

Synthesis of the Sum and Difference Patterns by a Hybrid Real/Integer-Coded Invasive Weed Optimization

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Abstract

The optimization of the sum and difference patterns for monopulse antennas by a hybrid real/integer-coded invasive weed optimization (IWO) is introduced in this paper. The whole array aperture is divided into several subarrays. The configuration and weight of each subarray are optimized. In order to reduce the difficulty of designing the feeding networks of the array antenna, the elements of the same subarray stay together. Since only the weight and elements number of each subarray is optimized, the number of the optimized parameters is reduced significantly which will reduce the complexity of the simulation procedure. Several numerical simulations are applied to validate the effectiveness of the proposed approach.

Keywords: *pattern synthesis, antenna arrays, monopulse antennas, sum and difference patterns, Invasive Weed Optimization*

1. Introduction

The sum and difference patterns have to be synthesized in the design of monopulse antennas. Considerable attention has been received in the design of the sum and difference beam formers for monopulse arrays. A number of methods have been developed in order to avoid the difficulty of the completely independent implementation of the two arrays [1-6].

The synthesis of subarrayed low-side-lobe sum and difference patterns is one of the most popular methods in the design of monopulse antennas. The whole array aperture is divided into several subarrays and the configuration and weight of each subarray are optimized. In [1], a hybrid genetic algorithm optimized the subarray size and subarray weights to minimize the maximum sidelobe level. The optimization of difference patterns of monopulse antennas is considered in [2]. The problem of synthesizing 'optimal' sum and difference patterns subject to arbitrary sidelobe bounds by means of a simple feeding network is dealt with in [3]. In [4], new methods of synthesizing low-side-lobe sum and difference patterns for linear arrays are described. The simultaneous optimization of the partition in subarrays and the subarray weights is reported in [5]. The maximization of the directivity of compromise difference patterns in sub-arrayed monopulse linear array antennas with an optimum sum mode is addressed by means of a two-stage excitation matching procedure in [6].

IWO is a global optimization method and has been effectively used in the design of antennas [7-8]. Usually, IWO outperforms the other global optimization methods in the convergence rate as well as the final error level [9]. In this paper, a hybrid IWO is used in the synthesis of sum and difference patterns for the monopulse antennas. Three kinds of arrays are considered in this paper. First, the subarray synthesized low-side-lobe sum

pattern is optimized. Second, the synthesis of subarrayed monopulse array providing a best compromise difference pattern is addressed. Finally, the subarrayed sum and difference patterns are synthesized.

The rest of the paper is organized as follows. The mathematical formulation of the synthesis problem is given in Section 2. The optimization steps are given in Section 3. The analysis aimed at describing the behavior of the proposed approach is presented in Section 4. Eventually, conclusions are drawn.

2. Mathematical Formulation

A linear array of $2N$ elements is considered. For this kind of array structure, the array factor $AF(\theta)$ is defined by

$$AF(\theta) = \sum_{\substack{n=-N \\ n \neq 0}}^N a_n \exp(jkx_n \cos \theta) \quad (1)$$

where a_n , $n = -N, \dots, -1, 1, \dots, N$, are the excitations of the radiation elements. $k = 2\pi/\lambda$ is the wave number. λ is the wavelength. x_n , $n = -N, \dots, -1, 1, \dots, N$, are the positions of the array elements. θ defines the angle respect to the array axis.

The required sum pattern is obtained by assuming the excitations to be symmetric (i.e. $a_{-n} = a_n$, $n = 1, \dots, N$). The array factor can be reduces to

$$AF_s(\theta) = 2 \sum_{n=1}^N a_n^s \cos(kx_n \cos(\theta)) \quad (2)$$

where a_n^s , $n = 1, \dots, N$ denotes the n th excitations for the sum mode which can be obtained by using Taylor or Dolph-Chebichev synthesis method.

In order to obtain a difference pattern, the excitations are anti-symmetric (i.e. $a_{-n} = -a_n$, $n = 1, \dots, N$). In this case, (1) reduces to

$$AF_d(\theta) = 2j \sum_{n=1}^N a_n^d \sin(kx_n \cos(\theta)) \quad (3)$$

where a_n^d , $n = 1, \dots, N$, denotes the n th excitations for the difference mode.

2.1. Subarrayed Sum Pattern

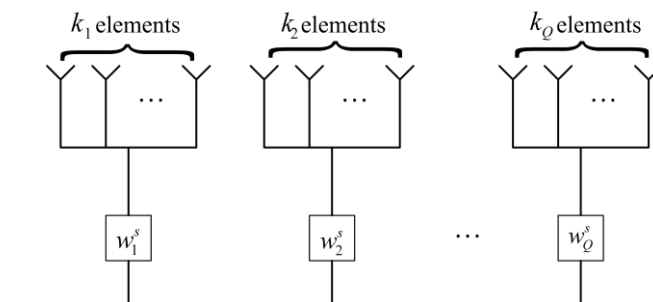


Figure 1. Configuration of Subarrayed Sum Pattern

In this section, the whole array aperture is divided into Q subarrays and sum radiation pattern with low-side-lobe level is obtained. Each subarray has the same excitation weight. The array structure is as shown in Figure. 1. The q th subarray has k_q , $q = 1, \dots, Q$, elements. The total element number of the considered array antenna can be given by

$$N = \sum_{q=1}^Q k_q \quad (4)$$

The sum radiation pattern of this kind of array structure can be given by

$$AF_s(\theta) = 2 \sum_{q=1}^Q w_q^s \sum_{n=1}^N \delta_{c_n^s, q} \cos(kx_n \cos(\theta)) \quad (5)$$

where $w_q^s \in [0,1]$, $q=1, \dots, Q$, is the weight of the q th subarray. δ_{ij} denotes the Kronecker function, i.e., $\delta_{ij} = 1$, if $i = j$, $\delta_{ij} = 0$, elsewhere. $c_n^s \in [1, Q]$, $n=1, \dots, N$, is a positive integer which denotes the element n belongs to the q th subarray, i.e., if $c_n^s = q$, the n th element belongs to the q th subarray. The minimum number of elements that are allowed in a subarray is represented by k_{\min} . k_{\min} has to be at least greater than or equal to 1. The maximum number of elements in a subarray is k_{\max} and $k_{\max} \leq N - (Q-1) \times k_{\min}$. For the sum radiation pattern, $w_1^s = 1.0$. The elements number of Q th subarray can be determined by $k_Q = N - \sum_{q=1}^{Q-1} k_q$. So, the optimized parameters vector can be given by

$\xi = [w_2^s, \dots, w_Q^s, k_1, \dots, k_{Q-1}]$. There are $N_p = 2Q - 2$ elements in the optimized vector. The vector is composed of two parts: one of length $Q - 1$ containing subarray weights and another of length $Q - 1$ denoting the number of elements that are allowed in a subarray.

2.2. Subarrayed Compromised Difference Pattern

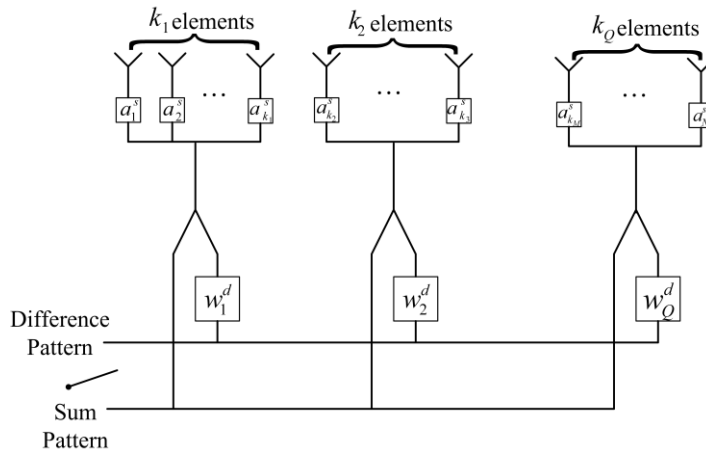


Figure 2. Configuration of Subarrayed Compromised Difference Pattern

To fulfill the synthesis of a compromised difference pattern, the array structure is depicted in Figure. 2. The best sum radiation pattern is obtained and the sum pattern excitations are fixed. Then, the excitations of the difference pattern can be obtained by the following relation

$$a_n^d = a_n^s \sum_{q=1}^Q \delta_{c_n^d, q} w_q^d, \quad n = 1, \dots, N \quad (6)$$

Then, the difference radiation pattern can be obtained by (3). The optimization vector can be given by $\xi = [w_1^d, \dots, w_Q^d, k_1, \dots, k_{Q-1}]$. There are $N_p = 2Q - 1$ elements in the optimization vector.

2.3. Subarrayed Sum and Difference Patterns

In this section, the subarrayed sum and the difference patterns are optimized. The structure of this kind of array is given in Figure. 3. The two subarrays have the same configuration but have different weights. The two array factors can be given by

$$AF_s(\theta) = 2 \sum_{q=1}^Q w_q^s \sum_{n=1}^N \delta_{c_n,q} \cos(kx_n \cos(\theta)) \quad (7)$$

$$AF_d(\theta) = 2j \sum_{q=1}^Q w_q^d \sum_{n=1}^N \delta_{c_n,q} \sin(kx_n \cos(\theta)) \quad (8)$$

where $w_1^s = 1.0$. So, the optimization vector can be given by $\xi = [w_2^s, \dots, w_Q^s, w_1^d, \dots, w_Q^d, k_1, \dots, k_{Q-1}]$. There are $N_p = 3Q - 2$ elements in the optimization vector and the vector is composed of two parts. The first $2Q - 1$ elements denote the subarray weights and the last $Q - 1$ elements denote the group membership of each array element.

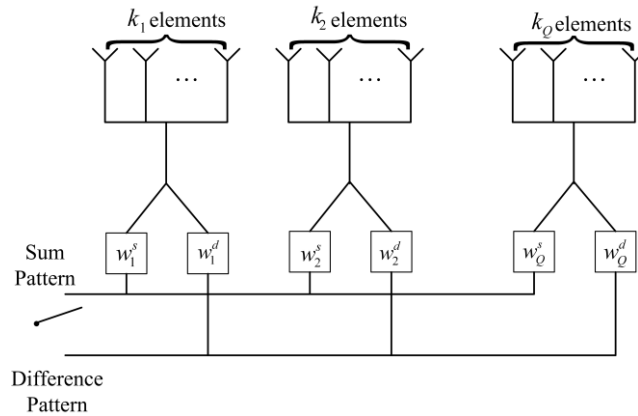


Figure 3. Configuration of Subarrayed Sum and Difference Patterns

3. Optimization Steps

In order to optimize the sum and difference patterns, the optimization procedure can be expressed as follows:

Step 1. The parameter values of the antenna arrays and IWO are given. The optimization vector can be given by

$$\xi = [w_1, \dots, w_{N_1}, k_1, \dots, k_{Q-1}], \quad i = 1, \dots, N_p \quad (9)$$

There are $N_p = N_1 + Q - 1$ parameters to be optimized. The first N_1 elements denote the subarray weights and the last $Q - 1$ elements denote the group membership of each array element. A K -dimensional matrix is chosen as the initial population to be optimized. Each dimension of the population can be depicted as $r_i^k, i = 1, 2, \dots, N_p, k = 1, 2, \dots, K$, and $r_i^k \in [0, 1]$. Let $iter = 1$.

Step 2. The first N_1 elements of r_i^k are the subarray weights. Next, the array configuration will be determined. Each subarray starts with k_{\min} elements. The remaining $k_r = N - Q \times k_{\min}$ elements are distributed among the subarrays. A new vector introduced which are depicted by N'_r . Each element of N'_r can be determined by

$$N'_r(1) = 0, N'_r(q) = \lfloor -0.49 + (k_r + 0.98) \times r_{N_r+q-1}^k \rfloor, q = 2, \dots, Q \quad (10)$$

Where $\lfloor a \rfloor$ denotes the integer part of a . N'_r is sorted in ascending order. N_r is sorted in ascending order and a new vector N_r is obtained. $N_r(q)$, $q = 1, \dots, Q$, represents the sum of total remaining elements number before the q th subarray, i. e., $N_r(1) = 0$, $N_r(q) = \sum_{i=1}^{q-1} (k_i - k_{\min})$, $q = 2, \dots, Q$. It can be proved from (10) that

$N_r(q) \in [0, k_r]$. So, the elements number of each subarray can be determined by

$$k_q = k_{\min} + (N_r(q+1) - N_r(q)), k_Q = N - \sum_{q=1}^{Q-1} k_q, q = 1, \dots, Q-1 \quad (11)$$

Step 3. After the subarray weights and configuration are determined, the array factor can be obtained by (3), (5), (6), (7) or (8) for different kinds of optimization problems. The cost function is given by

$$err(k) = \begin{cases} SLL(k) - SLL_d, & SLL(k) > SLL_d \\ 0, & \text{elsewhere} \end{cases}, k = 1, 2, \dots, K \quad (12)$$

Where $SLL(k)$, $k = 1, 2, \dots, K$, is the peak side lobe level (PSLL) value corresponding to the k th element of the population. SLL_d is the desired PSLL value. $SLL(k)$, $k = 1, 2, \dots, K$, can be computed by searching the side lobes outside the main-lobe region.

The fitness function is given by

$$f(k) = \frac{1.0}{err(k)}, k = 1, 2, \dots, K \quad (13)$$

The fitness value f increases with the decrease of cost function value. The optimized parameters that can produce best fitness are preserved as the ultimate result.

Step 4. The optimized parameters $r_i^k, i = 1, 2, \dots, N_p$, are updated by IWO which has been introduced in [9] and [11].

Step 5. Let $iter = iter + 1$, if $iter < it_{\max}$, go to step 2, otherwise, terminate iteration, where it_{\max} is the maximum iteration steps.

4. Optimization Results

In order to show the effectiveness and flexibility of the proposed approach, several simulation results are performed. As is shown in [9], The IWO parameters are given in Table 1. The number of sampling points for θ is 1801. A desired sidelobe level should be a few dBs lower than it can be realistically hoped. So, the desired sidelobe level SLL_d is chosen as -40dB. A linear array that the elements spaced d along the x-axis is considered. The position of each element can be given by $x_n = (n - 0.5)d$, $n = 1, \dots, N$. d is chosen as $\lambda/2$ in this paper. The algorithm is repeated 10 times and the best result is preserved.

Table 1. IWO Parameter Values

it_{\max}	P_{\max}	S_{\max}	S_{\min}	K	n	initial SD	final SD
3000	30	10	0	10	3	0.1	0.001

4.1. Simulation of Subarrayed Sum Pattern

As the first example, the subarrayed sum pattern is synthesized. Since the array is symmetric, only the right hand side of the array is considered. The number of the array elements is $N = 64$ and the number of the subarrays is chosen as $Q = 8$. So, as is shown in Section 2.1, the total number of the optimized parameters is 14.

By using the approach proposed above, the sum radiation pattern and the behavior of the cost function are depicted in Figure. 4. The PSLL of the optimized radiation pattern is -37.5dB. The optimized number of elements in each subarray along with the optimized subarray weights is given in Table. 2. From Table. 2, it can be found that $k_{\min} = 5$. For comparison, the same array configuration is synthesized in [1]. The minimum number of elements in each subarray is $k_{\min} = 4$ and the PSLL is -35.9dB. The PSLL optimized by the method proposed in this paper is with an improvement of nearly 1.6dB in terms of PSLL with respected to [1]. Also, the minimum elements number of subarrays increases by 1.

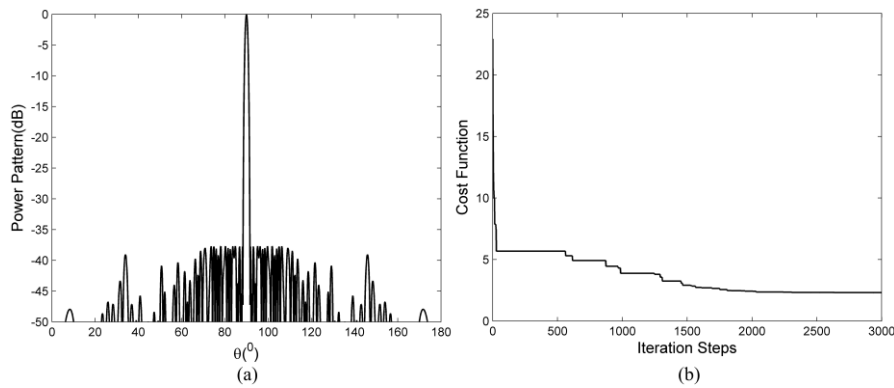


Figure 4. Simulation Result for Subarrayed Sum Pattern (Q=8): (a) Sum Radiation Pattern; (b) Behavior of the Cost Function

Table 2. Subarray Structure and Weights (Sum Pattern)

Subarray Number	1	2	3	4	5	6	7	8
Element Number	14	5	7	7	5	10	7	9
Subarray Weights	1.0	0.883	0.789	0.652	0.524	0.352	0.209	0.113

4.2. Simulation of Compromised Difference Pattern

In this example, the subarrayed compromised difference pattern is synthesized. The mathematical theory is depicted in Section 2.2. Considering the symmetry of the antenna array, the number of the array elements is $N = 50$. The number of the subarrays is chosen as $Q = 6$. The total number of the optimized parameters is 11.

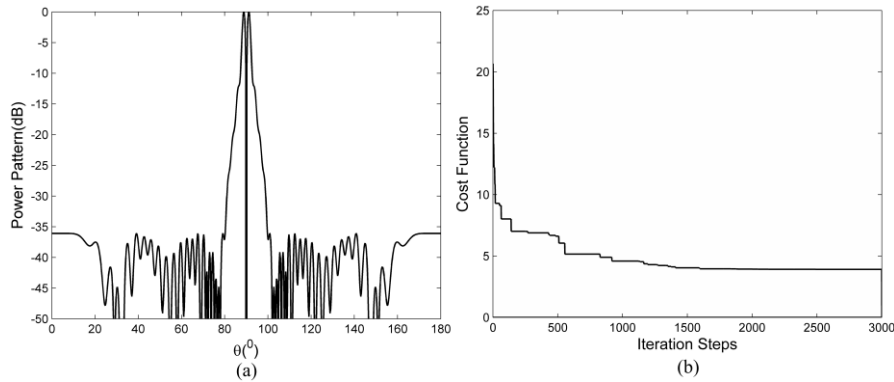


Figure 5. Simulation Result for Subarrayed Compromised Difference Pattern (Q=6): (a) Difference Radiation Pattern; (b) Behavior of the Cost Function

Table 3. SubArray Structure and Weights (Compromised Difference Pattern)

SubarrayNumber	1	2	3	4	5	6
Element Number	1	2	2	33	7	5
Subarray Weights	0.196	0.416	0.737	0.951	0.747	0.237

The sum pattern excitations a_n^s , $n = 1, \dots, N$, are chosen to produce a Taylor pattern [10] with $\bar{n} = 12$ and PSL = -35dB. Figure 5 (a) shows the optimized compromised difference pattern. Figure 5 (b) reports the plot of the cost function during the optimization procedure. The PSL of the optimized compromised difference pattern is -36 dB. The optimized number of elements in each subarray along with the optimized subarray weights is given in Table 3. A similar array structure is optimized in [2-3]. The number of the subarray is chosen as 4 and the optimized PSLs are -30dB and -33dB, respectively. Using the method proposed in this paper and choosing the subarray number as 4, the PSL is optimized as -32.5dB. Although the PSL is 0.5dB higher than that of calculated in [3], the elements of the same subarray stay together which will reduce the difficulty of designing the feeding networks of the array antenna significantly.

4.3. Simulation of Sum and Difference Patterns

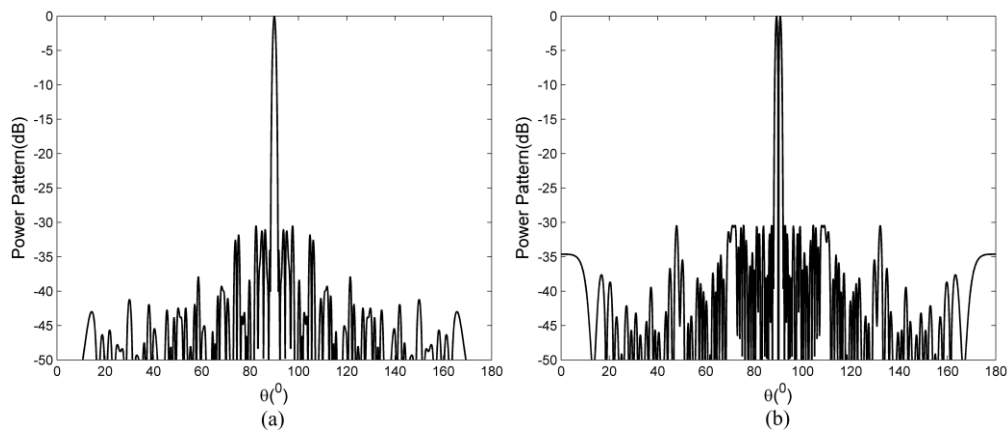


Figure 6. Simulation Result for Subarrayed Sum and Difference Pattern (Q=8): (a) Sum Radiation Pattern; (b) Difference Radiation Pattern

For the last example, the simulation result of subarrayed sum and difference patterns are given. The number of the array elements is $N = 64$. The number of the subarrays is chosen as $Q = 8$. The total number of the optimized parameters is 23.

The synthesized sum and difference patterns are shown in Figure. 6 (a) and Figure. 6 (b), respectively. The PSLLs for the sum and difference patterns are -30.4dB and -30.4dB, respectively. Figure. 7 gives the plot of the cost function during the optimization procedure. The elements number and weight of each subarray are depicted in Table. 4.

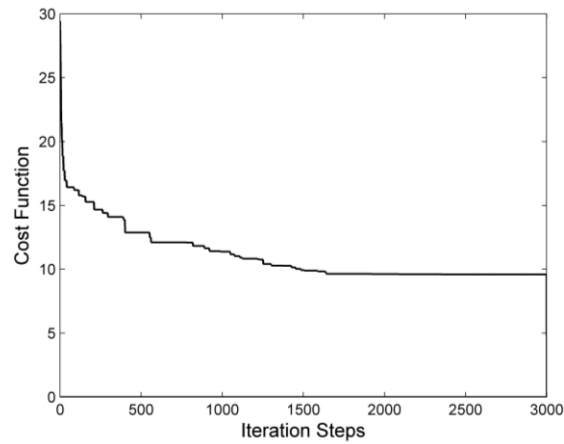


Figure 7. Behavior of the Cost Function for Sum and Difference Patterns

Table 4. Subarray Structure and Weights (Sum and Difference Patterns)

Subarray Number	1	2	3	4	5	6	7	8
Element Number	6	6	6	16	7	8	6	9
Subarray Weights (Sum Pattern)	1.0	0.953	0.896	0.701	0.450	0.292	0.171	0.113
Subarray Weights (Difference Pattern)	0.191	0.497	0.726	0.979	0.900	0.708	0.476	0.296

5. Conclusions

In this paper, the optimization of sum and difference radiation patterns is considered. A hybrid real/integer invasive weed optimization is used into the considered synthesis problem. The whole array aperture is divided into several subarrays and low side lobe level patterns are obtained. Compared with other relevant papers, the subarray structures optimized by the proposed method reduce the difficulty in designing the feeding networks of the array antenna.

References

- [1] R. L. Haupt, "Optimized weighting of uniform subarrays of unequal sizes", *IEEE Trans. Antennas and Propag.*, vol. 55, no. 4, (2007), pp. 1207-1210.
- [2] S. Caorsi, A. Massa, M. Pastorino and A. Randazzo, "Optimization of the difference patterns for monopulse antennas by a hybrid real/integer-coded differential evolution method", *IEEE Trans. Antennas and Propag.*, vol. 53, no. 1, (2005), pp. 372-376.
- [3] M. D'Urso, T. Isernia and E. F. Meliado, "An effective hybrid approach for the optimal synthesis of monopulse antennas", *IEEE Trans. Antennas and Propag.*, vol. 55, no. 4, (2007), pp. 1059-1066.
- [4] T. S. Lee and T. K. Tseng, "Subarray-synthesized low-side-lobe sum and difference patterns with partial common weights", *IEEE Trans. Antennas and Propag.*, vol. 41, no. 6, (1993), pp. 791-800.
- [5] P. Lopez, J. A. Rodriguez, F. Ares and E. Moreno, "Subarray weighting for different patterns of monopulse antennas: Joint optimization of subarray configurations and weights", *IEEE Trans. Antennas and Propag.*, vol. 49, no. 11, (2001), pp. 1606-1608.

- [6] L. Manica, P. Rocca and A. Massa, "Excitation matching procedure for sub-arrayed monopulse arrays with maximum directivity", *IET Radar, Sonar & Navigation*, vol. 3, no. 1, (2008), pp. 42-48.
- [7] G. G. Roy, S. Das, P. Chakraborty and P. N. Suganthan, "Design of Non-Uniform Circular Antenna Arrays Using a Modified Invasive Weed Optimization Algorithm", *IEEE Tran. Antennas Propag.*, vol. 59, no. 1, (2011), pp. 110-118.
- [8] S. H. Sedighy, A. R. Mallahzadeh, M. Soleimani and J. Rashed-Mohassel, "Optimization of Printed Yagi Antenna Using Invasive Weed Optimization (IWO)", *IEEE Antennas Wireless Propag. Lett.*, vol. 9, (2010), pp. 1275-1278.
- [9] S. Karimkashi and A. A. Kishk, "Invasive Weed Optimization and its Features in Electromagnetics", *IEEE Tran. Antennas Propag.*, vol. 58, no. 4, (2010), pp. 1269-1278.
- [10] C. A. Balanis, "Antenna Theory: Analysis and Design", Wiley, New York, (1982).
- [11] A. R. Mehrabian and C. Lucas, "A novel numerical optimization algorithm inspired from weed colonization", *Ecol. Inform.*, vol. 1, (2006), pp. 355-366.

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