International Journal of Signal Processing, Image Processing and Pattern Recognition
Vol.9, No.4 (2016), pp.27-36
http://dx.doi.org/10.14257/ijsip.2016.9.4.03

Detection Method of Weak Low-Frequency Electromagnetic Signal Based on Multi-layer Autocorrelation

Zheng Dou[1], Yu Tao[1], Yun Lin[1,2*1], Zhiqiang Wu[2], Dong Li[2] and Zhiping Zhang[2]

1.College of Information and Communication Engineering
Harbin Engineering University, Harbin, China
linyun@hrbeu.edu.cn
2.Department of Electrical Engineering
Wright State University, Dayton, Ohio, U.S.
zhiqiang.wu@wright.edu

Abstract

Low frequency electromagnetic signals play an important role in the detection of underground electrical structure. After propagation of long distance, signal will be attenuated to a certain extent and probably buried in noise, make it difficult to extract. In this paper, the method of multi-layer autocorrelation will be used to detect and extract the weak low frequency electromagnetic signal, and its performance will be compared with the narrow-band filter and the method of synchronous accumulation. Simulation results show that the correlation coefficient of signal obtained by multi-layer autocorrelation and source signal can achieve more than 0.8 when SNR is -18dB. However, the value of narrow-band filter and synchronous accumulation is lower than that mentioned above when SNR is only -14dB. Experimental verifies that multi-layer autocorrelation method can suppress noise more effectively and detect weak signals more accurately.

Keywords: *Weak signal detetion; Multi-layer autocorrelation; Narrow-band filter; Synchronous accumulation*

1. Introduction

Low frequency electromagnetic signals, which can penetrate strata of tens kilometers , have a unique advantage in the detection of crust structure. Through the highly accurate observation of electromagnetic signal on the earth's surface, we can detect the electrical structure of underground medium that plays an important role in seismic monitoring and forecast. The current ELF and interact resistivity technology mainly use artificial source and signal is high strength and wide coverage, as well as high stability frequency, so the two kind of electromagnetic observation method are potential and promising developments.

The ELF and interact resistivity technology mainly use low frequency AC signal source to transmit sine wave of single frequency. Although the transmitting power of signal source is great, after transmission of long distance, signal will be attenuated to a certain extent and probably buried in noise. Therefore, to extract useful weak signal is of great significance.

Weak signal detection method of traditional likelihood ratio has significant limitations. First of all, the object to detect must satisfy the assumption of Gaussian conditions, only in that case can we divide the observation space according to the criterion, and then make a judgment [3]. Second, if the condition of signal-to-noise ratio is very poor, the detection performance of system will drop sharply, makes it difficult to extract the weak signals. Compared with traditional method, the multi-layer autocorrelation detection technology

* Corresponding Author

ISSN: 2005-4254 IJSIP

not only has the characteristics of simple theory derivation and clear physical meanings, but also has a very high accuracy of frequency measurement. Meanwhile, the multi-layer autocorrelation detection takes advantage of the periodic characteristic of the signal itself to filter out the noise, so priori knowledge need not to be known can we extract the useful signal, which is more beneficial for the application in the engineering.

2. The Basic Principle of Weak Signal Extraction

2.1 Multi-Layer Correlation

When the external noise is a non-Gaussian white noise, the correlation detection method can be utilized to extract the periodic signal. The principle of correlation detection is to find the difference between the informative signals and noises, and use the correlation of the signal and the randomness of the noise to remove the noise[6]. The principle diagram of the autocorrelation method is as follow.

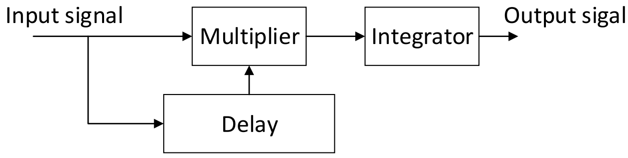


Figure 1. The Principle Diagram of the Autocorrelation Method

Assume that the input signal is

$$x_i(t) = s_i(t) + n_i(t) \tag{1}$$

$s_i(t)$ is the informative signal, $n_i(t)$ is interference, then we can get the autocorrelation function through time delay, multiplied and integral operation of input signal.

$$R_{xx}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x_i(t)x_i(t-\tau)dt = R_{ss}(\tau) + R_{sn}(\tau) + R_{ns}(\tau) + R_{nn}(\tau) \tag{2}$$

As the informative signal and noise are not related, so it can be seen from the nature of the correlation function that $R_{sn}(\tau)$ and $R_{ns}(\tau)$ tend to be zero, so

$$R_{xx}(t) = R_{ss}(\tau) + R_{nn}(\tau) \tag{3}$$

If the delay is long enough, the average value of noise will be zero, means that $R_{nn}(\tau)$ is zero, then

$$R_{xx}(t) = R_{ss}(\tau) \tag{4}$$

The formula above is the autocorrelation function of informative signal. $R_{ss}(\tau)$ contains certain frequency and amplitude information of informative signal $s_i(t)$, so we can extract the informative signal by means of $R_{ss}(\tau)$.

Then we take a concrete instance, assume that the input signal $s_i(t)$ is cosine signal, after adding it to noise, the total input signal $x(t)$ is

$$x(t)=s(t)+n(t)=Acos(\omega t+\varphi)+\ n(t) \tag{5}$$

$n(t)$ represents the noise, then its autocorrelation function is

$$R_{xx}(\tau)=\frac{A^2}{2}\cos(\omega\tau)+\frac{A^2}{2T}\int_0^T\cos\big(\omega(2t+\tau)+2\varphi\big)dt$$
$$+R_{nn}(\tau)+\frac{1}{T}\int_0^T n(t+\tau)dt\cdot\frac{1}{\tau}\int_0^T s(t)dt+\frac{1}{T}\int_0^T s(t+\tau)dt\cdot\frac{1}{T}\int_0^T n(t)dt \tag{6}$$

We make $y_1(t)$ represent $R_{xx}(\tau)$, and $n^{(1)}(t)$ represents the formula below

$$n^{(1)}(t)=\frac{A^2}{2T}\int_0^T\cos\big(\omega(2t+\tau)+2\varphi\big)dt+R_{nn}(\tau)$$
$$+\frac{1}{T}\int_0^T n(t+\tau)dt\cdot\frac{1}{\tau}\int_0^T s(t)dt+\frac{1}{T}\int_0^T s(t+\tau)dt\cdot\frac{1}{T}\int_0^T n(t)dt \tag{7}$$

$n^{(1)}(t)$ tends to zero in theory when T tends to infinite,

So the formula (6) can be rewritten as

$$y_1(t)=\frac{A^2}{2}cos(\omega t)+n^{(1)}(t) \tag{8}$$

Comparing Eq.(5) and Eq.(8) we can find that although the amplitude and phase of signal, as well as the function form of noise are changed, the frequency remains unchanged, and the variation of amplitude meets the analytical relationship, so we can get the frequency and amplitude characteristics of source signal by $R_{xx}(\tau)$, then recover the source signals. However, the delay τ is not infinite in the actual situation, causing that $R_{nn}(\tau)$ is not zero but a function of τ. In order to get a better detection performance, multi-layer autocorrelation method is put forward on the basis of it.

The basic principle of multi-layer autocorrelation and the autocorrelation detection is similar, the difference is that the output signals of multi-layer autocorrelation need to continue to do autocorrelation repeatedly, means that $y_1(t)$ is used as the input signal $y(t)$, and repeat the methods and procedures of autocorrelation detection. Moreover, the more the number of autocorrelation, the larger SNR improved, the stronger detection capabilities.

Similarly, after autocorrelation is repeated n times, the output signal is

$$y_n(t)=\frac{A^{2n}}{2^{2^n-1}}cos(\omega t)+n^{(n)}(t) \tag{9}$$

The frequency of output signal still remains unchanged with the input ones, and the variation of amplitude meets the analytical relationship, so we can recover the source signals by the output signals above.

In order to compare the detection performance, the brief theoretical analysis of filter and synchronous accumulation are given below.

2.2 Narrowband Filter and Synchronous Accumulation

The narrowband filter is applicable for the conditions that there is no overlap of spectrum between informative signal and noise. Its principle is to set the pass band of the filter that covers the bandwidth of the useful signal, and signals beyond the pass band will be filtered off. According to the different characteristics of signal, commonly used filter consists of low pass filter (LPF) and band pass filter (BPF).Low pass filter is generally used to signal whose frequency changes slowly, which can effectively restrain the high frequency noise, but the low-frequency noise can't be restrained. If the signal is the one with fixed frequency, bandpass filter can be used to inhibit noise at other frequencies out of passband. The narrower bandwidth, the better the effect.

The improvement of SNR is equal to the total bandwidth of white noise divided by the bandwidth of filtering system. Therefore, the narrower bandwidth of bandfilter, the better the effect.

In principle, using the filter with super narrow frequency band can obtain great effect, but the center frequency of filter must be fixed on the signal frequency, for the signal that cycle is not fixed or frequency is not absolute constant, the passband of filter cannot be too narrow, so the improvement of SNR cannot be so significant. However, correlation detection is equivalent to tracking filter, there is no such restrictions.

The narrowband filter is applicable for the conditions that there is no overlap of spectrum between informative signal and noise. Its principle is to set the pass band of the filter that covers the bandwidth of the useful signal, and signals beyond the pass band will be filtered off. According to the different characteristics of signal, commonly used filter consists of low pass filter (LPF) and band pass filter (BPF).Low pass filter is generally used to signal whose frequency changes slowly, which can effectively restrain the high frequency noise, but the low-frequency noise can't be restrained. If the signal is the one with fixed frequency, bandpass filter can be used to inhibit noise at other frequencies out of passband. The narrower bandwidth, the better the effect.

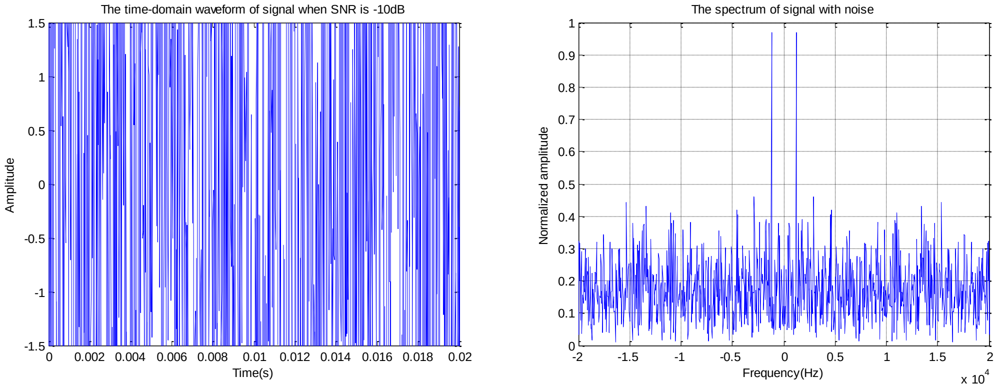
The improvement of SNR is equal to the total bandwidth of white noise divided by the bandwidth of filtering system. Therefore, the narrower bandwidth of bandfilter, the better the effect.

In principle, using the filter with super narrow frequency band can obtain great effect, but the center frequency of filter must be fixed on the signal frequency, for the signal that cycle is not fixed or frequency is not absolute constant, the passband of filter cannot be too narrow, so the improvement of SNR cannot be so significant. However, correlation detection is equivalent to tracking filter, there is no such restrictions.

When the spectrum of signal is particular wide or complicated, synchronous accumulation method can be used in signal detection. Synchronous accumulation is based on the stability of signal and stochastic of noise. For the aperiodic signal, repeatedly measured signal can be accumulatied at each corresponding time. Similarly, for the periodic signal, values in the same position of different period can be cumulated. Since the noise is random and mean zero, so after accumulation, the amplitude of informative signal will increase the number of times of accumulation, and the noise can be offset to some extent, then informative signal can be extracted. The more cumulative number, the obviously the SNR improve. However, improving the SNR of this method will increase the measuring time for the price, and make the actual operation more complex. Therefore, the synchronous accumulation is limited by accumulated times.

3. Simulations and Performance Analysis

Set the frequency of input source signal is 1.2kHz, then add Gauss white noise to the source signal, simulate under the condition of SNR is -10dB. The domain waveform and spectrum characteristics of source signal with noise are as follows.



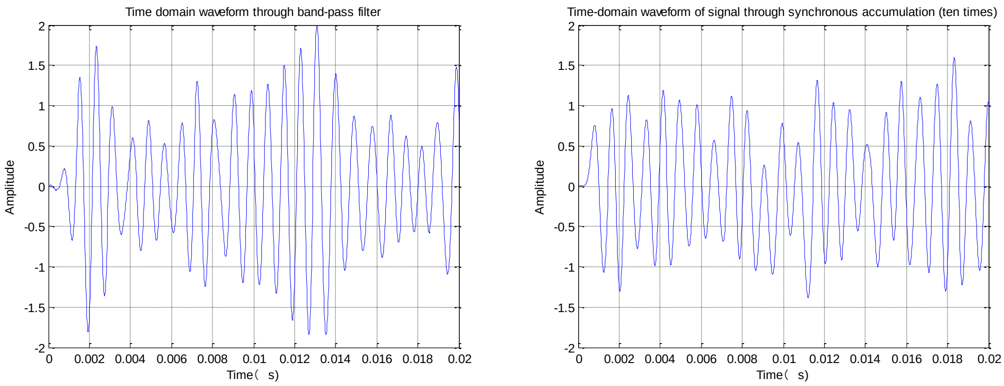
(a) (b)

Figure 2. The Domain Waveform and Spectrum of Source Signal with Noise

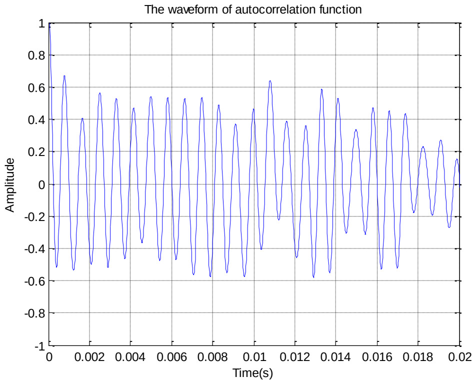
Using the three methods above to deal with the noise of weak signal respectively.

Simulation indicators are as follows. The center frequency of narrow band filter is 1.2kHz, bandwidth of 3dB is 500Hz. For the synchronous accumulation, the more cumulative number, the obviously the SNR improve, but the monitoring time is longer. Based on an overall consideration of various factors, the paper selects ten times of accumulative synchronous to simulate. Firstly, signal is input to a low-pass filter, accumulated ten times, then taking the average value of time domain. In order to compare and analyze the relationship between times of autocorrelation and extraction performance, we get once and times of waveform of autocorrelation respectively, and contrast the detection performance of them.

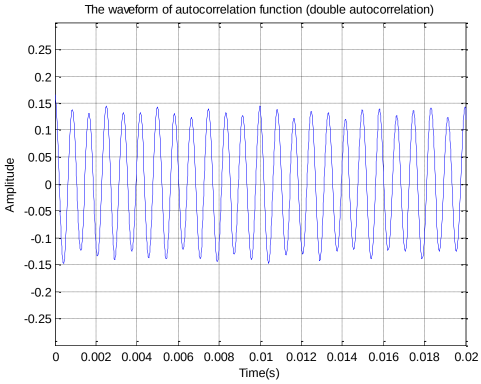
3.1 The Comparison of Time-Domain Waveform



(a) (b)



(c)

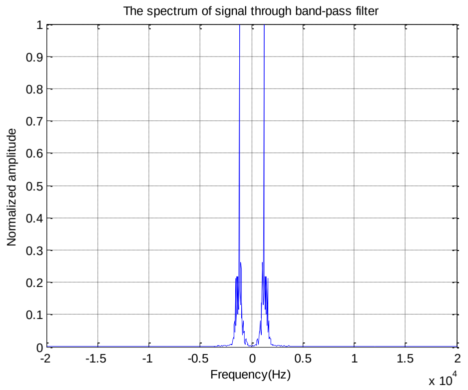


(d)

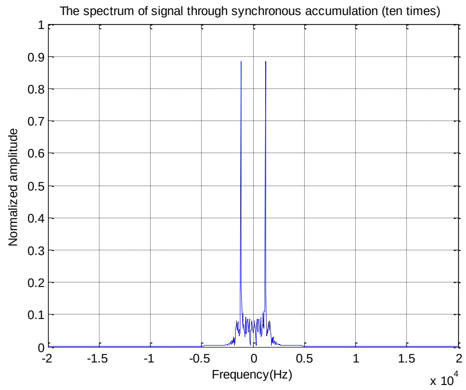
Figure 3. The Time-Domain Waveforms. (a) Narrow-Band Filter, (b) Synchronous Accumulation, (c) Autocorrelation, (d) Double Autocorrelation

Under the condition of -10dB SNR, Figure.(a) is the time domain waveform of signal through narrow-band filter. As we can see from the graph, the waveform of output signal became clear relative to the input ones, but the frequency and amplitude are not very stable. Fig.(b) is the time domain waveform of synchronous accumulation, compared with simple filtering, the performance of this method has certain improvement, the frequency of output signal is closer to the source signal. Figure.(c) is the autocorrelation function of input signal, and Figure.(d) is the function of double autocorrelation. From the theoretical analysis above can we know that the frequency of output signal remains unchanged with the input ones, and the variation of amplitude meets the analytical relationship, so source signal can be recovered by means of this characteristic. Specifically, the amplitude of input source signal is 1, so the amplitude of Figure.(c) is 0.5 in theory, similarly, Figure.(d) is 0.125, from that can we see that the performance of the autocorrelation method is proportional to the number of autocorrelation. The frequency and amplitude of output signal are very stable after double autocorrelation, we can conclude that the signal has been extracted completely.

3.2 The Comparison of Frequency Spectrum



(a)



(b)

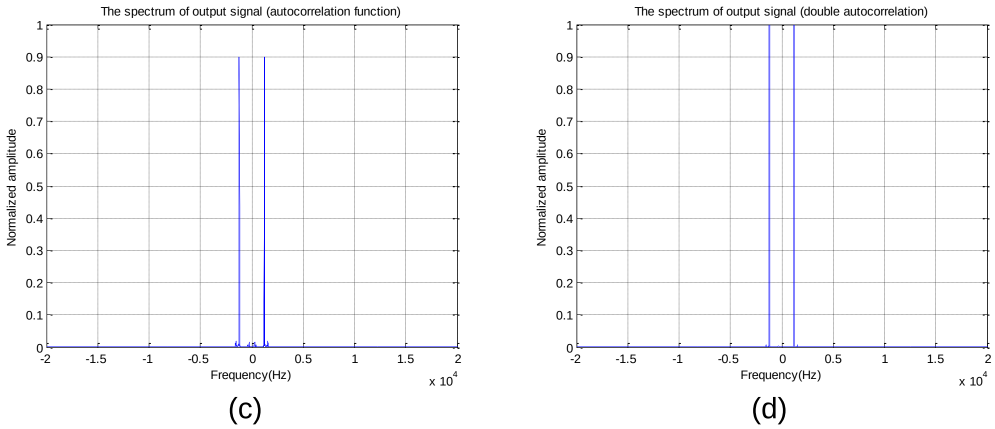


Figure 4. The Time-Domain Waveforms. (a) Narrow-Band Filter, (b) Synchronous Accumulation, (c) Autocorrelation, (d) Double Autocorrelation

Comparing with the spectrum of input signal, the noise spectrum of the output signal through narrow-band filter is filtered out to some extent, and the informative signal is preserved completely, SNR is improved. However, this method cannot effectively extract the useful signal when the spectrum of signal and noise are overlap, which makes it limited in engineering application. Figure.(b) is the spectrum of signal extracted by synchronous accumulation, as can be seen from the Figureure, after ten times of synchronous accumulation, the spectrum of useful signal is purer and noise is further suppressed. Since the cumulative times is limited by monitoring time, so we cannot infinitely increase it to improve the SNR. Figure.(c) is the signal spectrum of autocorrelation, Figure.(d) is the spectrum of double autocorrelation, when SNR is -10dB, signal extracted by autocorrelation still contains tiny amount of noise components, which is completely suppressed after processing of double autocorrelation, and the useful signal is completely preserved.

3.3 The Comparison of Correlation Coefficient

The correlation coefficient is an important index to judge the similarity degree of two different signals, so we can evaluate the performance of different algorithms by the magnitude of correlation coefficient between source signal and extracted signal. We carried out the signal simulation at -20dB to -10dB SNR, repeated extraction and took the average value to reduce the randomness brought by a single simulation. The simulation results are Table 1, and datas are plotted in Figure. 5..

Table 1. The Table of Correlation Coefficient with Source Signal

SNR（dB）	-20	-15	-10	-5	0	5	10
Narrow-band filter	0.5469	0.6962	0.7809	0.9386	0.9565	0.9861	0.9826
Synchronous accumulation	0.6809	0.7387	0.8907	0.9796	0.9917	0.9982	0.9988
Autocorrelation	/	0.6600	0.8004	0.8770	0.9028	0.9322	0.9670
Multi-layer autocorrelation	0.7335	0.8876	0.9333	0.9869	0.9958	0.9976	0.9896

Note: The slash means that current SNR has been lower than the limit of detection algorithm, correlation coefficient fluctuates greatly and cannot detect accurately.

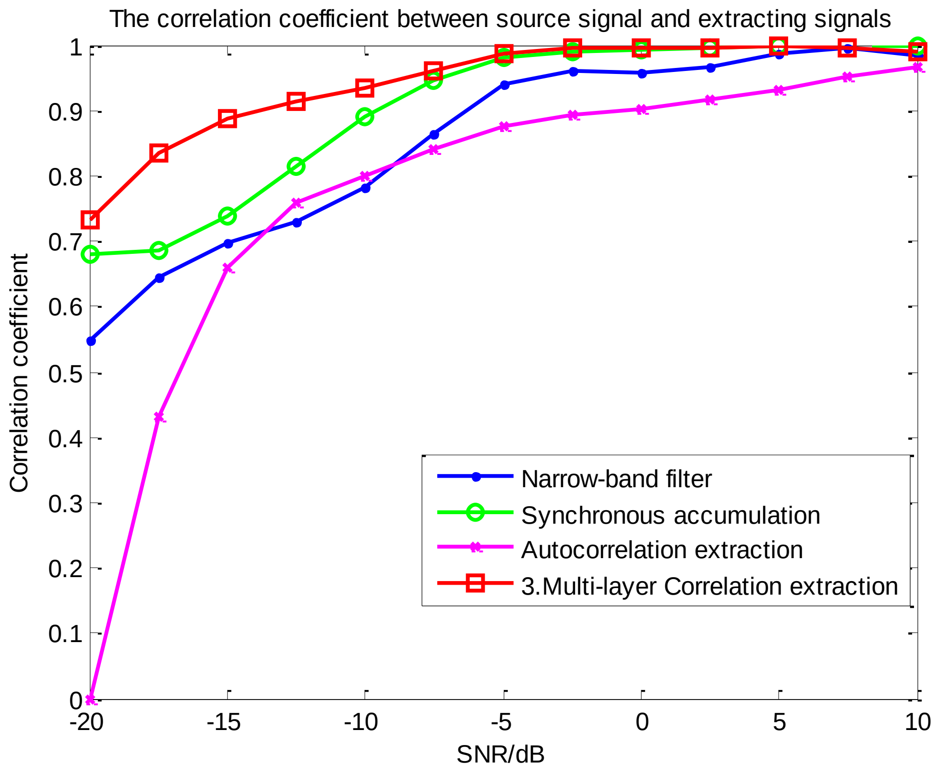


Figure 5. The Correlation Coefficient with Source Signal of Diffierent Methods

It can be seen from Table 1 and Figure.5 that the superiority of the performance is in turn as multi-layer autocorrelation, synchronous accumulation, narrow-band filtering and autocorrelation method when detecting weak sinusoidal signal under the condition of the SNR -20 dB~10 dB. At the same time, the autocorrelation method is highly affected by the SNR environment . The performance will significantly decline on the condition of low SNR and even cannot be measured when SNR is less than -15 dB, in this case the weak signal cannot be effectively extracted from noise. The detection performance of synchronous accumulation method changes slowly with the SNR alters. We need to increase the times of cumulation to improve the detection performance when the SNR is lower than -10 dB which is in price of sacrificing monitoring time. Single autocorrelation detection performance is poor and not suitable for weak signal detection under the condition of low SNR, however, multi-layer autocorrelation detection can significantly inhibited the noise and accurately extract the frequency of informative signal. Double autocorrelation method remains high correlation coefficient when the SNR is lower than -15 dB. The experimental results show that the extraction method of weak signal with multi-layer autocorrelation has a high detection performance and is suitable for weak signal detection under the condition of low SNR.

In theory, more than 3 times autocorrelation will get higher accuracy, but with the increasing times of autocorrelation, the rounding error in the process of program operation will have greater influence on the results, so we reduce the calculation accuracy to a certain extent. Therefore, choosing appropriate sampling frequency and sampling points, as well as taking the average value of several measurements is the best way to improve the accuracy of amplitude in the practical application.

4. Conclusions

Through the characteristic analysis of the noise which influences weak signal detection and in view of the randomness of noise and determinacy of signal, the narrow-band filter , the method of synchronous accumulation and multi-layer autocorrelation were put forward to be used in detection and extraction of weak single-frequency sinusoidal signal. The main indicators of simulation include the time-domain waveform, spectrum and the correlation coefficient with the source signal. Experiments show that the method of filter and autocorrelation detection are affected by SNR obviously and the minimum limit of the SNR is -15dB. The performance of multi-layer autocorrelation is the best, which provided with high accuracy in the frequency measurement and also can achieve the desired accuracy on the amplitude measurement through measuring average for times. The method can still effectively extract the weak signal under the condition of less than -20dB. The more autocorrelation times, the stronger the detection performance.

ELF is the electromagnetic observation method provided with the potential and future development currently in the study of earthquake monitoring and prediction, which can be accurately observed through surface to detect the electric property structure of underground medium and the chance of ionosphere. The transmitted power of ELF transmitter is pretty big, but it still would become small and submerged in strong background noise with the long-distance transmission. In order to accurately extract the amplitude and frequency of the sinusoidal signal information, we can use the research methods above for detection and suppressing noise.

Acknowledgements

This work was supported by the Key Development Program of Basic Research of China(JCKY2013604B001), the Nation Nature Science Foundation of China (No.61301095 and No.61201237), Nature Science Foundation of Heilongjiang Province of China (No. QC2012C069, F201408 and F201407) and the Fundamental Research Funds for the Central Universities (No. HEUCF1508).

Meantime, all the authors declare that there is no conflict of interests regarding the publication of this article.

References

[1] L. Jun and Z. Binzhen, "Weak Signal Detection Technology[M]", Beijing, Publishing House of Electronics Industry, **(2010)**, pp. 25.

[2] Z. Qiuzhan and H. Fengye, "Detection & of Weak Signal[M]", Beijing, Beijing Aeronautics and Astronautics Press, **(2011)**, pp. 108-109.

[3] L. Kun and S. Lixin, "Parameter analysis of weak signal measurement system based on Chaos Theory [J]", Journal of Harbin University of Science and Technology, no. 4, **(2007)**, pp. 66-69+73.

[4] M. CHEN and Z. LIU, "The Detection of Weak Sinusoidal Signal by Multi-Layer Auto Correlation, Noise and Vibration Control", no. 5, **(2011)**, pp. 28-30.

[5] L. Lizhong, "The Introduction of Detection in Weak Signal[M]", Beijing, China Metrology Publish House, **(2012)**, pp. 35.

[6] J. Tiejun, R. Lijun and Z.Aiming, "Coherent averaging weak signal detection method without synchronization signal [J]", Journal of Harbin University of Science and Technology, no. 3, **(2002)**, pp. 35-37.

[7] D. Banerjee, "PLL Performance,Simulation,and Design[M]", Dog Ear Publishig, **(2003)**, pp.1-158.

[8] B. G. SHERLOCK, Y. P. KAKAD, "Windowed discrete cosine and sine transforms for shifting data[J].Signal processing", vol. 81, no.7, **(2011)**, pp. 1465-1478.

[9] D. E. NEWLAND, "Ridge and phase indentification in the frequency analysis of transient signals by harmonic wavelets [J]", J of Vibration and Acoustics,Transactions of the ASME, vol.121, no. 2, **(1999)**, pp. 149-155.

[10] D. E. NEWLAND, "Harmonic wavelets in vibrations and acoustics [J]".Royal Soc Trans Philos:A, vol. 1760, no. 357, **(1999)**, pp. 2607-2625.

[11] W. ZHANG, X. WANG, L. GE and Z. ZHANG, "An Algorithm for Weak Signal Detection in Strong Noise Environment", Acta Metrologica Sinica, vol. 28, no. 1, **(2007)**, pp. 70-73.

[12] F. Xiaozhi, W.Changguang, H. Xiaohong and S. Xuejun, "AN INSPECTING TECHNOLOGY FOR WEAK SINUSOIDAL SIGNAL BASED ON WAVELET ANALYSIS AND MULTI-LAYER AUTOCORRELATION",Computer Applications and Software, **(2009)**, pp. 24.