The matching relation of the two feature points sets is obtained as follows [5]:

- 1) For every feature point P<sub>i</sub>, choose a circular neighborhood of radius R centered on this point and translation and scale normalize the circular neighborhood.
- 2) Construct a Zernike moments descriptor vector P<sub>z</sub> for each normalized circular neighborhood, as formula (4):

$$P_{z} = (|Z_{1,1}|, \dots, |Z_{p,q}|, \dots, |Z_{10,10}|)$$
(4)

Where  $|Z_{p,q}|$  is the magnitude of Zernike moments. The highest moments order used in the descriptor vector  $P_z$  (10 in the algorithm) is chosen to achieve a compromise between noise sensitivity and the information content of the moments.

3) The feature points matching process is performed by computing the correlation coefficients of two descriptor vectors. The matching points are those who give the maximum coefficient correlation value in both directions. The correlation coefficient Cof of two feature descriptor vectors  $P_{z1i}$  and  $P_{z2j}$  of two feature points respectively in reference image and distorted image is defined as shown in formula (5):

$$Cof = \frac{(P_{z1i} - m_1)^T (P_{z2j} - m_2)}{|P_{z1i} - m_1||P_{z2j} - m_2|}$$
(5)

Where  $m_1$  and  $m_2$  are the mean of  $P_{z1i}$  and  $P_{z2j}$  respectively.

Finally, we select the first three matching feature points pairs having the maximum correlation coefficient to make transformation parameter estimation.

## 2.3. The Existing Problems

There exist the following two problems in image registration algorithm based on NSCT and Zernike moments:

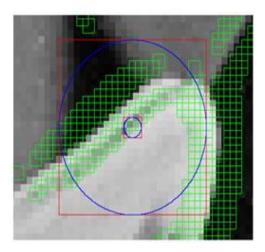
1) In the process of NSCT- based feature points extraction: To eliminate maxima that are very close to each other, in the last step, find a local maximum point in each block neighbourhood of size  $w\times w$  centred on each recorded maxima point. This will make a NSCT coefficients maxima point is the maximum point in the neighbourhood of size  $w\times w$  in the reference image, but is no longer the maximum point in the neighbourhood of size  $w\times w$  in the distorted image when there exists rotation transformation. An example is shown in Figure 4.

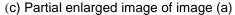


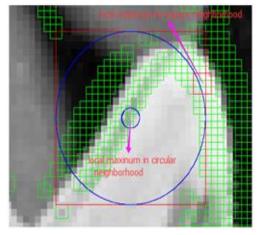
(a) "Lena" image marked by NSCT maxima > Th



(b) Rotated "lena" image marked by NSCT maxima > Th







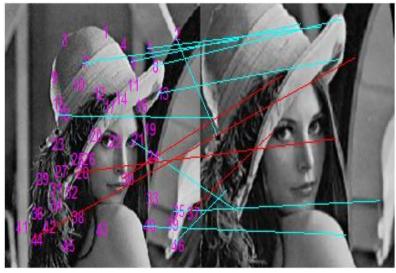
(d) Partial enlarged image of image (b)

Figure 4. The Results of Local Maximum Points Obtained using Different Neighborhoods. Where the Rotation Angle of the Rotated "lena" Image is 20°, and in (c) and (d), the Small Square and the Small Circle Respectively Represents the Local Maximum Point in the Square Neighborhood and in the Circular Neighborhood

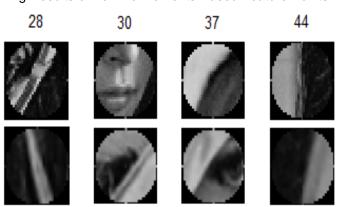
Known from Figure 4, at the same scene location, within a square neighborhood of size w×w centred on the NSCT maxima point, the centre NSCT maxima point is the local maximum point in the original "lena" image, but is no longer the local maximum point in the rotated "lena" image. While selecting a circular neighborhood can solve the problem.

Therefore, selecting square neighborhood is bad for feature points homologous extraction and matching when there exists rotation transformation.

2) In the process of Zernike moments-based feature points matching: Estimating matching relation between two feature points, the radius R of the selected circular neighborhood centered on feature point P<sub>i</sub> can be neither too small nor too large. If R is too small, can't enough reflect the image information near the feature point, and if R is too large, will cause mismatching due to more leakage of image information when there exists scaling transformation or due to the rotation invariance of Zernike moments. This will inevitably causes difficulties for the selection of the parameter R and affect the matching accuracy and precision. And a single circular neighborhood is difficult to guarantee the reliability of the matching similarity measure. An example of mismatching is shown in Figure 5. The original algorithm can find correct matching points used to make transform parameter estimation of images possessing obvious characteristics and with small scaling, rotation and translation transformation, but cannot succeed to find correct matching points for some registration of images related by larger scale, rotation, translation transformation or with not obvious characteristics. Therefore, further improving the reliability and effectiveness of the matching similarity measure is the core of the proposed improved registration algorithm.



(a) Matching Results of Zernike Moments-Based Feature Points Matching



(b) The Circular Neighborhood of Four Mismatching Feature Points Pairs Maked by Red Line

Figure 5. Analysis of Mismatching. Where the Distorted Image is the Result of the Original Image Scaled up 1.3 Times, Counterclockwise Rotated 10 Degrees, Shifted Left 20 Pixels, Shifted up 10 Pixels

## 3. Improved Image Registration Algorithm based on NSCT and Zernike Moments

## 3.1. Improved Feature Points Extraction based on NSCT

To improve the rotation invariance of NSCT-based feature points extraction method, we change the neighbourhood of finding a local maximum point from square neighbourhood of size w×w to circular neighborhood of diameter w. This is beneficial to feature points matching when there exists rotation transformation and increases the correct matching points pairs when there exists rotation transformation in the same condition.

Furthermore, in order to eliminate the influence of boundary points, in this paper, we eliminate feature points whose distances to the image boundary are less than w.

Taking "lena" image as an example, contrast of feature points extraction results before and after the NSCT-based feature points extraction method improved is shown in Figure 6. Where the distorted image is the result of the original image counterclockwise rotated 20 degrees, and feature points whose distances to the image boundary are less than w are eliminated both in the original and the improved method.

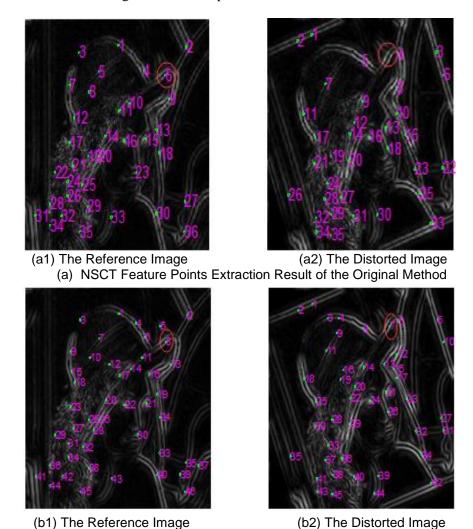


Figure 6. Contrast of Feature Points Extraction Results Before and After the NSCT-Based Feature Points Extraction Method Improved

(b) NSCT Feature Points Extraction Result of the Improved Method

The contrast results show that the improved method in which we adopt circular neighborhood is better for accurately extracting feature points when there exists rotation transformation. As an example marked in Figure 4, the feature point "6" in (a1) have no corresponding matching feature point in (a2) in the feature points extraction result of the original method, while the feature point "8" in (b1) at the same scene location is corresponding matched with feature point "7" in (b2) in the feature points extraction result of the improved method.

Taking "lena" and "ny\_256" image as examples, a comparison result of the number of correct matching points pairs in matching points pairs whose correlation coefficient is greater than thresholding CoTh and the matching accuracy of these matching points pairs when there exists rotation transformation is shown in Figure 7.

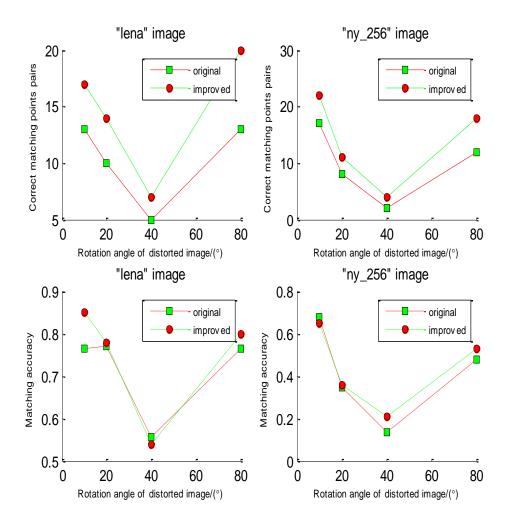


Figure 7. The Matching Performance Contrast before and after the NSCT-Based Feature Points Extraction Method Improved

Known from the analysis of Figure 7, in the same condition, using the improved method can increase correct matching points pairs and generally will not decrease the matching accuracy when there exists rotation transformation. Increasing of correct matching points pairs is better for screening out correct matching points pairs used to make transformation parameters estimation.

## 3.2. Improved Feature Points Matching based on Zernike Moments about Concentric Circular Neighborhoods

To improve the reliability and effectiveness of Zernike moments-based feature points matching, we propose an improved feature points matching based on Zernike moments about concentric circular neighborhoods.

In the improved algorithm, the correspondence is evaluated using two concentric circular neighborhoods of different sizes chosen successively and respectively centered on each feature point and each obtained first-matching point. First, match all feature points by Zernike moments descriptor vectors constructed using the large circular neighborhood and then accurately examine the similarity of every obtained first-matching points pair by Zernike moments descriptor vectors constructed using the small circular neighborhood. And in order to further eliminate the mismatching points, we consider the stability of matching points. In this paper, we called that the larger the ratio of maximum and submaximum of correlation coefficients between a feature point and every feature point in the corresponding image the more stable the feature point is. Thus the improved algorithm can not only improve the reliability and effectiveness of the original matching algorithm but also be beneficial to obtain more stable and accurate matching points.

The improved Zernike moments-based feature points matching algorithm can be summarized as follows:

- 1) For every feature point P<sub>i</sub>, choose a circular neighborhood of radius R centered on this point. Obtain first-matching points and their correlation coefficients of Zernike moments feature descriptor vectors according to the original Zernike moments-based feature points matching algorithm.
- 2) For every first-matching point whose Zernike moments feature descriptor vector correlation coefficient between the corresponding first-matching point is greater than thresholding CoTh, choose a circular neighborhood of radius Rs (R>Rs) centered on this point and translation and scale normalize the circular neighborhood to construct Zernike moments feature descriptor vector. For all first-matching points pairs, compute correlation coefficients of their Zernike moments feature descriptor vectors constructed using circular neighborhood of radius Rs and the first 50% pairs of first-matching feature points having the maximum correlation coefficient are the final obtained matching points pairs.

In addition, in order to further eliminate the mismatching points, we also consider the stability of matching points (measured by the ratio of maximum and sub-maximum of correlation coefficients between a feature point and every feature point in the corresponding image). Therefore, we can select the first two or three matching feature points pairs having best stability in the final matching feature points to make transformation parameters estimation so that the selected matching feature points pairs have both good correlation and good stability.

For easy analysis, in this paper, the original algorithm is also set the correlation coefficient thresholding CoTh. When the distorted image is the result of the original image scaled up 1.15 times (s=1.15), counterclockwise rotated 10 degrees ( $\theta$ =10°), shifted left 20 pixels (t<sub>x</sub>=-20 pixels), shifted up 10 pixels (t<sub>y</sub>=-10 pixels), the matching results contrasts of three groups of experiments of the original matching algorithm and the improved matching algorithm are shown in Figure 8.



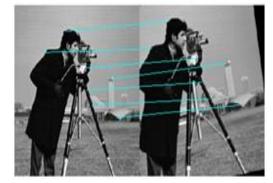


Matching results of original algorithm

Matching results of improved algorithm

(a) Matching results contrast of "lena" image



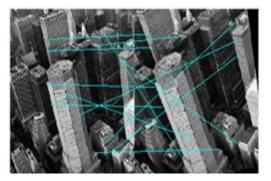


Matching Results of Original Algorithm

Matching Results of Improved Algorithm

(b) Matching Results Contrast of "cameraman" Image





Matching Results of Original Algorithm

Matching Results of Improved Algorithm

(c) Matching Results Contrast of "ny\_256" Image

Figure 8. The Matching Results Contrasts Before and After the Matching Algorithm Improved when using Different Images

Taking "lena" image as an example, on the basis of the original transformation parameters: s=1.15,  $\theta=10^{\circ}$ ,  $t_x=-20$  pixels,  $t_y=-10$  pixels, increase one of the transformation parameters, and the matching results contrasts of the original matching algorithm and the improved matching algorithm are shown in Figure 9:





Matching results of original algorithm

Matching results of improved algorithm

(a) Matching Results Contrast of "lena" Image when s=1.3





Matching Results of Original Algorithm

Matching Results of Improved Algorithm

(b) Matching Results Contrast of "lena" Image when θ=80°





Matching Results of Original Algorithm

Matching Results of Improved Algorithm

(c) Matching Results Contrast of "lena" Image when  $t_x$ =-120 pixels,  $t_y$ =-120 Pixels

Figure 9. The Matching Results Contrasts Before and After the Matching Algorithm Improved when Increasing one of the Transformation Parameters

The matching accuracy and related data of the three groups of experiments in Figure 8 and the three groups of experiments in Figure 9 are shown in Table 1.

Table 1. The Matching Accuracy Contrast Table before and after the Matching Algorithm Improved

The experiment group	Matching points pairs			ching points airs	Matching accuracy	
	Original	Improved	Original	Improved	Original	Improved
Fig.8_a	18	9	12	9	0.6667	1
Fig.8_b	15	7	10	7	0.6667	1
Fig.8_c	25	12	6	5	0.2400	0.4167
Fig.9_a	15	7	7	6	0.4667	0.8571
Fig.9_b	17	8	8	7	0.4706	0.8750
Fig.9_c	10	5	4	4	0.4000	0.8000

The matching correctness of the first three matching feature points pairs that screened out respectively according to correlation coefficient in the original matching algorithm and according to correlation coefficient and stability in the improved matching algorithm is shown in Table 2 (T represents correct matching and F represents mismatching).

Table 2. The Matching Correctness Comparison Table of the First Three Matching Feature Points Pairs Screened Out

The experiment group	Original matching algorithm	Improved matching algorithm		
Fig.8_a	ТТТ	ТТТ		
Fig.8_b	ттт	ТТТ		
Fig.8_c	FFF	ТТТ		
Fig.9_a	F T T	ТТТ		
Fig.9_b	ТТГ	ТТТ		
Fig.9_c	TFF	ТТТ		

It is clear from these results that the matching accuracy of matching points pairs and the matching accuracy of the first three matching feature points pairs screened out are obviously improved. For matching of images with no obvious characteristics, as the aerial image "ny\_256" and some matching of images with obvious characteristics but related by larger scale, rotation and translation transformation, the improved algorithm can successfully screen out correct matching points pairs to make transformation parameters estimation but the original algorithm can't achieve. Therefore, the effectiveness and robustness of the original algorithm are improved. And that the matching feature points pairs selected out by the improved matching algorithm about two concentric circular neighborhoods have both good correlation and good stability, which is beneficial to improving transformation parameter accuracy.

## 4. Transformation Model Selection and Parameters Estimation

Since there are only scaling, rotation, translation transformation between the distorted image and the reference image, linear conformal transformation model is chosen to estimate the transformation parameters. Linear conformal transformation model can be expressed as formula (6):

$$\begin{cases} u = s(x\cos\theta + y\sin\theta) + tx \\ v = s(-x\sin\theta + y\cos\theta) + ty \end{cases}$$
 (6)

Where, s,  $\theta$ , tx, ty respectively represent the transformation parameter of scaling, rotation (°), translation in x direction (pixels), translation in y direction (pixels), (x, y) represent selected matching point coordinate in reference image, (u, v) represent corresponding matching point coordinate in distorted image.

Due to the linear conformal transformation model contains four unknown parameters, so at least two pairs of matching points need to be selected. We select the first two matching feature points pairs having best stability in the matching feature points to make transformation parameters estimation.

However, due to existence of rotation transformation, it will affect the estimation of translation parameters. Therefore, backward correction for the distorted image is required using the obtained scaling and rotation parameters, and then put coordinates of matching points obtained by matching after backward correction into formula (6) again to obtain translation parameters. Also we can obtain translation parameters by comparing the relative position of the matching points in the backward corrected distorted image and in the reference image [7], as shown in formula (7):

$$\begin{cases} tx = (u - n_2/2) - (x - n_1/2) \\ ty = (v - m_2/2) - (y - m_1/2) \end{cases}$$
 (7)

Where  $m_1 \times n_1$  is the size of reference image,  $m_2 \times n_2$  is the size of the backward corrected distorted image, (x, y) represents the matching point coordinate selected in reference image, and (u, v) represents corresponding matching point coordinate in the the backward corrected distorted image. Because two pairs of matching points are selected, the final tx, ty are respectively the average value of tx, ty resulting from the two pairs of matching points. However, in order to be consistent with the practical application, in the proposed algorithm, the distorted image that is the same size as the reference image is cut out from the center part of a deformation image and the deformation image is generated from the reference image using some known geometrical transformations (scaling, rotation and translation). So formula (7) is adopted to obtain the translation parameters in the algorithm.

## 5. Experimental Results

The experimental results of this paper are acquired from MATLAB2012b. To evaluate the performance of the algorithm the robustness and registration accuracy need to be considered. In this paper, the distorted image is cut out the same size as the reference image from the center part of a deformation image and the deformation image is obtained by doing some known geometrical transformation (scaling, rotation and translation) to the reference image. The classic "lena" and "cameraman" image and aerial image "ny\_256" are used in the experiments, "lena" image rich in detail, foreground characteristics of "cameraman" image having some similarities and the characteristics of "ny\_256" image is not obvious, so there is certain registration difficulty. Their sizes are both  $256 \times 256$  pixels.

The experimental results were obtained according to the following settings: the NSCT decomposition of images, performed using the NSCT toolbox, with resolution levels N=4 and each level the number of directional subbands L=4; parameter c=1.2, the circular neighborhood diameter W = 25; the Zernike moments feature point descriptor, concentric circular neighborhoods radiuses R=15, Rs=12, scale normalization parameters  $\beta = 0.6 \cdot \frac{\pi R^2}{M \times N} \cdot m_{00}$  (M×N the size of reference image,  $m_{00}$  zero order regular moments of reference image),  $\beta s = \frac{Rs^2}{R^2} \cdot \beta$ ; thresholding CoTh = 0.94. The particular level-pair used to obtain "maxima of the NSCT coefficients" image is (2, 4). In order to reduce time-

consuming and do not affect the effect of the algorithm, the number of directional subbands of the third level, which is not used, is set to 2.

Taking "lena" and "ny\_256" image as examples, on the basis of the original transformation parameters: s=1.15,  $\theta=10^{\circ}$ ,  $t_x=-20$  pixels,  $t_y=-10$  pixels, change one of the transformation parameters, the comparison results of the obtained estimated parameters and the true parameters are as shown in Table 3.

Table 3. Comparison of the Parameters Estimated using the Improved Registration Algorithm with the True Ones

The true peremeters		The estimated parameters							
The true parameters		lena				ny_256			
S	θ	$t_x$	$t_{y}$	S	θ	$t_{x}$ $t_{y}$	S	θ	t <sub>x</sub> t
1.15	10	-20	-10	1.1547	10.0284	-19.25 -10.23	5 1.1449	10.0477	-19.25 -10
1.25	10	-20	-10	1.2462	9.7447	-20 -10.5	1.2472	10.1917	-20.25 -10
1.3	10	-20	-10	1.2925	9.1680	-20 -10.5	1.3087	9.7156	-19.5 -10
1.4	10	-20	-10	1.3838	9.8528	-20 -10.2	5 1.4045	9.7647	-21 -9.5
1.15	20	-20	-10	1.1534	19.5800	-19.75 -10	1.1505	19.7879	-19.5 -10.5
1.15	40	-20	-10	1.1513	39.5085	-19.5 -11	1.1451	39.4631	-19.5 -10.5
1.15	80	-20	-10	1.1461	79.8245	-20 -10	1.1473	79.8630	-20 -10
1.15	10	-20	-20	1.1523	8.9574	-19.75 -20.73	5 1.1469	9.7472	-20 -19.75
1.15	10	-40	-40	1.1539	10.1359	-40 -40.5	1.1543	9.9777	-40 -40
1.15	10	-80	-80	1.1506	9.4538	-80 -80.25	1.1686	10.4792	-79.5 -80
1.15	10	-120	-120	1.1532	10.1071	-120 -119	1.1545	10.2172	-121 -119

Known from Table 3, the estimated parameters of the improved registration algorithm are very close to the true transformation parameters of images. Registration for scaling transformation can reach up to 1.4 times, and registration for rotation and translation transformation that can be achieved are very high. This proves the robustness and high accuracy of the improved registration algorithm.

### 6. Conclusions

This paper proposes an improved image registration algorithm based on NSCT and Zernike moments, which further improves the effectiveness and robustness of the original algorithm. Experimental results show that the improved algorithm can achieve registration of images related by larger scaling, rotation and translation transformation, and it is a robust, high-accuracy image registration algorithm.

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