The Research of An Algorithm Based on Symmetric Symbols for ICI Self-Cancellation in OFDM System

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Abstract

The intercarrier interference (ICI) will occur when the orthogonality between subcarriers is destroyed in orthogonal frequency division multiplexing (OFDM) communication system. ICI caused by frequency offsets is analyzed, and an algorithm based on symmetric symbols is proposed after researching on the typical ICI self-cancellation scheme. The simulation results show that, compared with the self-cancellation scheme, this algorithm not only reduces the intercarrier interference (ICI) coefficient, but also greatly improves the carrier-to-interference power ratio (CIR) and the performance of the system.

Keywords: OFDM; ICI; ICI self-cancellation; ICI coefficient; CIR carrier-to -interference power ratio

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a kind of multi-carrier modulation (MCM). Its main idea is to make a given channel into a number of orthogonal sub channels in frequency domain, and each sub channel uses one subcarrier which is transmitted in parallel for modulation [1]. OFDM has caught the widespread attention because of its outstanding ability of anti-jamming and high frequency spectrum utilization ratio. And with the development of digital circuit, IFFT/FFT becomes easier to implement, which speeds up the application of OFDM. However, OFDM system requires accurate frequency synchronization to maintain the orthogonality between subcarriers, the time-varying wireless channel and Doppler frequency shifts will make symbol frequency offset, resulting in intercarrier interference (ICI) and impacting the system performance [2].

At present, the methods for suppressing ICI have included channel estimation and frequency-domain equalization [3], time-domain windowing [4], and the ICI self-cancellation [5]. Among them, self-cancellation algorithm does not need to estimate the carrier frequency offset, so it has the advantages of low complexity, good effect and wide consideration. Based on this algorithm, people conduct many further researches and improvements [6-7]. The improved self-cancellation algorithms are presented in references [6-7], where both the algorithms are based on matrix transformation. But the computational complexity has increased because of designing the matrix transformation.

The paper proposes a new ICI self-cancellation algorithm which based on symmetric symbols after studying a variety of approaches for suppressing ICI. Different from the typical self-cancellation algorithm which modulates the opposite symbols onto adjacent subcarriers, the new algorithm modulates onto the symmetric subcarriers. At the receiver, by subtracting the corresponding demodulated symbols, the interferences are weakened,

so it can achieve the purpose of suppressing intercarrier interference.

2. Analysis of ICI and Self-Cancellation Algorithm Principles



2.1. OFDM System



The structure of the OFDM system has been shown in figure 1. In the OFDM system, the high speed data is divided into low bit rate streams through S/P conversion. Then the low bit data becomes the frequency-domain complex values after modulating by different types of digital modulations, such as phase shift keying (PSK) and quadrature amplitude modulation (QAM). Finally, the complex values modulate the subcarriers which can be realized by Inverse Fast Fourier Transform (IFFT). Because the subcarriers are orthogonal and transmitted parallel, the spectrums are overlap, which can make full use of the limited spectrum resource [8], which can be shown in figure 2. At the receiver, the demodulation is achieved by FFT.



Figure 2. Spectrums of the OFDM Aignal

2.2. ICI Mechanism

In this OFDM system, the number of subcarriers is N, the frequency-domain symbols u(i) modulated by different types of digital modulations are transformed into time-domain symbols x(n), after IFFT operation, they are sent to the channel. The cyclic prefix and multipath channel are not considered in this analysis in order to make it clear.

The symbols at the output of the IFFT can be expressed as:

$$x(n) = \frac{1}{N} \sum_{i=0}^{N-1} u(i) e^{j\frac{2\pi}{N}in}$$
(1)

In an OFDM system, if the frequency offset is caused by the frequency synchronization error between the transmitter and the receiver or the instability of the working frequency of the oscillator, then the system can be considered in the quasi static environment, and the effect of carrier frequency offset on the data transmitted is equivalent to that the channel is multiplied by a fixed attenuation. The attenuation coefficient is given by [9]:

$$h(n) = e^{j\frac{2\pi}{N}n\varepsilon}$$
(2)

Where ε is the normalized frequency offset, it is the ratio of the frequency offset to the subcarrier separation space. The following figure 3 shows the effect of the normalized frequency offset to the standard OFDM system. The digital modulation BPSK is considered and the number of the subcarriers N = 256.

It can be seen that as the normalized frequency offset ε increases, the system bit-error rate (BER) has also raised. For small frequency offsets $0 < \varepsilon < 0.1$, the BER increases slowly, and the system has not been affected much. Then for medium frequency offsets in the range $0.1 < \varepsilon < 0.3$, the values of BER increases sharply. When the frequency offsets up to the values $0.3 < \varepsilon < 0.5$, the BER has reached its maximum, and the system can not operate normally. So the frequency offsets have great influence on the OFDM system.



Figure 3. BER Performance for Standard OFDM System

At the receiver, it is assumed that the frequency and the timing are both synchronous accurately. w(n) is representative of the white Gaussian noise(AWGN), which applies to the most types of noise in wireless communication systems. The received time-domain symbols can be written as[10]: $s(n) = x(n) \times h(n) + w(n)$

$$=\frac{1}{N}\sum_{i=0}^{N-1}u(i)e^{j\frac{2\pi}{N}n(i+\varepsilon)}+w(n)$$
(3)

After FFT, the frequency-domain symbols on subcarrier l become:

$$y(l) = \sum_{n=0}^{N-1} s(n) e^{-j\frac{2\pi}{N}nl}$$

= $\sum_{n=0}^{N-1} [x(n) \times h(n) + w(n)] e^{-j\frac{2\pi}{N}nl}$
= $\frac{1}{N} \sum_{n=0}^{N-1} \sum_{i=0}^{N-1} u(i) e^{j\frac{2\pi}{N}(i+\varepsilon-l)n} + W(l)$
 $l = 0, 1, ..., N-1$
(4)

Where W(l) is the white Gaussian noise in frequency-domain and it can be represented as:

$$W(l) = \sum_{n=0}^{N-1} w(n) e^{-j\frac{2\pi}{N}nl}$$
(5)

In order to simplify analysis, we introduce the following formula:

$$\sum_{n=0}^{N-1} e^{j2n\phi} = \frac{1 - e^{j2N\phi}}{1 - e^{j2\phi}} = \frac{\sin N\phi}{\sin \phi} e^{j(N-1)\phi}$$
(6)

Then the equation (4) can be further derived by (6), which gives:

$$y(l) = u(l)I(0) + \sum_{i=0, i \neq l}^{N-1} u(i)I(i-l) + W(l)$$
(7)

Where I(i-l) can be expressed as:

$$I(i-l) = \frac{\sin[\pi(i+\varepsilon-l)]}{N\sin[\pi(i+\varepsilon-l)/N]} \exp(j\pi(i+\varepsilon-l)(1-1/N))$$
(8)

The first term of (7) is the useful signal, and the second is interference signal. I(i-l) is considered as the ICI coefficient between *i* th and *l* th subcarriers. The self-cancellation algorithm is to make its value as small as possible, so as to reduce the influence of the interference signal. When there is no frequency error, that is the normalized frequency offset $\varepsilon = 0$, I(0) gets its maximum value I(0)=1. But with ε increasing, I(0) decreases gradually, the coefficient of interference signal increases. Figure 4 shows this point.



Figure 4. ICI Coefficient for Standard OFDM System

$(N = 16, l = 0, \varepsilon = 0.05, 0.2, 0.3)$

In order to quantitatively study the effects of the interference signal on the system, the carrier-to-interference ratio (CIR) is used to be an indicator of the ICI power level, which can be defined as the ratio of the useful signal power and the interference signal power in the system. Assuming that the transmitted data symbols have zero mean and are independent, and the additive white Gaussian noise is omitted, so the CIR can be represented as:

$$CIR = \frac{|I(l)|^{2}}{\sum_{i=0, i \neq l}^{N-1} |I(i-l)|^{2}} = \frac{|I(0)|^{2}}{\sum_{i=1}^{N-1} |I(i)|^{2}}$$
(9)

Figure 5 shows the performance of the CIR for standard OFDM system, where the number of the subcarriers N = 256. The simulation result suggests that with the normalized frequency offset ε increasing, the values of the CIR have decreased. That is, the power of the useful signal reduces and the interference signal increases. It also can be learned that though the CIR is the function of N and ε , the values of CIR mainly depend on the normalized frequency offset ε , but varies very little because of the N.



Figure 5. CIR for Standard OFDM System

2.3. ICI Self-Cancellation Algorithm

In the OFDM system, ICI self-cancellation algorithm can be divided into two parts: ICI self-cancellation modulation and ICI self-cancellation demodulation. By the self-cancellation modulation, a pair of frequency-domain symbols which has the same values but opposite is modulated onto two adjacent subcarriers. At the receiver, the self-cancellation demodulation makes the received symbols a linear combination to further reduce the subcarrier interference coefficient. Then the ICI self-cancellation coding is extended in [11], where the frequency-domain coding polynomial $P(D) = (1-D)^{L-1}$ is proposed for modulating and demodulating symbols at the transmitter and the receiver. This coding algorithm requires the weighting coefficients of the symbols which for the modulation, the weighting coefficients should satisfy the polynomial $P(D) = (1-D)^{2(L-1)}$, where L is the number of the adjacent position, D represents the unit delay in the discrete frequency-domain. The typical ICI self-cancellation algorithm will be analyzed as following:

At the transmitter, the N/2 frequency-domain symbols and their corresponding opposite symbols are all mapped onto the adjacent subcarriers, that is : u(1) = -u(0), u(3) = -u(2), ..., u(N-1) = -u(N-2).

At the receiver, the received symbol on l th subcarrier is:

$$y(l) = \sum_{\substack{i=0\\i=even}}^{N-1} u(i) [I(i-l) - I(i+1-l)] + W(l)$$
(10)

and on l+1 th subcarrier is given by:

$$y(l+1) = \sum_{\substack{i=0\\i=even}}^{N-1} u(i)[I(i-l-1) - I(i-l)] + W(l+1)$$
(11)

After the self-cancellation demodulation, the symbol can be written as:

$$y'(l) = y(l) - y(l+1)$$

= $\sum_{\substack{i=0\\i=even}}^{N-1} u(i)[-I(i+l-1) + 2I(i-l) - I(i-l-1)] + W(l) - W(l+1)$ (12)

In this case, the ICI coefficient becomes^[12]:

$$I'(i-l) = -I(i+l-1) + 2I(i-l) - I(i-l-1)$$
(13)

The corresponding CIR then can be represented as:

$$CIR' = \frac{\left|2I(0) - I(-1) - I(1)\right|^2}{\sum_{\substack{i=2\\i=even}}^{N-1} \left|2I(i) - I(i-1) - I(i+1)\right|^2}$$
(14)

The main idea of the self-cancellation algorithm is that the interference coefficient changes a little between the adjacent subcarriers, thus the use of its similarity can decrease the influence of the ICI. The paper [5] has already simulated and verified that in OFDM system, by using the ICI self-cancellation algorithm, the ICI coefficient has been significantly reduced. The shortcoming of this algorithm is the utilization factor of frequency band has also been reduced, but it can be compensated through improving the

order number of the modulation.

3. Improved Self-Cancellation Algorithm

Figure 3 shows the similar ICI coefficients exist not only between the adjacent subcarriers but also the subcarriers which are symmetric by the central position of the subcarriers. Based on this point, this paper proposes an improved algorithm, and the main process is to mapping the data symbols which have the same values but are opposite onto the symmetric subcarriers instead of the adjacent subcarriers. Then the transmitted symbols are: u(N-1) = -u(0), u(N-2) = -u(1), ..., u(N/2) = -u(N/2-1).

At the receiver, after FFT, the received symbol on subcarrier l becomes (the white Gaussian noise is not considered for the simple analysis):

$$y'(l) = \sum_{l=0}^{N/2-1} u(l) [I(i-l) - I(N-i-l-1)]$$
(15)

and on subcarrier (N-1-l) is:

$$y'(N-1-l) = \sum_{l=0}^{N/2-1} u(l)[I(i-N+l+1) - I(l-i)]$$
(16)

Similar with the typical ICI self-cancellation algorithm, the self-cancellation demodulation can be expressed as:

$$y''(0) = y'(N-1) - y'(0), y''(1) = y'(N-2) - y'(2),...,$$

$$y''(N/2) = y'(N/2-1) - y'(N/2)$$

(17)

Then the finally received symbol is:

$$y''(l) = y'(N-1-l) - y'(l)$$

=
$$\sum_{i=0}^{N/2-1} u(l)[I(i-N+l+1) - I(l-i) - I(i-l) + I(N-i-l-1)]$$
(18)

In such case, the ICI coefficient becomes:

$$I''(i-l) = I(i-N+l+1) - I(l-i) - I(i-l) + I(N-i-l-1)$$
(19)

The corresponding CIR then can be derived as:

$$CIR'' = \frac{\left|I(1-N) - 2I(0) + I(N-1)\right|^2}{\sum_{i=1}^{N/2^{-1}} \left|I(i-N-1) - S(-i) - S(i) + S(N-i-1)\right|^2}$$
(20)

4. Simulations

The following simulations give the comparisons of the ICI coefficient for the new algorithm, standard OFDM system, and typical ICI self-cancellation algorithm. Where the numbers of the subcarriers are 64 and 256, the values of the normalized frequency offset are 0.05, 0.2, and 0.3.



Figure 6. ICI Comparison for the New and Typical Algorithm

10⁰ standard OFDM system - ICI self-cancellation proposed method 10⁻² 10 ||(i-l)| 10⁻⁶ 10-8 250 50 100 150 200 0 k

 $(N = 64, l = 0, \varepsilon = 0.3)$

Figure 7. ICI Comparison for the New and Typical Algorithm

 $(N = 256, l = 0, \varepsilon = 0.3)$

Figure 6 and Figure 7 show that in the case of $\varepsilon = 0.3$, the proposed algorithm compared to the ICI self-cancellation algorithm can further reduce the ICI interference coefficient, and with the increase of the number of carriers, the ICI interference coefficient also decreases gradually. But for the number of carriers, more is not necessarily better; it causes the increasing of the system bit-error rate (BER). So the appropriate number of subcarriers should be chosen according to the different channel conditions.



Figure 8. ICI Comparison for the New and Typical Algorithm

 $(N = 64, l = 0, \varepsilon = 0.2)$



Figure 9. ICI Comparison for the New and Typical Algorithm

 $(N = 256, l = 0, \varepsilon = 0.2)$

Figure 8 and Figure 9 show that the proposed algorithm can also reduce the ICI interference coefficient and the coefficient is smaller than $\varepsilon = 0.3$ in the case of $\varepsilon = 0.2$. Continually decreasing ε , the simulation results are shown in Figure 10 and Figure 11 where the ICI interference coefficient has been greatly diminished.



Figure 10. ICI Comparison for the New and Typical Algorithm



 $(N = 64, l = 0, \varepsilon = 0.05)$

Figure 11. ICI Comparison for the New and Typical Algorithm

 $(N = 256, l = 0, \varepsilon = 0.05)$

The simulations show for the most of i-l, the ICI coefficients derived by using the new algorithm proposed in this paper are smaller than that of the typical ICI self-cancellation algorithm. Besides, the formula (15) only takes half i values, which causes the number of the interference signals to half. Then either the number of the ICI signals or the values are much smaller than those in (7). When the normalized frequency offset is small, the proposed algorithm greatly reduces the ICI interference coefficient, but with the increasing of ε , the coefficient gradually closes to the ICI self-cancellation algorithm.

In order to analyze the improvement of the OFDM system performance by using the

new algorithm, the CIR simulation result is given. The typical ICI self-cancellation algorithm and the standard OFDM system have been considered for comparisons. In the simulation, there are 256 subcarriers and the normalized frequency offset ranges from 0 to 0.3. Figure 12 shows in the case of small ε , CIR has been significantly improved by using the new algorithm.



Figure 12. CIR Comparison for the New and Typical Algorithm

5. Conclusions

In OFDM system, the frequency offset is caused by the time-varying wireless channel and Doppler frequency shifts. Simulations show that the coefficient of interference signal increases, the CIR decreases and the system bit-error rate also rises with the normalized frequency offset ε increasing.

The paper analyzes the mechanism of the ICI and then proposes a new self-cancellation algorithm based on the symmetric symbols. The new algorithm based on symmetric symbols uses the similarity of the ICI coefficients on the symmetric subcarriers which are modulated by transmitted symbols to reduce the interference signals further. The simulations have proved that the new algorithm based on symmetric symbols has the smaller ICI coefficient compared to typical self-cancellation algorithm under the same frequency offset. The new algorithm based on symmetric symbols also improves the CIR performance, and the maximum improvement can reach 30 dB.

The new algorithm based on symmetric symbols maps the data symbols which have the same values but opposite onto the symmetric subcarriers, and the typical algorithm maps data symbols onto the adjacent subcarriers, so the new algorithm has the same complexity with the typical algorithm.

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