

Improved Computing Method of Mutual Information in Medical Image Registration

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Abstract

Mutual information based medical image registration is one of the most common registration method in registration domain currently. This paper studies the mutual information of the calculation process for mutual information calculation speed is slow, low efficiency of registration, registration can not meet real-time requirements, an improved gray compression algorithm, by reducing the gray-scale of images to improve image the registration efficiency and gradient similarity combined with mutual information as a new registration measure, so that the registration in improving efficiency while meeting the requirement of registration accuracy. The comparative experiments this article made show that, under the premise of registration accuracy to ensure that the method can significantly improve the speed of image registration.

Keywords: *Gray-scale compression; Medical image registration; Mutual information;*

1. Introduction

In medical diagnosis, planning therapy, guidance therapy and treatment disease surveillance, Medical images are playing an increasingly important role [1]. Medical image processing technology has been extensively and deep researched by the researchers. Among them, the research of medical image registration has made significant progress [2]. Medical image registration refers to make one medical image to match another medical image at the corresponding point by some space transformation [3]. Nowadays, we have more kinds of medical image and images of different modal have their own characteristics. Such as, CT and MRI provide anatomical information of organ at high spatial resolution and PET and SPECT provide metabolic function information of organ at low spatial resolution. In order to provide enough information for the doctor, these images of different mode need to be fused. And the image registration technology is the basis of image fusion [4].

In recent years, the study of the medical image registration is more in-depth, detailed and systematic. Applying the mutual information technology to image registration has become the one of research hotspots in the field of medical image registration [5-6]. Since the end of last century, the researchers studied the applications of information theory to medical image registration, this way is completely registration on the basis of fully automatic, need not treat do some complicated registration image preprocessing, and the registration method is quite high. Among them, Wells [7] and Collignon [8] were the first to put forward applying the mutual information method to the medical image registration. Plum [9] introduced space information on the basis of mutual information to improve the registration accuracy. After that, researchers replaced the normalized mutual information to the maximum mutual information as the similarity measure, to improve the affect of two images' coverage degree [10-11].

In this paper, we research the algorithm of medical image registration based on mutual information. And we found in the process of floating image transformation the mutual information values are requires to calculate largely and continually. Thus, it leads to lower registration efficiency. Aiming at this defect, we propose a new algorithm for the calculation of mutual information. The algorithm can speed up the calculation of mutual information and improve the efficiency of image registration. Considering the algorithm would affect the accuracy of registration, we introduce the gradient measurement on the basis of the mutual information. By combining the mutual information measurement and gradient, the speed of registration is improved, at the same time, the accuracy of registration is guaranteed.

2. The Principle of Medical Image Registration

Mutual information is a basic concept of information theory. It is generally used to represent the statistical correlation between the two systems, or the amount of information that a system contains another system. The basis of applying the concept of mutual information to medical image registration is: when the two medical images have been achieved registration, the mutual information of the two images reach to be maximized.

The value of mutual information is represented by entropy. We use A and B to represent two images respectively. And the mutual information of A and B is defined as follows:

$$I(A, B) = \begin{cases} H(A) + H(B) - H(A, B) \\ H(A) - H(A | B) \\ H(B) - H(B | A) \end{cases} \quad (1)$$

$H(A)$, $H(B)$ represent entropy of A and B respectively. $H(A, B)$ represents their combination entropy. $H(A | B)$ and $H(B | A)$ represent respectively conditional entropy of A when B is known and conditional entropy of B when A is known.

The above all kinds of entropy can be expressed as respectively.

$$H(A) = -\sum_a p_A(a) \log_2 p_A(a) \quad (2)$$

$$H(B) = -\sum_b p_B(b) \log_2 p_B(b) \quad (3)$$

$$H(A, B) = -\sum_{a,b} p_{AB}(a, b) \log_2 p_{AB}(a, b) \quad (4)$$

$p_A(a)$, $p_B(b)$ represent marginal probability distribution of A and B respectively. They can be determined by the normalized histogram of the image. $p_{A,B}(a, b)$ represent joint probability distribution and it can be determined by the normalized joint histogram.

We put formula (1) in formula (2) (3) (4), then we can get the computational formula of mutual information:

$$I(A, B) = -\sum_a p_A(a) \log_2 p_A(a) - \sum_b p_B(b) \log_2 p_B(b) + \sum_{a,b} p_{AB}(a, b) \log_2 p_{AB}(a, b) \quad (5)$$

From the definition of the normalized histogram and the joint histogram normalization, we can get the marginal probability distribution of A and B is represented by their joint probability distribution:

$$p_A(a) = \sum_b p_{AB}(a, b) \quad (6)$$

$$p_B(b) = \sum_a p_{AB}(a, b) \quad (7)$$

We put formula (6) (7) in formula (5), then we can get the more simple calculation formula of mutual information:

$$I(A, B) = \sum_{a,b} p_{AB}(a, b) \log_2 \frac{p_{AB}(a, b)}{p_A(a) \times p_B(b)} \quad (8)$$

The registration based on the maximum mutual information method depends on the information of image itself only. It need not any hypothesis or prior medical knowledge, and also need not to pre-process the image such as feature point extraction and organization classification. And it can reach registration of high precision of sub-pixel level. So it is widely used in CT/MR, PET/MR and so on in the medical image registration.

3. The Improved Computational Algorithm of Mutual Information

3.1. Histogram Method to Calculate the Mutual information

From the principle of mutual information above, we can get that: the computation of mutual information is, in fact, the image grayscale statistics and calculation. Gray-level histogram or histogram for short shows an image in the overall distribution of gray scale. It is often used to calculate the probability distribution of image gray level. The process that using histogram to calculate the probability distribution of the image process is as follows:

Step 1: Calculate the image edge of joint probability distribution and probability distribution of A and B.

Set the histogram of A and B as $h_A(a)$ and $h_B(a)$ respectively. Then the corresponding probability distribution respectively is as follows.

$$p_A(a) = \frac{h(a)}{\sum_A h(a)} \quad (9)$$

$$p_B(b) = \frac{h(b)}{\sum_B h(b)} \quad (10)$$

Set the joint histogram of the image as $h_{AB}(a,b)$. Then joint probability distribution is as follows:

$$p_{AB}(a,b) = \frac{h_{AB}(a,b)}{\sum_{a,b} h_{AB}(a,b)} \quad (11)$$

Step 2: According to the formula (4) and formula (11) to calculate the joint entropy of the image A and B refer as $H(A,B)$.

Step 3: According to the formula (2) (3) (9) (10) to calculate the edge-entropy of the image A and B denoted by $H(A)$ and $H(B)$ respectively.

Step 4: Then according to the formula (1) to calculate the mutual information of image A and B refer as $I(A,B)$.

3.2. The Improved Gray-Scale Compression Algorithms

According to the calculation process of mutual information above, the number of calculation and calculation of the mutual information is closely related to the image grayscale. Mutual information is actually gray statistics and calculation process. With the decrease of grayscale, image information is reduced. And the registration precision is affected. Therefore, we can speed the calculation of mutual information through gray-scale compression.

When we compress the image grayscale, the traditional calculation method of gray-scale compression is that proportionately mapping each pixel gray value of image to each pixel of a new image. Its computation formula can be represented as follows.

$$G_n(i, j) = G_o(i, j)/a \quad (12)$$

i, j represent the horizontal ordinate of pixel of image. G_o represents gray value of different pixel before gray-scale compression. G_n represents gray value of different pixel after gray-scale compression. a represents the ratio of gray-scale compression.

When applying this method to compress gray-scale, the compressed image would lose lots of information based on the actual result, especially when the image pixels are more similar. The reason is that this method doesn't take into account that the different image pixel gray value of each has a different distribution.

Aiming at the existing problem of the traditional linear compression method, we propose an improved gray scale compression algorithm. This algorithm can dynamically map the grey value to the new image according to the different proportion of each pixel in the image grey value. This method can make the pixels which have a large proportion had more decision-making power. The compressed image which got by this method has more highlighted details than the one which got by traditional method. It can improve the accuracy of registration. The implementation of the algorithm process is as follows.

- (1) Initialization: calculate the gray statistical histogram gray image.
- (2) Sort of pixel: grey value of gray statistical histogram sorting according to sizes.
- (3) Evaluation of gray value range: set n as the grayscale of new image, and get the pixel values of the number of top $n-1$. Set them as $G_1, G_2, G_3, \dots, G_{n-2}, G_{n-1}$ respectively. And the compressed image grey value of the interval is as follows:

$$[0, G_1], [G_1, G_2], [G_2, G_3] \dots [G_{n-2}, G_{n-1}], [G_{n-1}, G_n].$$

(4) Grey value mapping: map each pixel to the new image according to the original image in different range respectively. Set the grey value of the mapping to the new image as consecutive natural number: $0,1,2,\dots,n-1$.

3.3. The Result of Gray-Scale Compression

This experiment adopts the image source for Simulated Brain Database. We download the two formats of MRI brain images from this research institution. Among them, the image format of figure 1 is MRI-T1. The slice thickness of the image is 1 mm and the noise is 3%. The intensity non-uniformity of the image is 20% and the gray-scale value is 256. By using the gray-scale compression algorithms to process the image, its gray-scale value turn into 128 as figure 2. The image format of figure 3 is MRI- PD. The slice thickness of the image is 1 mm and the noise is 3%. The intensity non-uniformity of the image is 20% and the gray-scale value is 256. By using the gray-scale compression algorithms to process the image, the gray-scale value turn into 128 shown as figure 4.

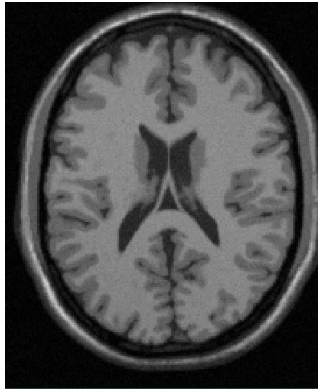


Figure1. 256 Gray-Level T1

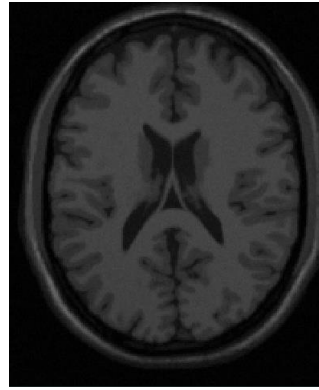


Figure2. 128 Gray-Level T1

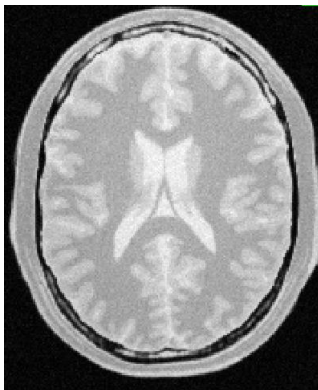


Figure3. 256 Gray-Level PD

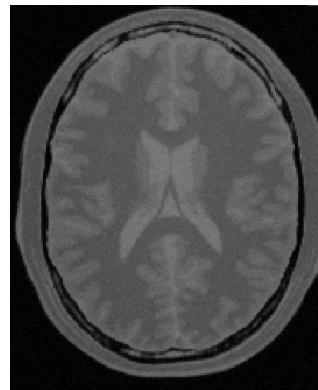


Figure4. 128 Gray-Level PD

4. Combining the Gradient Similarity with Mutual Information as the Registration Measure

After compressing the gray-scale of the image, although we can speed up image registration, image registration precision also is affected. In the registration measure, combining the space gradient information of images and mutual information of images can improve the accuracy of registration [12].

The gray scale, resolution and size of the same organ gained by different imaging mode will vary. But the boundary of the same organ is certain which will not obviously change with different imaging mode. Most of the corresponding pixel gradient on the same organ border is the same or opposite direction. Set one point of the reference images of A as a. Its corresponding point in floating image B is b. Set the gradient vector of point a and point b as $grad\ a$ and $grad\ b$. Then the included angle of the two points is as follows.

$$\alpha_{a,b} = \arccos \frac{grad\ a \bullet grad\ b}{|grad\ a| |grad\ b|} \quad (13)$$

In order to detect the direction of the gradient similarity, we define the gradient direction similarity measure as follows.

$$\omega(\alpha_{a,b}) = \frac{\cos(2\alpha_{a,b}) + 1}{2} \quad (14)$$

When the images are aligned, the gradient direction similarity of the corresponding point pair on the same organ boundary is 1.

Set the ratio of gradient modulus value of point pair as the modulus value similarity

$$\text{measure: } g_{a,b} = \begin{cases} \frac{\min(|grad\ a|, |grad\ b|)}{\max(|grad\ a|, |grad\ b|)} & \max(|grad\ a|, |grad\ b|) \neq 0 \\ 1 & \max(|grad\ a|, |grad\ b|) = 0 \end{cases} \quad (15)$$

In the multimode image registration, in view of the same anatomical structure, there is large difference between gray level information of different imaging models. So the same anatomical boundaries in a modal image have larger gradient amplitude and its gradient amplitude on other modal image may be smaller. The measure function is multiplied by the gradient in the direction of gradient amplitude of smaller value measure to calculate the gradient factor. The gradient of the two image similarity S can be expressed as follows.

$$S(A, B) = \sum_{(a,b) \in (A \cap B)} \omega(\alpha_{a,b}) g_{a,b} \quad (16)$$

By combining with the calculation formula of mutual information, we can get the mutual information measure of gradient similarity.

$$I_{GSI}(A, B) = I(A, B) \bullet S(A, B) \quad (17)$$

5. Experiments and Analysis

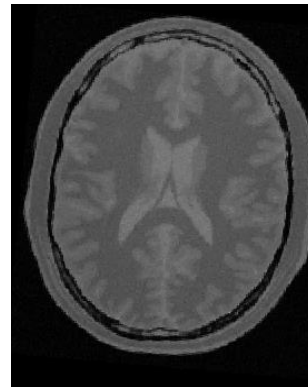
To analyze the improved gray-scale compression algorithm, we carried out the registration experiment according to the mutual information of gradient similarity measure of the efficiency and precision of image registration. The hardware environment of the experiment is that CPU is Intel-Core-i3-M350 (2.27 GHz), the internal storage is 2GB DDR3 SDRAM and the graphics card is ATI/AMD Mobility Radeon HD 4570 with video memory of 512MB and frequency of 220.0MHz. We used MATLAB R2010a to precede the related algorithm application programming.

The images which our experiment adopted are the previous gray-scale compression images. We put MRI-T1 as the reference image that its spatial position is invariable. And we put MRI-PD as the floating Image. We respectively translated the spatial parameter of

images of 256 gray-level and 128 gray-level. The two images are clockwise rotated 5 degree angle and X and Y directions are not moved. The transformed images are as follows.



**Figure5. Transformed
256 Gray-Level**



**Figure6. Transformed
128 Gray-Level**

The experiment was divided into two parts. The first part is the registration experiment of the original image of 256 gray-level. Registration optimization algorithm adopted the Powell algorithm and mutual information measure is the mutual information which not combined with the gradient information. Then we used the 128 gray-level image which was compressed by the improved gray algorithm to conduct the experiment. Registration optimization algorithm adopted the Powell algorithm and mutual information measure is the mutual information which combined with the gradient information. The second part is the contrast experiment. Experiment adopted the image which is compressed by using the traditional gray algorithm and the other were in accordance with the first part. Images of the two parts of the experiment of the mutual information calculated respectively and the optimization of the experimental results as shown in table 1 and table 2.

Table 1. The Registration Data Contrast Based on Traditional Gray-Scale Compression

Gray-level	Mutual information measure	Registration parameters (x,y,angle)	Time-consuming/s
256	0.4835	0,0,5	76
128	0.4001	0,0,4	30

Table 2. The Registration Data Contrast Based on Improved Gray-Scale Compression

Gray-level	Mutual information measure	Registration parameters (x,y,angle)	Time-consuming/s
256	0.4835	0,0,5	76
128	0.4126	0,0,5	48

By the results of comparison experiment we can find that when we used the method of the traditional gray-scale compression, the image has less number gray level and it contains less information. Although the speed of registration is fast, the registration parameters are inconsistent with the actual situation. When we used the method of the improved gray-scale compression, because we used the mutual information which combined with the gradient information as its registration measure, the registration parameters are accurate and the speed of registration is also fast.

6. Conclusion

This paper introduces an improved gray-scale compression algorithm to accelerate the calculation of mutual information. And we combine the gradient similarity with mutual information as the registration measure. The experimental results show that the method can greatly reduce the computational time of mutual information and improve the effect of the efficiency of the registration. And this improved algorithm also can make the registration precision to be not affected.

Though the experiment had a good experiment effect, but our registration search strategy is the traditional Powell algorithm. If we can improve the registration search algorithm, the registration result will be better. So next step we can combine the method this paper propose with registration search optimization algorithm. And it will realize that the registration is in real time on the premise of registration accuracy.

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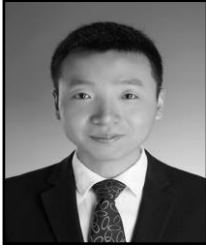
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